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Value-at-Risk Analysis in the Presence of Asymmetry and Long Memory: The Case of Turkish Stock Market

Mesut BALIBEY
(Corresponding Author)
Faculty of Economics and Administrative Sciences,
Tunceli University, Tunceli, Turkey.
Email: mstblby@hotmail.com

Serpil TURKYILMAZ
Bilecik Şeyh Edebali University,
Faculty of Arts & Sciences, Bilecik, Turkey.
Email: serpil.turkyilmaz@bilecik.edu.tr

ABSTRACT: Value-at-Risk (VaR) is a standard tool for measuring potential risk of economic losses in financial markets. In this study, we examine the convenience of the FIGARCH (1, d, 1) and FIAPARCH (1, d, 1) models in evaluating asymmetry features and long memory in the volatility of the Turkish Stock Market. Furthermore, we investigate the performances in-sample and out-of-sample Value-at-Risk (VaR) analyses based on Kupiec-LR test by using FIGARCH(1, d, 1) and FIAPARCH (1, d, 1) models with the normal, student-t and skewed student-t distributions. For these analyses, we take into account both short and long trading positions. The empirical results display that the FIAPARCH (1, d, 1) model with skewed student-t distribution is more accurate for in-sample and out-of-sample Value-at-Risk (VaR) analysis for short and long trading positions. In addition, the FIAPARCH(1, d, 1) model with skewed student-t has better accuracy results in capturing stylized facts in the volatility of Turkish Stock Market. Consequently, evaluating of asymmetry and long memory property in volatility of the returns can ensure suitable Value-at-Risk (VaR) model selection for performance of risk management in the Turkish financial markets. The findings can be evaluated by portfolio managers, investors, regulators and financial risk managers.

Keywords: Value-at-Risk; FIAPARCH Model; Long Memory; Volatility
JEL Classifications: C58; C13; G10; G15; G17

1. Introduction

Measuring risk has become a crucial issue for many portfolio managers and investors. In recent years, finance literature has focused on risk management. So, Value-at-Risk (VaR) analysis has been a matter of great concern for financial risk management. VaR analysis has been extensively used to measure the possible maximum amount of loss for an asset (or portfolio) in a specific period of time at a given confidence level by portfolio managers, regulators and practitioners. In other words, VaR has measured the maximum loss in value of a portfolio over a predetermined time perod for a given confidence level. The RiskMetrics model developed by the risk management group (the J.P. Morgan) in 1994, which is a benchmark for measuring the market volatility risk of asset portfolios under the assumption of normality, has become one popular method (Jorion, 2001). The main drawback of the RiskMetrics model is that model ignores the presence of fat-tailed and skewed characteristics in the return distributions. The other drawback is also that this model disregards many financial return series exhibit long memory property(Kang and Yoon, 2008).

The empirical studies in finance literature emphasize some stylized facts such as excess volatility, volatility clustering, fat-tails of return distributions, long memory and asymmetry in the asset prices. In order to evaluate these properties in returns, symmetric or asymmetric Generalized Autoregressive Conditional Heteroscedasticity (GARCH)-type models are commonly used in the literature. Especially, time varying volatility in returns and high frequency data have been modeled by the GARCH-type models which only capture the short-term dependencies. Furthermore, Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) model proposed by Baillie et al. (1996) investigates the long memory property in volatility of financial time series. Although the FIGARCH model can capture long-term dependencies in conditional variance, model assumes that positive (good-news) and negative (bad-news) shocks have same impacts on the volatility(Tse, 1998; Yoon and Kang, 2007). To evaluate both asymmetry and long memory in the conditional variance, Tse (1998) proposes a Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model. The researchs on market risks widely use the Value-at-Risk (VaR) approach based on the GARCH-type models and FIGARCH-type models.

Recently, empirical studies in finance literature have focused on examining the volatility dynamics, risk management and Value-at-Risk analysis. For example; Yoon and Kang (2007) show that long memory models with skewed student-t distribution model produces more accurate VaR estimations than normal and student-t distribution models for the daily return series of Japanese financial data (Nikkei 225 index and JPY-USD exchange rate). Wu and Shieh (2007) examine the volatility of T-Bond futures returns by using two long memory models such as FIGARCH(1, d, 1) and HYGARCH (1, d, 1) with normal and skewed student-t distributions and calculate the Value-at-Risk by these models. Their findings display that the HYGARCH (1, d, 1) with skewed student-t innovations based on Kupiec LR test gives model accuracy Value-at-Risk estimations.

Kang and Yoon (2008) examine the performance of RiskMetrics and two long memory Value-at-Risk (VaR) models with the normal, student-t and skewed student-t distribution assumptions in Korean shares. They display that VaR analysis with the skewed student-t distribution innovation provides more accurate VaR estimations.

Kasman (2009) investigates the long memory properties of Turkish Stock Index futures market using FIGARCH(1, d, 1) model and calculates the Value-at-Risk. He presents that FIGARCH (1, d, 1) models with skewed student-t distribution produces more better VaR estimations than normal distribution. Liu et al. (2009) examine the daily Value-at-Risk (VaR) for returns of the Taiwan Stock Exchange by using GARCH and GJR-GARCH models. They propose GJR-*t*/GARCH-H*T* model as a useful downside risk measure in volatile markets.

Demireli (2010) is modeled Istanbul Stock Exchange (ISE) index returns by using various symmetric and asymmetric GARCH-type models. In addition, the accuracy of one-day-ahead Value-at-Risk student-t and skewed student-t distributions. He finds that student-t FIAPARCH modeling the leverage and long memory properties in ISE index returns provides efficient VaR values. Bee and Miorelli (2010) present a backtesting exercise involving several VaR models for measuring market risk. They introduce three different stochastic processes for the losses such as GARCH-type models and EWMA-type model. Mighri at al. (2010) investigate the effects of asymmetric long memory volatility models on the accuracy of stock index return VaR estimates. They find that in-sample and out-of-sample VaR values computed using asymmetric long memory volatility models have better accuracy than the symmetric FIGARCH model.

Yoon et al. (2011) investigate the performance of in-sample and out-of-sample Value-at-Risk (VaR) analyses using the FIAPARCH model with the normal, student-t and skewed student-t distribution innovations for Shanghai Stock Market. They display that skewed student-t VaR models of long and short trading positions in Shanghai stock market give most accurately VaR estimations.

Stavroyiannis and Zarangas (2013) present the efficiency of an econometric model where the volatility is modeled by a GARCH (1, 1) process, and the innovations follow a standardized form of the Pearson type-IV distribution. In addition, they explore the accuracy of model by a variety of Value-at-Risk methods. They display that proposed model is a valid and accurate model for Value-at-Risk estimations. Abad et al. (2014) present a theoretical review of the existing literature on Value-at-Risk (VaR) specifically focussing on the development of new approaches for its estimations.

Chkili et al. (2014) investigate effects of asymmetry and long memory in modeling and forecasting the conditional variance and market risk of four widely traded commodities such as natural gas, gold, crude oil and silver. They find that in-sample and out-of-sample results indicate that FIAPARCH model is the best suited model for estimating the VaR forecasts. Demiralay and Ulusoy (2014) examine the Value-at-Risk estimations of four precious metals such as gold, silver, platinum and palladium with various long memory models under normal and student-t innovations distributions.

They find that the long memory volatility models under student-t distribution perform well in forecasting Value-at-Risk for both short and long positions. Jeremić and Terzić (2014) estimate normal and student-t VaR for different significance level for Belgrade stock exchange. They display that the normality assumption for higher significance levels can seriously underestimate VaR.

The primary aim of this study is to reconsider the volatility persistence for daily return series of Turkish Stock Market, and to compare the performance of various VaR models with normal, student-t and skewed student-t distribution. In this regards, the study provides a major contribution about understanding the volatility features of the Turkish Stock Market returns which is an important determinant in measuring Value-at-Risk for portfolio managers, regulators and investors. The other contribution, the study compares the performance of Value-at-Risk estimations by using long memory models such as FIGARCH and FIAPARCH model. Firstly, we examine volatility persistence in the Turkish Stock Market by using these long memory models. Secondly, we analyze the asymmetric impacts of the positive and negative shocks on the volatility of returns. Third, we compare Value-at-Risk estimations by using long memory volatility models such as FIGARCH and FIAPARCH according to the in-sample and out-of-sample performances in estimating market risk of return prices for both short and long trading positions. Finally, Value-at-Risk analyses offer the results of models with normal, student-t and skewed student-t distribution to evaluate asymmetry and fat-tail property for both short and long trading positions.

The remainder of the study is organized as follows: The methodology is summarized in Section 2. Section 3 provides the statistical characteristics of data set used and reports the empirical results. Conclusions are presented in Section 4.

2. Methodology

2.1. Symmetric Long Memory Volatility Models: FIGARCH (p, d, q) Model

Baillie et al. (1996) develop FIGARCH model which allows to model long memory property in volatility. The long memory is characterized by very slow decay in the autocorrelations of absolute and squared returns. According to Baillie et al. (1996) , the impacts of shocks (good-news/bad-news) on the volatility of returns are not finite. FIGARCH (p, d, q) model which is the extended version of squared errors in ARFIMA model notation is introduced as follows (Türkyılmaz and Balıbey, 2014):

$$\phi(L)(1-L)^d\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{1}$$

where, L denotes the lag or backshift operator. ε_t are serially uncorrelated errors having zero mean. ε_t^2 are squared errors of GARCH process. The process of $\{v_t\}$ is integrated for conditional variance σ_t^2 as variations. $v_t = \varepsilon_t^2 - \sigma_t^2$. It is assumed that all roots of $\phi(L)$ and $[1-\beta(L)]$ stayed out of unit circle. The parameter d is the fractional integration parameter showing the degree of long memory or persistence of shocks to conditional variance. d parameter satisfies the condition 0≤d≤1. If 0<d<1, the model indicates an intermediate range of long memory. It means that volatility shocks die hyperbolically. If d=0, then the process of FIGARCH (p,d,q) is reduced to the process of GARCH (p, q). If d=1, then the process of FIGARCH becomes an integrated process of GARCH (IGARCH). The Model (1) can be reformed as follows (Baillie et al., 1996):

$$[1-\beta(L)]\sigma_t^2 = w + [1-\beta(L) - \phi(L)(1-L)^d]\varepsilon_t^2, \tag{2}$$

where σ_t^2 which is conditional variance of ε_t^2 is displayed with;

$$\sigma_t^2 = \frac{\omega}{[1-\beta(L)]} + \lambda(L)\varepsilon_t^2, \tag{3}$$

where,

$$\lambda(L) = 1 - \frac{\phi(L)}{[1-\beta(L)]}(1-L)^d. \tag{4}$$

In addition, $(1-L)^d$ can be expressed by the Maclaurin series expansion and thus defined as:

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)L^k}{\Gamma(k+1)\Gamma(d-k+1)} \tag{5}$$

$$=1-dL-\frac{d(1-d)}{2!}L^2-\frac{d(1-d)(2-d)}{3!}L^3-...$$

$$=1-\sum_{k=0}^{\infty}c_k(d)L^k$$

with $c_1(d)=d$, $c_2(d)=\frac{1}{2}d(1-d)$, $c_3(d)=\frac{1}{6}d(1-d)(2-d),...$etc. Where $\Gamma(.)$ denotes the gamma function. Since $\Gamma(k-d)/\Gamma(k+1)\approx k^{-d-1}$ if k is large, the coefficients in the above infinite polynomial decay hyperbolically (Mighri et al., 2010).

FIGARCH(p, d, q) model assumes that the effects of positive and negative news (good-news/bad-news) on the volatility are symmetric. So, Tse(1998) developed the FIAPARCH model.

2.2. Asymmetric Long Memory Volatility Models: FIAPARCH (p, d, q) Model

Tse (1998) extended the FIGARCH model by adding the function $(|\varepsilon_t|-\gamma\varepsilon_t)^{\delta}$ of the Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) model to evaluate asymmetry and long memory property in the conditional variance. The FIAPARCH (p, d, q) model proposed by Tse (1998) can be expressed as follows:

$$\sigma_t^{\delta}=w+\left\{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\right\}(|\varepsilon_t|-\gamma\varepsilon_t)^{\delta}, \quad (6)$$

where δ,γ are the parameters of model. δ is power parameter. Furthermore, $\lambda(L)\equiv\sum_{i=1}^{\infty}\lambda_i L^i$, with $\lambda(L)$ an infinite summation. d parameter is the long memory parameter and when $0<d<1$, conditional variance has long memory property. It means that effect of a shock on the conditional variance decays at a hyperbolic rate(Demireli, 2010; Kang and Yoon, 2008; Mighri et al., 2010). When asymmetry parameter $\gamma>0$, negative shocks cause higher volatility than positive shocks, and visa versa. The FIAPARCH model is reduced the FIGARCH model, when $\delta=2$ and $\gamma=0$. Hence, it can be said that FIAPARCH model is superior to the FIGARCH model because it can evaluate both asymmetry and long memory in the volatility.

2.3. Value-at-Risk (VaR)

Value-at-Risk (VaR) is defined as the maximum loss over a given time horizon at a given confidence level. It is widely used to measuring potential risk of economic losses in financial markets.

The price of an asset in time t is denoted as P_t. Let $R_t=\ln(P_t/P_{t-1})*100$ be daily returns. The data generating process of the returns is as follows(Mighri et al., 2010):

$R_t=\mu_t+\varepsilon_t$, $t=1,\ldots,T$.

Furthermore, returns can be heteroscedastic. A multiplicative process for ε_t is as follows:

$R_t=\mu_t+\sigma_t z_t$, $t=1,\ldots,T$, where $\varepsilon_t=\sigma_t z_t$.

Mathematically, a k-day VaR on day t is expressed by(Demiralay and Ulusoy, 2014):

$$P(P_{t-k}-P_t\leq VaR(t,k,\alpha))=1-\alpha$$

Usually, portfolio managers, investors and traders focus on longer forecasting horizons. Value-at-Risk are computed on a 1-day 95% and 99% confidence level. It denotes that the loss is more than the reported Value-at-Risk of a portfolio in only 5% and 1% of the cases. Portfolio managers, traders and investors must evaluate not only long trading position, but also short trading positions. In case of long trading position (the left tail of distribution), the risk of a loss occurs when the traded asset price decreases. However, in case of short trading position (the right tail of distribution), the risk of a loss occurs when the traded asset price increases (Demiralay and Ulusoy, 2014; Yoon and Kang, 2007).

The VaR of α quantile for long and short trading positions are estimated as follows (Demireli, 2010): Under the normal distribution assumption;

$VaR_{long}=\hat{\mu}_t-z_{\alpha}\hat{\sigma}_t$

$VaR_{short}=\hat{\mu}_t+z_{\alpha}\hat{\sigma}_t$

Where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are conditional mean and conditional variance, respectively. z_{α} is the left or right quantile at α% for the normal distribution.

Under student-t distribution,

$VaR_{long} = \hat{\mu}_t - st_{\alpha,\upsilon}\hat{\sigma}_t$

$VaR_{short} = \hat{\mu}_t + st_{\alpha,\upsilon}\hat{\sigma}_t$

where $st_{\alpha,\upsilon}$ is the left or right quantile at α% for the student-t distribution.

Under skewed student-t distribution,

$VaR_{long} = \hat{\mu}_t - skst_{\alpha,\upsilon}\hat{\sigma}_t$

$VaR_{short} = \hat{\mu}_t + skst_{\alpha,\upsilon}\hat{\sigma}_t$

Where skst is the left or right quantile at α% for the skewed student-t distribution with υ degrees of freedom and γ asymmetry coefficient. If $\gamma<1$, the VaR value for long trading positions is bigger than that of short trading position, and vice versa (Kasman, 2009). To evaluate the performance of calculated VaR at pre-specified significance level of α ranging from 5% to 0.25%, a likelihood-ratio test namely Kupiec LR was developed by Kupiec (1995) (Giot and Laurent, 2003; Tang and Shieh, 2006). Kupiec LR (1995) test relies on the failure rate. The failure rate is expressed as the ratio of the number of times (x) in which returns exceed the forecasted VaR to the sample size (T). Tested hypothesis are $H_0{:}f=\alpha$ versus $H_1{:}f\neq\alpha$, where f is failure rate. If the VaR model is correctly specified, the failure rate should be equal to the pre-specified significance level of α. This test is called as Kupiec LR test. The test is defined as follows:

$$LR = -2\ln\left[(1-\alpha)^{N-x}(\alpha)^{x}\right] + 2\ln\left[(1-\hat{f})^{N-x}(\hat{f})^{x}\right] \sim \chi^2_{(1)} \qquad (7)$$

where $\hat{f}$ is the failure rate. x is the number of observations the forecasted VaR. N is the sample size. LR test statistics is asymptotically distributed as $\chi^2_{(1)}$.

3. Data and Empirical Analysis

3.1. Preliminary Analysis of Data

This study considers time series data set of Turkish financial market (Borsa İstanbul Stock Exchange-BIST). The data consists of daily closing price, and covers the sample period from August 24, 2010 to August 28, 2014. Our in-sample period runs from August 24, 2010 to July 18, 2013, while the out-of-sample period runs from July 19, 2013 to August 28, 2014. The daily price series are defined as the logarithmic difference of the daily closing index prices. The returns at time t are obtained by;

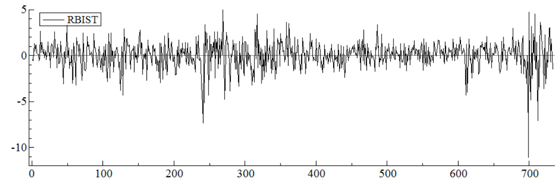
$$RBIST_t = \ln(P_t / P_{t-1})x100, \; t=1,2,\ldots,T. \qquad (8)$$

where P_t is current index price and P_{t-1} is the previous day's index price. $RBIST_t$ is the return in percent. The descriptive graphs of the daily closing index price and return series are presented in Figure 1: (a) level of price, (b) returns, (c) histogram and descriptive statistics of $RBIST_t$.

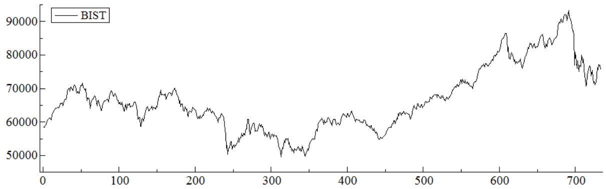
Figure 1a presents the sample data covering the daily closing prices. From Figure 1b, it can be said that the conditional variances of returns display volatility clustering which change over time, and they are not independent. In other words, volatility clustering is clearly observable in the graphs. Figure 1c summarizes the descriptive statistics of the $RBIST_t$ series. As shown in Figure 1c, the Jarque-Bera statistics rejects the null hypothesis of normality. In addition, the measures of skewness and kurtosis display that the distribution of returns is leptocurtic and negatively skewed. Descriptive statistics for the returns are resummarized in Table 1.

According to the results in Table 1, as mentioned before, it can be said that RBIST return series show asymmetric properties and platycurtic and fat tail compared to the normal. Moreover, Ljung-Box statistics (Q and Q^2) in various delays are estimated for independency test of return error and squared return error series. Return errors and squared return error series has not i.i.d. (independent and identically distributed) process because of RBIST return and squared return errors highly correlated up to 50^{th} delay.

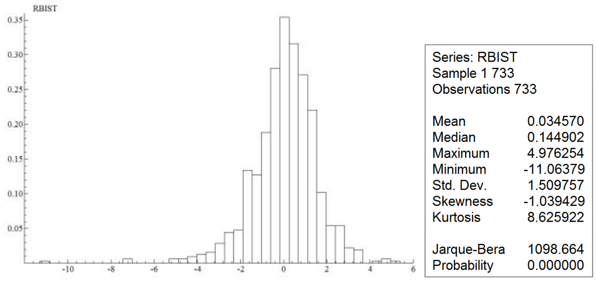
Figure 1. Descriptive Graphs for Sample Data



(a) Level of Price



(b) Returns



(c) histogram and descriptive statistics of $RBIST_t$

Table 1. Descriptive Statistics of Return Series

	RBIST
Mean:	0.03457
Standard Deviation:	1.5087
Skewness:	-1.0394
Kurtosis:	5.6259
Minimum:	-11.064
Maximum:	4.9763
J-B: **Prob.**	1098.7 (0.0000)
ARCH (2):	13.279**
ARCH (5):	9.4772**
ARCH (10):	6.0068**
Q(5):	15.2847 **
Q(10):	19.5965**
Q(20):	31.5018**
Q(50):	68.6886**
Q^2(5):	65.3826**
Q^2(10):	90.0444**
Q^2(20):	129.415**
Q^2(50):	147.626**
Lo R/S Test Statistics for Return	1.25167
Hurst-Mandelbrot R/S Test Statistics for Return	1.23829
Lo R/S Test Statistics for Squared Return	2.09316**
Hurst-Mandelbrot R/S Test Statistics for Squared Return	2.25126**
** shows statistical significantly at level 5%. Lo R/S and Hurst-Mandelbrot R/S Test Statistics: 95%, (0.809-1.862)	

Furthermore, statistical value of Ljung-Box statistics (Q^2) in 50^{th} delay, in high degrees displaying extensive effect of volatility clustering in RBIST stock market returns, is also statistically significant. From the Table 1, the results of ARCH-LM test in various lags imply the existence of significant ARCH effects in standardized errors.

At the 5% significance level, the null hypothesis of a short memory process is rejected if the modified Lo (R/S) and Hurst-Mandelbrot (R/S) statistic does not fall within the confidence interval [0.809, 1.862]. According to results in Table 1, test statistics indicate an evidence of long memory property in squared return series. Table 2 presents the Augmented Dickey Fuller(ADF), Phillips-Perron(PP) and Kwiatkowski-Phillips-Schmidt-Shin(KPSS) unit root test results. According to the results in Table 2, unit root tests are supported stationary for return series.

Table 2. Unit Root Tests for Return Series

Tests	RBIST
ADF	-27.5823**
PP	-27.5776**
KPSS	0.084165
** indicates the refusal of unit root null hypothesis in the significance level at %5. (McKinnon Critical Value is [-2.865], Kwiatkowski Critical Value is [0.463000])	

3.2. Long Memory Property and Asymmetry for RBIST

In order to examine symmetric long memory property in volatility of return series of Turkish Stock Market, FIGARCH models are estimated for different lags (p, q) under assumption of Normal(N), Student-t(ST) and Skewed Student-t(SST) distributions. The different FIGARCH(p, d, q) models as p,q=0,1,2 for RBIST return series are estimated and compared in terms of Akaike (AIC) and Schwarz (SIC) Information Criteria. Table 3 presents estimation results of most appropriate model FIGARCH(1, d, 1) for RBIST. Similarly, asymmetric long memory property in volatility of return series of Turkish stock market is investigated by using FIAPARCH model. The results of most appropriate FIAPARCH(1, d, 1) model are also given in Table 3.

According to Table 3, long memory d parameter for FIGARCH model is significantly different from zero for RBIST return series. It can be said that the volatility of return has long memory property. According to Ljung-Box test statistics (Q and Q^2), return series demonstrate i.i.d. property. The results of Pearson Goodness of Fit Test indicate that different distributions are also appropriate for RBIST return series. Furthermore, tail parameter "v" is statistically significant for all of the distributions (ST, SST). Asymmetry parameter $\ln(\zeta)$ supports negatively skewed distribution. Jarque-Bera statistics is also an indicator referring standardized errors have distributions different from normal distribution.

According to the results of FIAPARCH(1, d, 1) model in Table 3, long memory parameter d of the model is statistically significant at 5% level. The coefficient of the asymmetric response of volatility to news γ is positive and statistically significant at 5% level. It means that unexpected negative shocks cause more volatility than unexpected positive shocks of the same magnitude. This asymmetric effect can give hints to investors about that decreasing price movements in returns are determinants of uncertainty in stock market. Furthermore, the power parameter δ ranges in value from 1.593830 to 2.224163. It means that a squared error term fits the conditional variance specification for returns. The tail parameters (v) of the FIAPARCH(1, d, 1) model with student-t and skewed student-t distributions are statistically significant at 5% level. In other words, standardized residuals have fat-tail density. The results of FIAPARCH (1, d, 1) model with skewed student-t distribution present superior results in terms of AIC and SIC information criteria. Furthermore, Ljung-Box test statistics (Q and Q^2) fail to reject the null hypothesis of no autocorrelation in standardized residuals and squared residuals. ARCH-LM tests point out no remaining also ARCH effects.

Comparing the FIGARCH (1, d, 1) and FIAPARCH (1, d, 1) models, the skewed student-t FIAPARCH(1, d, 1) model is the best for modeling asymmetric long memory volatility process according to the lowest value of AIC and SIC information criteria, the insignificant values of Ljung-Box statistics and ARCH-LM.

Table 3. The Results of FIGARCH and FIAPARCH Models

p=1, q=1	FIGARCH			FIAPARCH		
	N	ST	SST	N	ST	SST
ω	2.826530** (1.4107) [0.0455]	2.174891** 0.68903 [0.0017]	2.251318** (0.63430) [0.0004]	1.492453** (0.46785) [0.0015]	0.148722 (0.46508) [0.7492]	1.302804** (0.22666) [0.0000]
β_0	0.185916** (0.16438) [0.02584]	0.204296 (0.23832) [0.3916]	0.189201 (0.22180) [0.3939]	0.202380 (0.16355) [0.2163]	0.543144*** (0.31706) [0.0871]	-0.478224 (0.31935) [0.1347]
β_1	0.416610** (0.15786) [0.0085]	0.418005 (0.28336) [0.1406]	0.390576** (0.25834) [0.0310]	0.259255 (0.17948) [0.1490]	0.548537*** (0.32008) [0.0870]	-0.459006*** (0.33961) [0.0769]
(Aparch)γ	-	-	-	0.816995*** (0.42489) [0.0549]	0.807063** (0.20800) [0.0001]	0.821142** (0.30321) [0.0069]
(Aparch) δ	-	-	-	1.593830** (0.43620) [0.0003]	2.224163** (0.45009) [0.0000]	1.784524** (0.37297) [0.0000]
d	0.363342** (0.10048) [0.0003]	0.300050** (0.086578) [0.0006]	0.277674** (0.073092) [0.0002]	0.185606** (0.071754) [0.0099]	0.071950** (0.028200) [0.0109]	0.113000** (0.025431) [0.0000]
v	-	7.002334** (1.7814) [0.0001]	6.838949** (1.6481) [0.0000]	-	7.982288** (2.4406) [0.0011]	6.751100** (1.6688) [0.0001]
ln(ζ)	-	-	-0.196079** (0.051337) [0.0001]	-	-	-0.207893** (0.060235) [0.0006]
Log(L)	-1286.564	-1261.722	-1254.192	-1269.699	-1250.646	-1244.537
AIC	3.521321	3.456268	3.438450	3.480761	3.431504	3.417564
SIC	3.546408	3.487626	3.476080	3.518391	3.475405	3.467738
Skewness	-0.83308	-0.91372	-0.94198	-0.55123	-0.67381	-0.68935
Excess Kurtosis	4.6305	5.0210	5.1751	2.7123	3.5328	3.5848
J-B	739.65	871.97	926.35	261.80	378.67	450.54
Q(5)	9.22969	9.44594	9.43186	8.61528	9.35352	8.42067
Q(10)	12.6196	12.7904	12.8019	12.0990	12.7619	12.0016
Q(20)	19.6476	20.0704	20.2330	17.6256	18.9486	17.9394
Q(50)	46.9495	48.1435	48.8652	44.0220	46.4767	45.7055
$Q^2(5)$	2.88542	3.79472	4.27254	2.14792	3.66417	2.92499
$Q^2(10)$	6.26612	6.82712	7.31207	8.76830	8.76886	7.78058
$Q^2(20)$	15.0478	14.9915	15.6113	25.8843	21.7482	25.3448
$Q^2(50)$	32.4776	30.0814	30.2504	53.3918	43.7432	46.7105
ARCH(5)	0.56604 [0.7261]	0.74850 [0.5873]	0.83921 [0.5220]	0.42018 [0.8348]	0.70987 [0.6161]	0.56922 [0.7237]
ARCH(10)	0.63759 [0.7821]	0.68206 [0.7417]	0.72381 [0.7024]	0.92565 [0.5087]	0.90287 [0.5300]	0.82504 [0.6045]
P(40)	61.3793	39.2237	27.2183	56.9045	40.9700	31.5839
P(50)	68.8417	52.1978	36.7817	60.2469	52.1978	50.5607
P(60)	70.9018	71.3929	52.2387	90.8745	72.7026	60.0969

, * indicate statistically significant 5% and 10% respectively. ( ) indicates standard error, [ ] indicates p-values. P(40), P(50) ve P(60) indicate, Pearson Goodness of Fit for 40, 50, 60 cells.

3.3. Long Memory and Asymmetry in Value-at-Risk(VaR)

Portfolio managers, investors and regulators may deal with the suitable model to forecast the VaR of their asset portfolios. In this subsection, we produce the VaR estimates by using FIGARCH and FIAPARCH models with normal, student-t and skewed student-t distributions.

In-sample VaR estimates and out-of-sample VaR calculations for both short and long trading positions are produced for various α levels ranging from 5% to 0.25%. The accuracy of FIGARCH and FIAPARCH models is examined by using the Kupiec LR test. The test compares the failure rate for both the right and the left tails to the prespecified VaR.

The best VaR model implies the model which the empirical failure rate is equal to the prespecified significance level α. The investors and portfolio managers can accurately forecast their possible trading losses by using the best VaR model(Chkili et al., 2014).

3.3.1. In-Sample Value-at-Risk (VaR) Analysis

The in-sample VaR estimations for the Turkish Stock Market returns by using FIGARCH(1, d, 1) and FIAPARCH(1, d, 1) models are summarized in Table 4 and Table 5.

Table 4 displays failure rates, Kupiec-LR test statistics and p-Values for long and short trading positions according to three different distributions (normal, student-t and skewed student-t). If VaR model is estimated accurately, it should be explain the actual observations very well(Tang and Shieh, 2006).

Table 4. In-Sample VaR Calculated by FIGARCH Model for RBIST

Short Positions				Long Positions			
α	Failure Rate	Kupiec LR	P-value	α	Failure Rate	Kupiec LR	P-value
VaR Results with Normal Distribution							
0.9500	0.96317	2.9320***	0.086841	0.0500	0.049113	0.012203	0.91204
0.9750	0.97817	0.31587	0.57410	0.0250	0.034106	2.2430	0.13422
0.9900	0.99181	0.25984	0.61023	0.0100	0.015007	1.6088	0.20466
0.9950	0.99318	0.43854	0.50783	0.0050	0.010914	3.8456**	0.049876
0.9975	0.99591	0.62445	0.42940	0.0025	0.0081855	5.9217**	0.014955
VaR Results with Student-t Distribution							
0.9500	0.95771	0.96529	0.32586	0.0500	0.054570	0.31342	0.57559
0.9750	0.98363	2.5452	0.11063	0.0250	0.031378	1.1331	0.28711
0.9900	0.99318	0.84210	0.35880	0.0100	0.012278	0.35833	0.54944
0.9950	0.99864	2.7421***	0.097738	0.0050	0.0068213	0.43854	0.50783
0.9975	0.99953	0.50042	0.00000	0.0025	0.0054570	1.9163	0.16626
VaR Results with Skewed Student-t Distribution							
0.9500	0.94270	0.78673	0.37509	0.0500	0.045020	0.39531	0.52952
0.9750	0.97544	0.0059461	0.93854	0.0250	0.021828	0.31587	0.57410
0.9900	0.98636	0.88203	0.34765	0.0100	0.0068213	0.84210	0.35880
0.9950	0.99318	0.43854	0.50783	0.0050	0.0054570	0.029882	0.86276
0.9975	0.99864	0.45459	0.50017	0.0025	0.0013643	0.45459	0.50017

(**and *** denote statistically significantly at 5% and 10% level, respectively)

According to results in Table 4, the null hypothesis (f=α) that failure rate equals to prescribes quantiles in the normal FIGARCH is rejected by the Kupiec LR test for α value of 0.0050 and 0.0025 for long position, and is rejected for α values of 0.9500 for short position. Similarly, the null hypothesis (f=α) in the student-t FIGARCH is rejected by the Kupiec LR test only for α value of 0.9950 for short trading position. On the other hand, FIGARCH model with skewed student-t significantly improve on the in-sample VaR performance for both short and long trading positions. This result implies that FIGARCH VaR model with the skewed student-t provides more accurate crucial loss than FIGARCH model with normal and student-t distribution in the in-sample VaR analysis. According to Table 5, the FIGARCH models with the student-t and skewed student-t distribution produce lower Kupiec LR test values in contrast to normal distribution.

Table 5 displays the in-sample VaR results calculated by the FIAPARCH model with normal, student-t and skewed student-t distributions. From Table 5, it can be said that the model with the normal and student-t distributions produce poor performance for long and short trading positions. For FIAPARCH model with normal distribution, the failure rates significantly exceed the prescribed quantiles for α values of 0.9500 and 0.9975 for short position, and for α values of 0.0100 and 0.0025 for long positions. Similarly, for FIAPARCH model with student-t distribution, the null hypothesis (f=α) is rejected by the Kupiec LR test only for α value of 0.9975 for short trading position. However, FIAPARCH model with skewed student-t fails to reject the null hypothesis (f=α) by the Kupiec LR test for all the cases.

Table 5. In-Sample VaR Calculated by FIAPARCH Model for RBIST

Short Positions				Long Positions			
α	Failure Rate	Kupiec LR	P-value	α	Failure Rate	Kupiec LR	P-value
VaR Results with Normal Distribution							
0.9500	0.96453	3.6096***	0.057448	0.0500	0.049113	0.012203	0.91204
0.9750	0.98226	1.7633	0.18421	0.0250	0.027285	0.15257	0.69609
0.9900	0.98363	2.5205	0.11238	0.0100	0.017735	3.6017***	0.057719
0.9950	0.99045	2.4044	0.12099	0.0050	0.0095498	2.4044	0.12099
0.9975	0.99318	3.7163***	0.053883	0.0025	0.0068213	3.7163***	0.053883
VaR Results with Student-t Distribution							
0.9500	0.96180	2.3313	0.12680	0.0500	0.053206	0.15550	0.69333
0.9750	0.98090	1.1383	0.28602	0.0250	0.030014	0.71108	0.39908
0.9900	0.99045	0.015235	0.90177	0.0100	0.010914	0.060075	0.80638
0.9950	0.99591	0.12931	0.71915	0.0050	0.0054570	0.029882	0.86276
0.9975	0.99921	0.53276**	0.00000	0.0025	0.0013643	0.45459	0.50017
VaR Results with Skewed Student-t Distribution							
0.9500	0.94816	0.051746	0.82005	0.0500	0.042292	0.96529	0.32586
0.9750	0.97408	0.025202	0.87387	0.0250	0.020464	0.65896	0.41693
0.9900	0.98499	1.6088	0.20466	0.0100	0.0054570	1.8298	0.17615
0.9950	0.99181	1.2527	0.26305	0.0050	0.0013643	2.7421	0.19773
0.9975	0.99591	0.62445	0.42940	0.0025	0.0013643	0.45459	0.50017

(**and *** denote statistically significantly at 5% and 10% level, respectively)

3.3.2. Out-of-Sample Value-at-Risk(VaR) Analysis

In previous subsection, we compared the past performance of the Value-at-Risk by using two different long memory models. In this subsection, following the analysis procedure of Tang and Shieh (2006), we compute VaR values for period from July 19, 2013 to September 28, 2014. Out-of-sample VaR analysis provides information to investors, portfolio managers and financial instituations about the biggest loss they will experience (Kasman, 2009). The empirical results of the out-of sample VaR analyses for Turkish Stock Market returns by using FIGARCH and FIAPARCH models are presented in Table 6 and Table 7, respectively.

Similar to in-sample VaR analysis, the null hypothesis is that failure rate equals to prescribed quantiles (f=α) for both long and short trading positions. According to Table 6, the null hypothesis (f=α) in normal distribution FIGARCH model is rejected when α is 0.0050 for long position, whereas the FIGARCH model with the student t and skewed student-t distribution fail to reject the null hypothesis (f=α) for all cases. Hence, the out-of sample VaR analysis implies that FIGARCH models with student-t and skewed student-t distributions are better than normal distribution for asymmetry and tail-fatness in return distributions.

Similar to in-sample VaR analysis, the null hypothesis is that failure rate equals to prescribed quantiles (f=α) for both long and short trading positions. According to Table 6, the null hypothesis (f=α) in normal distribution FIGARCH model is rejected when α is 0.0050 for long position, whereas the FIGARCH model with the student t and skewed student-t distribution fail to reject the null hypothesis (f=α) for all cases. Hence, the out-of sample VaR analysis implies that FIGARCH models with student-t and skewed student-t distributions are better than normal distribution for asymmetry and tail-fatness in return distributions.

As can be seen from Table 7, FIAPARCH models with normal, student-t and skewed student-t distributions for both long and short positions fail to reject null hypothesis (f=α) for all cases. The empirical results indicate the FIGARCH and FIAPARCH models with skewed student-t distributions are suitable for the Turkish Stock Market returns. Generally, the findings prove the price series of Turkish Stock Market are skewed student-t distributed and fat-tailed according to asymmetry parameter ln(ζ) and tail parameters (v) of the models. Furthermore, the skewed student-t model performs accuracy estimations for the in-sample and out-of-sample VaR calculations based on Kupiec LR test than the other symmetric distributions.

Table 6. Out-of-Sample VaR Calculated by FIGARCH Model for RBIST

Short Positions				Long Positions			
α	Failure Rate	Kupiec LR	P-value	α	Failure Rate	Kupiec LR	P-value
VaR Results with Normal Distribution							
0.9500	0.95486	0.14790	0.70055	0.0500	0.048611	0.011800	0.91350
0.9750	0.98264	0.77076	0.37998	0.0250	0.027778	0.088050	0.76667
0.9900	0.98611	0.39244	0.53102	0.0100	0.017361	1.2923	0.25563
0.9950	0.98958	1.2923	0.25562	0.0050	0.013889	3.0761***	0.079448
0.9975	0.99306	1.5323	0.21577	0.0025	0.0069444	1.5323	0.21577
VaR Results with Student-t Distribution							
0.9500	0.95486	0.14790	0.70055	0.0500	0.048611	0.011800	0.91350
0.9750	0.98611	1.7340	0.18790	0.0250	0.027778	0.088050	0.76667
0.9900	0.98958	0.0049825	0.94373	0.0100	0.013889	0.39244	0.53102
0.9950	0.99653	0.15139	0.69721	0.0050	0.0034722	0.15139	0.69721
Short Positions				Long Positions			
VaR Results with Student-t Distribution							
α	Failure Rate	Kupiec LR	P-value	α	Failure Rate	Kupiec LR	P-value
0.9975	1.0000	.NaN	1.00000	0.0025	0.0034722	0.097281	0.75512
VaR Results with Skewed Student-t Distribution							
0.9500	0.94792	0.025977	0.87196	0.0500	0.048611	0.011800	0.91350
0.9750	0.98264	0.77076	0.37998	0.0250	0.027778	0.088050	0.76667
0.9900	0.98958	0.0049825	0.94373	0.0100	0.010417	0.0049825	0.94373
0.9950	0.99306	0.19511	0.65870	0.0050	0.0034722	0.15139	0.69721
0.9975	1.0000	.NaN	1.00000	0.0025	1.00000	.NaN	1.00000

(**and *** denote statistically significantly at 5% and 10% level, respectively and NaN represents the statistics is not available).

Table 7. Out-of-Sample VaR Calculated by FIAPARCH Model for RBIST

Short Positions				Long Positions			
α	Failure Rate	Kupiec LR	P-value	α	Failure Rate	Kupiec LR	P-value
VaR Results with Normal Distribution							
0.9500	0.95833	0.44527	0.50459	0.0500	0.045139	0.14790	0.70055
0.9750	0.97569	0.0057501	0.93955	0.0250	0.027778	0.088050	0.76667
0.9900	0.98264	1.2923	0.25563	0.0100	0.013889	0.39244	0.53102
0.9950	0.98958	1.2923	0.25562	0.0050	0.010417	1.2923	0.25562
0.9975	0.99306	1.5323	0.21577	0.0025	0.0069444	1.5323	0.21577
VaR Results with Student-t Distribution							
0.9500	0.95139	0.011800	0.91350	0.0500	0.052083	0.025977	0.87196
0.9750	0.97917	0.21726	0.64113	0.0250	0.024306	0.0057501	0.93955
0.9900	0.98958	0.0049825	0.94373	0.0100	0.010417	0.0049825	0.94373
0.9950	0.99306	0.19511	0.65870	0.0050	0.0034722	0.15139	0.69721
0.9975	0.99653	0.097281	0.75512	0.0025	0.0034722	0.097281	0.75512
VaR Results with Skewed Student-t Distribution							
0.9500	0.95139	0.011800	0.91350	0.0500	0.052083	0.025977	0.87196
0.9750	0.97917	0.21726	0.64113	0.0250	0.020833	0.21726	0.64113
0.9900	0.98958	0.0049825	0.94373	0.0100	0.010417	0.0049825	0.94373
0.9950	0.99306	0.19511	0.65870	0.0050	0.0034722	0.15139	0.69721
0.9975	0.99653	0.097281	0.75512	0.0025	0.0034722	0.097281	0.75512

4. Conclusions

Financial time series frequently exhibit stylized facts such as asymmetry, strong volatility, fat-tail characteristics and long memory. Hence, financial econometrics literature focuses on distributional and statistical properties of financial return series. In this study, we have investigated stylized facts of the Turkish Stock Market returns. For modeling the volatility, asymmetric effects and long memory, the FIGARCH and the FIAPARCH models with normal, student-t and skewed student-t distributions have been employed.

According to model results, long memory parameters d are statistically significant at 5% level indicating that volatility of Turkish Stock Market has persistence. Furthermore, the FIAPARCH model outperforms the FIGARCH model for modeling of asymmetric long memory volatility process in terms of AIC and SIC information criteria. It is well known that accurate volatility modeling is an important determinant of market risk management and portfolio management. So, we have examined the performance of the VaR models by using the FIGARCH(1,d,1) and FIAPARCH(1,d,1) models with the normal, student-t and skewed student-t distributions for both long and short positions. From the results of VaR analyzed based on Kupiec LR test in-sample and out-of-sample, we have generally concluded that the FIGARCH and FIAPARCH models distributed the skewed student-t outperform the other symmetric distributions. It can be concluded that the volatility models with skewed student-t distribution are more suitable for VaR calculations.

Consequently, long memory models perform more efficient results than the traditional short memory models for Value-at-Risk analysis. In summary, it can be said that Turkish Stock Market returns exhibit asymmetry, fat-tails and long memory property. Comparing the results of two long memory models, FIAPARCH(1,d,1) with skewed student-t distribution is preferable a model to analyze the Value-at-Risk for long and short trading positions. In this sense, the findings of study can be evaluated by financial risk managers, investors, regulators and academicians.

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