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Islamic Financial Intermediation Compared to Ribaoui Financial Intermediation: A Theoretical and Mathematical Analysis

Khadidja Khaldi[1], Amina Hamdouni[2]*

[1]Department of Finance and Investment, College of Economics and Administrative Sciences, Al-Imam Muhammad Ibn Saud Islamic University, Riyadh, Saudi Arabia, [2]Department of Finance and Investment, College of Economics and Administrative Sciences, Al-Imam Muhammad Ibn Saud Islamic University, Riyadh, Saudi Arabia. *Email: aminahamdouni@yahoo.fr

ABSTRACT

The present research uses mathematical analysis instruments for a comparative study between the Islamic financing system and the Ribaoui financing system usually called the classical financing scheme. The former system deals with contracts based on variable returns while the second i-e the ribaoui system is based on fixed return contracts. Using the principles of partial equilibrium theory for risk adverse investors, the study demonstrates that returns are higher with contracts based on profit and loss sharing.

Keywords: Interest Rate, Classic Bank, Islamic Bank, Profit Sharing System, Return, Speculation, Mathematical Modelling
JEL Classifications: G21, G3

1. INTRODUCTION

Riba is a way of making money, but it is an illicit gain in Islam. Classic Banks may make money via Illegal and/or Haraam businesses. Many financial intermediaries around the world now operate intermediation subject to the requirements of Islamic jurisprudence (Honohan, 2001). As such, they neither charge interest (riba) nor do they receive it.

Riba is mentioned and condemned in several different verses in the Qur'an. It is also mentioned in many ahadith (Abu Zahra, 1985). The word was used by the Arabs prior to Islam to refer to an "increase."

In classical Islamic jurisprudence, the definition of riba was "surplus value without counterpart." The word "riba" literally means "excess" or "addition." The literal translation of the Arabic word riba is the increase, addition, growth or development (Ibn Mandour, 1968), though it is usually translated as "usury." While English speakers usually understand usury as the charging of an exploitative interest rate, the word "riba" in Arabic applies to a wider range of commercial practices.

One of the most important forms of riba is that a person makes some money to another person (as a loan), in return for that particular money being returned plus a predefined increase. This is the same as what interest banks are doing today.

The majority of scholars (Al-Masri. 1990, Amin, 2003, Darwish, 1995) concluded that banking interest in all its forms is one of the riba that is haraam and is absolutely forbidden the Qur'an and Sunnah of Prophet (peace and blessings of Allaah be upon him)[1].

Perhaps the wisdom of forbidding usury in Islam is because it is a guaranteed gain in any case without being exposed to risk factors and loss (Afifi, 1980). This is what signifies injustice and causes the breakdown of cooperation and social solidarity called for by Islamic legislation. Therefore, riba has social, economic and moral effects that are devastating and destructive (von Bethmann, 1993).

1 The prohibition on interest (riba) is not the only defining feature of Islamic finance. Also precluded are recklessness or unnecessary risk (gharar), gambling (maysir), and the exploitation of ignorance (jahl). In addition, Islamic law prohibits investing in businesses that are considered unlawful or Haraam (such as businesses that sell alcohol or pork, or businesses that produce media such as gossip columns or pornography, which are contrary to Islamic values) (Mastoor, 2014).

2. LITERATURE REVIEW

2.1. Islamic Banking System

The term "Islamic banking" refers to a system of banking or banking activity that is consistent with Islamic law (Shariah) principles and guided by Islamic economics (Mastoor, 2014).

Islamic banking has the same purpose as conventional banking[2]: To make money for the banking institute by lending out capital while adhering to Islamic law. Because Islam forbids simply lending out money at interest, Islamic rules on transactions (known as *Fiqh al-Muamalat*) have been created to prevent it (Mastoor, 2014). The basic principle of Islamic banking is based on Profit and loss – sharing which is a component of trade rather than risk-transfer which is seen in conventional banking.

Islamic banking has the same purpose as conventional banking except that it operates in accordance with the rules of Shariah. Islamic bank is an institution which receives deposits and leads all banking activities except the collection and payment of interest. Islamic bank incites all the parts with a transaction to share risk and profit-and-loss (Khaldi and Hamdouni, 2011).

If Islam has forbidden riba, he has permitted other aspects to make money and invest it. In the wholly Qur'an four verses are about Riba (interest) revealed on different occasions (Hanif, 2011). The first verse is in Surah Al-Rum 30:39 whereby displeasure of Allah is disclosed for interest-based practices. The second verse is in Surah An-Nisaa 4:161 where interest charging was disclosed as a sinful act of Jews. The third verse is part of Surah Al-i-'Imran 3:130 whereby prohibition of Riba (interest) was declared "O those who believe do not eat up riba doubled and redoubled." The last verse revealed is reported in Surah Al-Baqarah 2:275 whereby severe punishment is declared for those dealing with interest:

> *"Those who take the interest will not stand but as stands whom the demon has driven crazy by his touch. That is because they have said: 'Trading is but like riba'. And Allah has permitted trading and prohibited riba. So, whoever receives an advice from his Lord and stops, he is allowed what has passed, and his matter is up to Allah. And the ones who revert back, those are the people of Fire. There they remain forever"* (translation from Usmani, 1999[3]).

The two basic categories of financing permitted in Islam are: (1) profit-and-loss-sharing, also called participatory modes, i.e., joint venture (musharakah) and mudharabah (with profit sharing concept) and (2) purchase and hire of goods or assets and services on a fixed-return basis, i.e., cost plus (murabaha), istisna'a, salam and leasing (Ijar), safekeeping (Wadiah).

In accordance with the principle of sharing of profits and losses, mudharabah is a form of passive participation treated as a limited partnership or a corporation. In a mudharabah financing, only the bank (Rab al-mal or capital provider) provides capital while the client (Mudharib or entrepreneur) manages the business. Mudharib brings its experience and expertise, manages the activity or business under and provide the labor needed to use these funds and does not guarantee the capital invested and the realization of profit. The bank cannot interfere in the day-to-day running of the business (Siddiqui, 1985a), (Khan and Mirakhor, 1994). Any profit is shared. The client is not paid a salary, and if he or she does not make a profit, the client loses all the time and effort expended on the venture and the bank absorbs losses. The distribution of profits and losses is fixed in the contract (Khaldi and Hamdouni, 2011)[4].

In musharakah (a form of active participation), both bank and client contribute capital and agree to a profit-sharing ratio (profit or loss) (Siddiqui, 1985a). However, there are cases in which it is difficult to use mudharabah and musharakah. Other formulas can be used for the same principles as Murabaha, Istisna'a. This is not limited to this formula, where shariah allowed a great deal of flexibility between the parties in the field of financial transactions, with the prohibition of injustice, gharar, ignorance, and riba (Khan and Mirakhor, 1994).

2.2. Islamic Financial Intermediation

Banks in the Islamic system have the same functions as conventional banking, although they do not deal with riba. They manage the system of payments and financial intermediation. However, Islamic banks do not get a fixed return on the financing they provide (as in conventional banking).

Instead, Islamic Banks participate in the profit and loss of businesses that provide them with financial resources. Similarly, those who entrust their savings to Islamic banks also participate in the profits and losses of these banks and do not guarantee them a predetermined return on the nominal value of their deposits in the bank (as in conventional banking). Islamic banks can find differences in return between surplus and deficit units, providing a higher return for deficit units, and gives lower returns to surplus units (Khan and Mirakhor, 1994).

2 Hanif (2011) present study that address the perceptional issues by identifying the similarities and differences in Islamic and conventional banking. Evidence suggest Islamic banking is very much practiced like modern conventional banking with certain restrictions imposed by Sharia and addresses a large number of business requirements successfully hence perceiving Islamic banking as totally foreign to the business world is not correct. It is further found in the study that Islamic banking is not a mere copy of conventional practices rather major differences exist in the operations of Islamic Financial Institutions (IFIs) in comparison with conventional banking. IFIs have succeeded in creating trust in the eyes of depositors and receive deposits on profit and loss sharing basis, however, investment and financing options available to Islamic banks are limited in comparison to conventional banks.

3 Cited in Hanif (2011).

4 The research presented by Khaldi and Hamdouni (2011) illustrated principles and activities that define the Islamic banking system, allowing the latter to be more efficient and more equitable. The first model is based on the Mudharabah (deposit, investment funds); the second concerns the Mudharabah for the deposit only while for the investment we need the Musharakah; The third model is based on the Mudharabah for deposits but introduces the debt and quasi-debt instruments (Murabahah, Istisna, Salam, Ijara); The fourth model is based on Mudharabah for deposits and Mutajarah on the assets side. Results show that the first model is more efficient than the others, particularly the third which is paradoxically largely adopted. The fourth is not recommended for its negative impact on trade.

2.3. The Nature of the Islamic Banking Intermediation

The relationship between depositors and the Islamic Bank is an agency relation and the best contract is a mudharabah contract. However, intellectuals disagreed about this relationship on the assets side as follows:

2.3.1. Firstly: Mudharabah on the liability side and on the assets side

Some thinkers (Siddiqui, 1998) believe that the best contracts that should govern the relationship between the bank and borrowers are the Mudharaba contract. In this case, the bank is on one side mudharib (with depositors) and on the other side rab al-mal (with entrepreneurs): The “mudharib yudharib.” With this model, the primary function of the Islamic banking sector appears as financial intermediation (Khaldi and Hamdouni, 2011).

Advantages of this model:

- The return of the mudharabah is linked to productivity and quality of the project, thus the yield is a close association with the return in the real economy. The balance between the real economy and the financial economy in Islamic transactions based on Mudharabah is assured since this form is systematically linked to real assets as opposed to the “ribawi” system.
- Justice affects efficiency by influencing the generation of savings and increasing their good use and allocation.

Inconveniences of this model:

- Mudharabah is a contract of passive partnership also considered a participatory contract based on sharing of profits and losses. In such contracts, moral hazard and adverse selection can be greatly prejudicial for the bank in an agency relationship. With this model, the bank appears as a principal and entrepreneurs as an agent. According to agency theory, there is a typical agency relationship between the bank and its entrepreneurs since there is a separation between ownership and control. The entrepreneur may act contrary to the interests of the bank that provides full funding of the investment project. The bank cannot control the actions of the entrepreneur and faces problems of asymmetric information and moral hazard.

2.3.2. Secondly: Mudharabah on the liability side and Musharakah on the assets side

While other thinkers (Al-Swailem, 1998) argue that musharakah (a form of active participation) is the best contract that can be used on the Assets Side, and the banking intermediation, in this case, becomes a Mudharab on the Liability side.

Advantages of this model:

- Hedging increases
- Lower the possibility of moral hazard and the adverse selection.

Inconveniences of this model:

- This model assumes direct investment and this is not the financial intermediary functions.
- The reluctance of some businessmen to finance their projects by musharakah.

2.3.3. Thirdly: Mudharabah on the Liability side and debt and quasi-debt Instruments on the assets side (Murabahah, Istisna, Salam, Ijhara)

While other thinkers believe that Islamic banks should adopt all forms of Islamic finance in terms of employment. Islamic bank uses mudharabah on the liability side, but on the assets side, she makes use of debt and quasi-debt instruments for the employment of those funds collected by deposits. The bank will be conducted to practice murabahah, or istisna, or ijara and may thus reduce the risks related to mudharabah and musharakah. Remuneration of the bank is directly related to these debt and quasi-debt instruments concerning a direct use of funds from the bank without recourse to entrepreneurs or without investing in capital projects (Qahaf, 2001).

Advantages of this model:

- The capital and income are guaranteed and do not support risk.

Inconveniences of this model:

- Disables the ability to take second mudharabah risks and get a larger return.
- Exposure to commercial risks.
- Exposure to the suspicion of some forbidden products.

2.3.4. Fourthly: Mudharabah on the liability side and Mutajarah on the assets side

Islamic banks conduct commercial operations directly, and most thinkers believe that this model is not a good model for financial intermediation.

The disadvantages of this model:

- Exposure to business risk.
- The principle of a bank is financial intermediation, and with this model, Islamic bank no longer belongs to this category of financial intermediary, but rather to the category of direct investors.

As a result, the best form of Islamic financial intermediation is the model of mudharabah on both sides of the balance sheet because it is the model that guarantees equity and efficiency for the whole banking system. Mudharabah is called pure financial intermediation (Siddiqui, 1998).

Mudharabah on both sides is more efficient and fair than Islamic financial intermediation, which depends on Mudharabah on the Liability side and debt and quasi-debt Instruments on the assets side (Murabahah, Istisna, Salam, Ijhara) (Al-Swailem, 1998). It is also more legitimate, more efficient and fair than interest-based financial intermediation (Al-Swailem, 1998).

2.4. The Efficiency of the Islamic Banking System

Many studies have shown by mathematical models that the return on savings can be higher in the Islamic system than in the interest system (Masood, 1989), (Shahid and Bashir, 1999), (Haque and Mirakhor, 1986), (Shahid, 2000; 2002), (John and John, 1994).

In the light of these studies, we will try to present a mathematical model that analyzes the relationship of depositors with Islamic banks, which is usually a speculative relationship (Mudharabah) based on profit sharing and comparing the return achieved from mudharaba with the return from the riba-based loan.

Since the value of the profit attributable to the Mudharaba deposit holder in the Islamic Bank is undefined, and that the depositor may face the risk of loss, the uncertainties will arise.

In the area of financial investment, the diversification of the portfolio is among the important decisions. Many dealers, do not accept risk higher than average risk, in order to obtain returns higher than average returns (Avery and Elliehausen, 1986). Most dealers can be considered a risk-averse person (Baily et al., 1980).

On the other hand, the financial theory recognizes in advance that risk-taking or acceptance is compensated. In order to obtain the highest rate of return, the investor must accept the highest level of risk (Hanna and Chen 1997).

The investor has to determine how much risk he accepts. This type of decision for the investor is important and decisive. The Normative model is the ablest to help decision-making under uncertainty, which expects utility (Schoemaker, 1982).

Most studies of optimal behavior under uncertainty, and where investor aversion to risk assumes that the expected utility must be maximized, the utility is a function of the argument wealth (Hanna and Chen 1997).

Maximizing the expected utility is the widely used approach to optimal portfolio analysis (Markowitz, 1952). The optimal portfolio maximizes the expected utility, among the available portfolios.

Among the types of utility functions expected in the analysis of investment decisions is the utility function with the stability of constant relative risk aversion (CRRA) (Samuelson, 1990). The analysis below will be used using the partial equilibrium theory (maximizing the expected utility) for the risk-averse investor. The mathematical model was inspired by a study of Shahid and Bachir (1999).

However, we disagreed with the researchers in several points, including:

- Inflation rate: The researchers took into consideration to find the current values for consumer utility at time t = 1 the discount coefficient that includes the expected inflation rate, so, the futures values have been linked to the inflation rate or to the change in the general price level (indexation).

However, by reference to Islamic jurisprudence, the question of linking future rights and obligations to change in the general price level has been rejected. Some people legalize interest on account of inflation and decrease in purchasing power of the lent money. They say that the borrower must indemnify the lender in case of depreciation of the loan amount due to inflation. They criticize the theory of Islamic finance as it does not accept indexation of financial liabilities with gold, baskets of goods or any stable currency. But the owners of the four schools of thought in Islam: The Maaliki, the Shafa'i, Hanbali and Hanafi agreed that the borrower must pay back dinars and dirhams, and debt liability cannot increase due to inflation[5].

This ruling applies to the banknotes in circulation in our time because they replaced the dirhams and dinars in circulation because:

- Linkage such as usury: The fuqaha have categorically rejected the concept of linking on the basis that it contradicts the prohibition of usury in Islam (Zaman, 1985).

In case the loan or debt is linked to change in the general price level, the linkage leads to obtaining a return, even if this return is not real cash, and this is like a "*Riba al-nasi'ah*"[6]. There are also jurisprudential objections to linking transactions at prices as a kind of "*Riba al-fadhl*"[7].

- Linking includes ignorance (Jahl or Jahala)[8]: One of the terms of the deferred payment contract according to Islamic law is the definition of the obligation at the time of the contract. If this obligation is not known, it is like "Jahala" and therefore the contract becomes voidable. In the case of linkage, the amount of the obligation is not known to be specified or confirmed only at maturity. The linking of future obligations to changes in the general price level means reserve (against inflation), and this situation may arise or not arise, and is considered as "gharar" (Zaman, 1985), which nullifies the contract.
- Linking restrain risk: Linking future liabilities to change in the general price level in a period of continuous inflation means that lenders have a profit without risk (Siddiqui, 1985). This restrains risk of trade in favor of bank investments.
- Linking increases the severity of inflation: Over the past two decades, the linkage experience has shown that inflation remains. The main reason for this is that linkage tends to weaken resistance against inflation because it temporarily

5 Lending in Islam is a virtuous activity since the lender has to give away the lent goods/money to the borrower for the period of the loan without seeking any compensation. If the value of that loan decreases due to inflation, it is as if the lender has done a larger virtue. The Holy Qur'an encourages giving extra time to borrowers who are in difficulty or faced with financial constraint. Therefore, the Fiqh Academy of the OIC has categorically ruled out as strictly forbidden the commonly suggested solution of the indexation of a lent amount of money to the cost of living, interest rates, GNP growth rates, the price of gold or some other commodities. However, one can lend in terms of gold or any other currency which is considered not vulnerable to inflation. In this case, too, debt liability cannot increase due to inflation (Ayub, 2007).

6 "*Riba al-nasi'ah*" is a type of riba that exists in, or results from, a sale transaction which unduly benefits one the counterparties in the form of a surplus or extra amount due to delay of delivery of his side of the transaction. More specifically, riba al-nasi'ah arises in loan transactions (on the basis of future repayment of more than the principal) as well as sale transactions (on the basis of deferred price).

7 "*Riba al-fadhl*" is the riba of surplus/riba of excess; a type of riba that exists in, or results from, a sale transaction whose underlying is a ribawi item. Riba of surplus or riba al-fadhl comes into existence in a sale transaction that involves the exchange of one ribawi commodity/ribawi item (such as dates, wheat, etc) for the same type of commodity but the different amount or weight.

8 "*Jahl or Jahala*:" Ignorance, lack of knowledge; indefiniteness in a contract, non-clarity about the parties or their rights and obligations, the goods/subject matter or the price/consideration – leading to "*Gharar*."

eases the troubles caused by inflation, relieving pressure on governments to pursue sound policies, and maintaining sustained inflation. Moreover, the linkage leads to the acceleration of inflation rates and does not achieve the desired purpose (Siddiqui, 1985a).

- Connectivity is an unfair tool: The single index of linkage is not based on the different consumption patterns. In fact, it does not represent the consumer habits of the majority of individuals, and here it negates the principle of social and economic justice.
  - The researchers have only order determined the first conditions to maximize the expected utility of consumption. Under these conditions, the financial asset valuation parameter can be determined and we will add the first order conditions for maximization.
  - Examining the changes in the financial asset valuation parameter according to the factors determined for it.
  - Non-binary distribution for possible profit and loss.

3. MODEL

Consider a two-step model: t = 0, t = 1, and the allocations (wealth) for an economic trader, are W_0, W_1 in the two periods, t = 0 and t = 1, respectively.

Rational trader makes try to invest part of W_0 after consuming a certain part C_0.

Assuming that the only option available to the trader is to invest his remaining funds in the Mudharaba formula by placing his money as a deposit (Mudharaba deposit) with an Islamic bank. After investing this deposit, the Islamic Bank achieves the value P for its project in period t =1, the Bank will pay to the client (in case of profit) the face value of his deposit, plus a specified and agreed percentage of the increase achieved on the basis of the lowest liquidity value. The basic conditions of the first order to maximize the expected benefit of the client determine the coefficient of valuation of the financial asset (Mudharaba deposit).

The analysis below maximizes the expected benefit of the client's wealth within his budget.

3.1. The Objective Function

The agent behavior is a maximization problem. It means making the most of his limited resources to maximize the utility expected of his wealth:

$$\max_{Q0,c0,c1} \left\{ U(C_0) + \rho\, U(C_{1a}) + (1-\rho)\, U(C_{1b}) \right\}$$

$$0 \prec \rho \prec 1 \quad (1)$$

The budget constraints are:

$$G_1 = W_0 - Q_0 - C_0 = 0 \quad (2)$$

$$G_2 = W_1 + Q_0\,(1+\theta\,\delta_a) - C_{1a} = 0 \quad (3)$$

$$G_3 = W_1 + Q_0\,(1+\theta\,\delta_b) - C_{1b} = 0 \quad (4)$$

by assuming:

$$\delta_a \succ 0, \delta_b \prec 0$$

Where: $E_0\{\ \}$: Expectation for period t = 1.
U ( ): Utility function.
C_0 and C_1 represent the consumption of the trader in periods t = 0 and t = 1, respectively.
ρ: Represents the probability of profit.
P: Represent liquidity value of the project in the period t = 1.
P': Represent the minimum liquidity value of the project in t = 1.
Q_0: The nominal value of the deposit.
W_0 and W_1 are the allocations (wealth) in periods t = 0 and t = 1, respectively.
θ: Participation in increase parameter.
$\delta = \dfrac{p' - p}{p'}$: Represent the increase in the value of the project on the minimum liquidity value.

To solve the maximization of utility equation under the previous budget, the Lagrange function L is written as follows:

$$L=U(C_0)+\rho U(C1_a)+(1-\rho)\, U(C1_b)+\lambda_1 G_1+\lambda_2 G_2+\lambda_3 G_3$$

Where λ_1, λ_2 and λ_3 represents Lagrange multipliers, and to solve Lagrange's function, the equation is derived based on variables C_0, Q_0, C_{1a}, C_{1b}, as follows:

- Adequate conditions for maximization (first order conditions from the Lagrangian):

$$\frac{\partial L}{\partial C_0} = U'(C_0) - \lambda_1 = 0 \qquad \Rightarrow U'(C_0) = \lambda_1 \quad (5)$$

$$\frac{\partial L}{\partial C_{1a}} = \rho U'(C_{1a}) - \lambda_2 = 0 \ \Rightarrow \rho U'(C_{1a}) = \lambda_2 \quad (6)$$

$$\frac{\partial L}{\partial C_{1b}} = (1-\rho)U'(C_{1b}) - \lambda_3 = 0 \ \Rightarrow (1-\rho)U'(C_{1b}) = \lambda_3 \quad (7)$$

$$\frac{\partial L}{\partial Q_0} = -\lambda_1 + \lambda_2(1+\theta\delta_a) + \lambda_2 Q_0(1+\theta\delta_b) = 0$$

$$\Rightarrow -U'(C_0) + \rho U'(C_{1a})(1+\theta\delta_a) + (1-\rho)U'(C_{1b})\, Q_0\, (1+\theta\delta_b) = 0 \quad (8)$$

$$\frac{\partial L}{\partial \lambda_1} = W_0 - Q_0 - C_0 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = W_1 + Q_0(1+\theta\delta_b) - C_{1b} = 0$$

$$\frac{\partial L}{\partial \lambda_3} = W_1 + Q_0(1+\theta\delta_b) - C_{1b} = 0$$

- Study the basic conditions for maximization (Second order conditions): Second order conditions are found in Appendix 01.

3.2. Solution of the Model

We have reached through first order conditions from the Lagrangian that:

$$U'(C_0)+\rho U'(C_{1a})(1+\theta\delta_a)+(1-\rho)U'(C_{1b})Q_0(1+\theta\delta_b)=0 \quad (9)$$

When equation (2), (3) and (4) is computed in equation (9) we obtain:

$$\rho(1+\theta\delta_a)U'(W_1+Q_0(1+\theta\delta_a)) +(1-\rho)(1+\theta\delta_b)U'(W_1+Q_0(1+\theta\delta_b))-U'(W_0-Q_0)=0 \quad (10)$$

By giving the exact configuration for utility function, and values for W_0, W_1, Q_0, δ_a and δ_b we obtain a value of θ from Equation (10).

3.3. Testing Hypothesis Through Simulations

Simulations methodology leads to resolve a model for the risk-averse investor under the following assumptions:

1. As mentioned earlier, for risk-averse agent, Relative Risk Aversion is CRRA: Called constant-relative risk aversion utility functions CRRA.

Assuming that CRRA is: $U_{(C_i)}=(\frac{[C_i]^{1-\alpha}}{1-\alpha})$

The derivation of this function is: $U'_{(C_i)}=\frac{1}{[C_i]^{\alpha}}$

This allows us to write equation (9) as follows:

$$\frac{1}{(W_0-P')^{\alpha}}=\frac{\rho(1+\theta\delta)}{(W_1+P'(1+\theta\delta_a))^{\alpha}}+\frac{(1-\rho)(1+\theta\delta)}{(W_1+P'(1+\theta_b))^{\alpha}} \quad (11)$$

2. In the case of interest-bearing fixed interest loan, Equation (11) is written as follows:

$$\frac{1}{(W_0-Q_0')^{\alpha}}=\frac{1+i}{(W_1+Q_0'(1+i))^{\alpha}} \quad (12)$$

3. The Bank's project offers:
   - A non-binary distribution of the probability of profit or loss, where:

ρ* Represents the probability of profit.
$(1-\rho)$* Represents the probability of loss.

The solution:

- By giving values for W_0, W_1, ρ, Q_0, α, δ_a, δ_b (in equation 11) and using Maple 9.5, we obtain θ values and hence the return on Mudharabah financing.
  - And when compensating the same values (Q_0, W_0, W_1, α) (in equation 12)) and using Maple 9.5, we find values for (i) and where we get the return from interest financing.
  - We will study a solution with to the changes in θ in terms of: W_0, Q_0, ρ and β where β is the amount of money invested relative to the primary wealth $W_0\beta = Q_0$. We have:

$$\rho(1+\theta\delta_a)U'(W_1+Q_0(1+\theta\delta_a))+(1-\rho)(1+\theta\delta_b)U'(W_1+Q_0(1+\theta\delta_b)) -U'(W_0-Q_0)=0 \quad (13)$$

Assuming:

$$\delta_a \succ 0, \delta_b \prec 0$$

- The constant-relative risk aversion utility functions CRRA is of the form:

$$U(x)=\frac{x^{1-\alpha}}{1-\alpha}, 0\prec\alpha\prec 1, x\succ 0,$$

$$\Rightarrow U'(x)=x^{-\alpha}$$

$$\Rightarrow U''(x)=-\alpha x^{-\alpha-1}$$

$$\Rightarrow U''(x)\prec 0$$

Simulation No. 01:

Case 1: Study change in θ in terms of Q_0:

$\theta = H(Q_0)$

We study the derivative signal: $H'(Q_0)$ with assuming the stability of: W_0, W_1, δ_a, δ_b, α and ρ. After studying the derivative signal (Appendix 2), we find that:

θ is an increasing function for Q_0, if: $\rho \succ\succ \frac{-\delta_b}{\delta_a-\delta_b}$

θ is a decreasing function for Q_0, if: $\rho \prec \frac{-\delta_b}{\delta_a-\delta_b}$

Case A: $\rho \succ\succ \frac{-\delta_b}{\delta_a-\delta_b}$

We can obtain the Figure 1a with assuming:

$\alpha = 0.3$, $W_1 = 1$, $W_0 = 10$, $Q_0 = 1\ldots5$, $\rho = 0.6$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$.

Case B: $\rho \prec \frac{-\delta_b}{\delta_a-\delta_b}$

We can obtain the Figure 1b with assuming:

$\alpha = 0.3$, $W_1 = 1$, $W_0 = 10$, $Q_0 = 1\ldots5$, $\rho = 0.14$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$.

Case 2: Study change in θ in terms of W_0:

Assuming that Q_0, W_1, δ_a, δ_b, α and ρ are constants, and after studying the derivative signal (Appendix 3), we find that:

θ is a decreasing function for W_0, if: $\rho \succ\succ \frac{-\delta_b}{\delta_a-\delta_b}$

θ is an increasing function for W_0, if: $\rho \prec \frac{-\delta_b}{\delta_a-\delta_b}$

Simulation No. 02:

Case A: We can obtain the Figure 2a with assuming:

$\alpha = 0.3$, $W_1 = 1$, $Q_0 = 2$, $W_0 = 1\ldots90$, $\rho = 0.6$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$.

Figure 1: (a) Comparison of profit rates with interest rates in terms of invested funds in case: $\rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b}$. (b) Comparison of profit rates with interest rates in terms of invested funds in case: $\rho \prec \frac{-\delta_b}{\delta_a - \delta_b}$

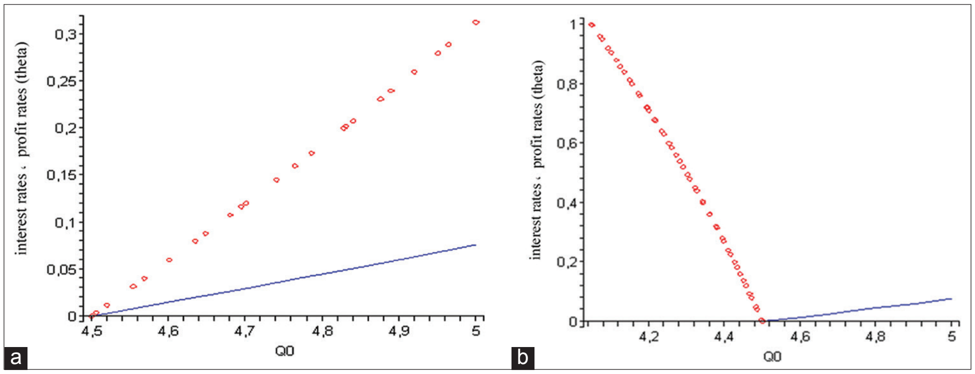
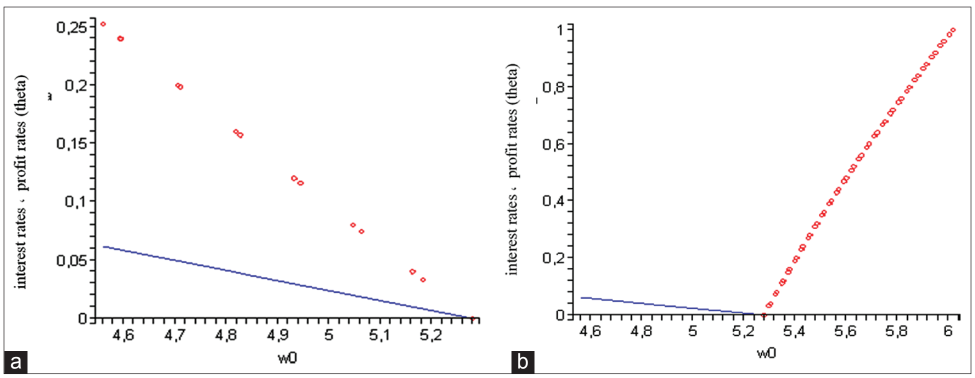


Figure 2: (a) Comparison of profit rates with interest rates in terms of primary wealth in case: $\rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b}$. (b) Comparison of profit rates with interest rates in terms of primary wealth in case: $\rho \prec \frac{-\delta_b}{\delta_a - \delta_b}$



Case B: We can obtain the Figure 2b with assuming:

$\alpha = 0.3$, $W_1 = 1$, $Q_0 = 2$, $W_0 = 1\ldots90$, $\rho = 0.14$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$

Case 3: Study change in θ in terms of β:

$Q_0 = W_0\beta$, Where $0 \prec \beta \prec 1$ and W_0, W_1, δ_a, δ_b, α and ρ are constants and after studying the derivative signal (Appendix 4), we find that:

θ is an increasing function for β, if: $\rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b}$

θ is a decreasing function for β, if: $\rho \prec \frac{-\delta_b}{\delta_a - \delta_b}$

Simulation No. 03:

Case A: We can obtain the Figure 3a with assuming:

$\alpha = 0.3$, $W_1 = 1$, $W_0 = 10$, $\beta = 0\ldots1$, $\rho = 0.6$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$

Case B: We can obtain the Figure 3b with assuming:

$\alpha = 0.3$, $W_1 = 1$, $W_0 = 10$, $\beta = 0\ldots1$, $\rho = 0.14$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$

Case 4: Study change in θ in terms of ρ:

W_0, W_1, δ_a, δ_b, α and Q_0 are constants. After studying the derivative signal (Appendix 5), we find results in simulation no.4:

Simulation No. 04:

Case A: We can obtain the Figure 4 with assuming:

$\alpha = 0.3$, $W_1 = 1$, $W_0 = 10$, $\rho = 0\ldots1$, $Q_0 = 5$, $\delta_a = 0.547$, $\delta_b = -0.2$, $\theta = 0\ldots1$

It should be noted that the obvious turning point in all curves corresponds to low-profit potential:

$$\rho = \frac{-\delta_b}{\delta_a - \delta_b} \Rightarrow 1 - \rho = \frac{\delta_a}{\delta_a - \delta_b}$$

Meaning that the maximum probability of loss = $\rho^* \frac{\delta_a}{\delta_a - \delta_b}$

This probability is minimal if:

$$\delta_a - \delta_b \succ\succ 0 \Rightarrow \delta_a \succ\succ \delta_b$$

Figure 3: (a) Comparison of profit rates with interest rates in terms of percentage of invested funds to primary wealth in case: $\rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b}$.
(b) Comparison of profit rates with interest rates in terms of percentage of invested funds to primary wealth in case: $\rho \prec \frac{-\delta_b}{\delta_a - \delta_b}$

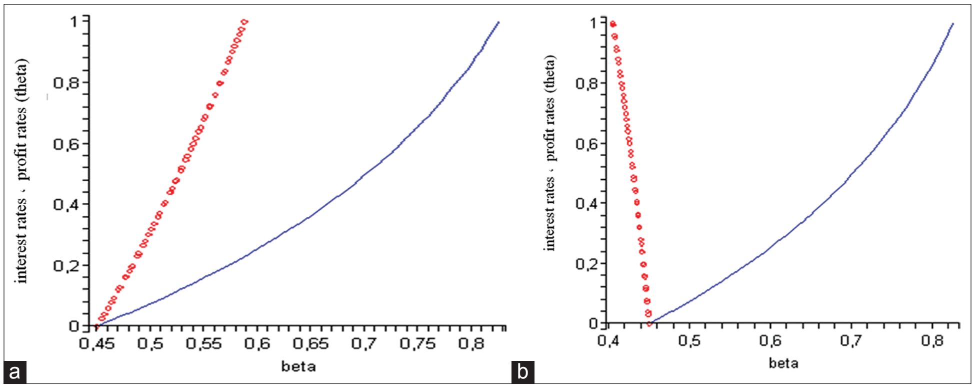
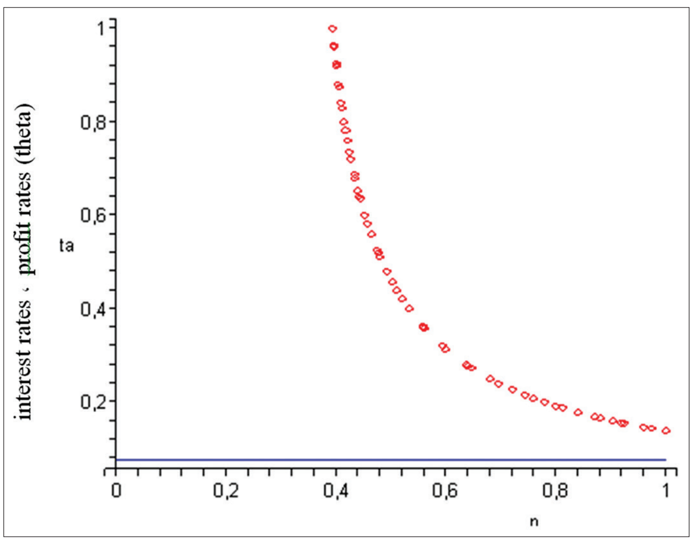


Figure 4: Comparison of profit rates with interest rates in terms of profit probability



This means that if the expected profit from the project is much higher than the expected losses, the maximum probability of loss is minimal. And this makes the Mudharaba contract more efficient as it yields greater returns and less risk.

Model results:
- On the whole, the model demonstrates the effectiveness of Islamic finance compared to interest financing. In all cases studied, the expected return from Islamic finance exceeds the return of the riba-based loan and the maximum probability of loss is minimal.
- The profit sharing parameter θ is positively proportional to the variables: Q_0, β and is inversely proportional to ρ and W_0.
- There are limits to the values of both Q_0, β, ρ and W_0. The investor should not exceed them either as a minimum or a maximum as described through simulation.
- This will help the investor to make a decision, giving him an idea of the θ values that will maximize his expected utility within his capabilities, which should be negotiated with the other party organizer.

4. STUDY RESULTS AND CONCLUSION

The previous study leads to the following results:
- The return levels of the Mudharaba contract may be greater than the return levels of the riba-based loan, which will support the ability of Islamic banks to attract more investment deposits, thereby contributing to increased levels of savings. This conclusion contradicts Priyor (1985) opinion: "… the removal of the nominal interest rate will reduce savings in the Islamic economic system, assuming the stability of other factors."
- The return on financing in the Islamic system is determined on the basis of the actual results of the project. The return is linked to production in the real sector. Thus, the correlation between the rates of return in the financial and real sectors of the Islamic system is stronger than in the interest system. This means that the cycle capacity (volatility) in different stages of the business cycle will be lower in an Islamic economy than in a capitalist economy.
- The previous observation provides self-stabilizing factors in the investment process, as demonstrated by Khan and Mirakhor (Chishti, 1985). It also proves that investment decisions are based on financing considerations (How the entity is funded), unlike the theory of the capital structure for Modigliani and Miller (1958).
- Treating losses of investment activity of Islamic banks as if they reflect an erosion of the nominal value of deposits, reflects the ability of the Islamic banking system to adapt to the shocks caused by banking crises and the disruption of the functioning of the State's payment system. When such a shock occurs, banks can absorb these shocks immediately by changing the nominal values of deposits held by the public at the bank. Therefore, the true values of the assets and liabilities of the Islamic Bank will be equal at every moment. But in the interest system, the nominal value of deposits is fixed (guaranteed). Such shocks can lead to a divergence between real assets and real liabilities. This recession in the interest system may lead to instability (Khan, 1986).
- Islamic financing using Mudharaba can be very effective in development, by helping to create innovators and finding

them, because it mates between those with money and those with ideas in the world of investment and development, and this is especially important in the economies based on surplus labor (Khan, 1986).

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APPENDIX

Appendix 1

The second partial derivatives: $\dfrac{\partial^2 L}{\partial {C_0}^2} = U''(C_0)$

$$\frac{\partial^2 L}{\partial {C_{1a}}^2} = \rho U''(C_{1a})$$

$$\frac{\partial^2 L}{\partial {C_{1b}}^2} = (1-\rho)U''(C_{1b})$$

$$\frac{\partial^2 L}{\partial {Q_0}^2} = 0$$

$$\underset{C_0,C_{1a},C_{1b},Q_0}{Hess(L)} = \begin{pmatrix} U''(C_0) & 0 & 0 & 0 \\ 0 & \rho U''(C_{1a}) & 0 & 0 \\ 0 & 0 & (1-\rho)U''(C_{1b}) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\nabla G_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \nabla G_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ (1+\theta\delta_a) \end{pmatrix}, \nabla G_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \\ (1+\theta\delta_b) \end{pmatrix}$$

$$V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix}, V \perp \nabla G_1, V \perp \nabla G_2, V \perp \nabla G_3$$

$$\begin{cases} V \perp \nabla G_1 \\ V \perp \nabla G_2 \\ V \perp \nabla G_3 \end{cases} \Rightarrow \begin{cases} \langle \mathrm{V}, \nabla \mathrm{G}_1 \rangle = -v_1 - v_4 = 0 \\ \langle \mathrm{V}, \nabla \mathrm{G}_2 \rangle = -v_2 + v_4(1+\theta\delta_a) = 0 \\ \langle \mathrm{V}, \nabla \mathrm{G}_3 \rangle = -v_3 + v_4(1+\theta\delta_b) = 0 \end{cases} \Rightarrow \begin{cases} v_1 = -v_4 \\ v_2 = v_4(1+\theta\delta_a) \\ v_3 = v_4(1+\theta\delta_b) \\ v_4 = v_4 \end{cases} \Rightarrow V = v_4 \begin{pmatrix} -1 \\ (1+\theta\delta_a) \\ (1+\theta\delta_b) \\ 1 \end{pmatrix}$$

$$\mathrm{S} = \begin{pmatrix} -1 \\ (1+\theta\delta_a) \\ (1+\theta\delta_b) \\ 1 \end{pmatrix}$$

$$\Rightarrow S^{\mathrm{T}} \underset{C_0,C_{1a},C_{1b},Q_0}{Hess(-L)}\, S = \begin{pmatrix} -1 & (1+\theta\delta_a) & (1+\theta\delta_b) & +1 \end{pmatrix} \begin{pmatrix} -U''(C_0) & 0 & 0 & 0 \\ 0 & -\rho U''(C_{1a}) & 0 & 0 \\ 0 & 0 & -(1-\rho)U''(C_{1b}) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ (1+\theta\delta_a) \\ (1+\theta\delta_b) \\ +1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & (1+\theta\delta_a) & (1+\theta\delta_b) & +1 \end{pmatrix} \begin{pmatrix} +U''(C_0) \\ -(1+\theta\delta_a)\rho U''(C_{1a}) \\ -(1+\theta\delta_b)(1-\rho)U''(C_{1b}) \\ 0 \end{pmatrix}$$

$$= -U''(C_0) - \rho(1+\theta\delta_a)^2 U''(C_{1a}) - (1-\rho)(1+\theta\delta_b)^2 U''(C_{1b}) \geq 0$$

So, function F is a super function in the points that make the constraints.

Appendix 2

Case 1: Study change in θ in terms of Q_0:

$\theta = H(Q_0)$

We study the derivative signal: $H'(Q_0)$ with assuming stability of: W_0, W_1, δ_a, δ_b, α and ρ.

$$\rho(1+H(Q_0)\delta_a)\, U'(W_1+Q_0(1+H(Q_0)\delta_a))+(1-\rho)(1+H(Q_0)\delta_b)*$$

$$U'(W_1+Q_0(1+H(Q_0)\delta_b)) - U'(W_0-Q_0)=0$$

$$T_a=(W_1+Q_0(1+H(Q_0)\delta_a))$$

$$T_b=(W_1+Q_0(1+H(Q_0)\delta_b))$$

$$\Rightarrow \rho H'(Q_0)\delta_a U'(T_a)+\rho(1+\mathrm{H}(\mathrm{Q}_0)\delta_a)\{(1+\mathrm{H}(\mathrm{Q}_0)\delta_a)+\mathrm{Q}_0\mathrm{H}'(\mathrm{Q}_0)\delta_a\}U''(T_a)$$
$$+(1-\rho)H'(Q_0)\delta_b U'(T_b)+(1-)\left(1+\mathrm{H}(\mathrm{Q}_0)\delta_b\right)\{(1+\mathrm{H}(\mathrm{Q}_0)\delta_b)+\mathrm{Q}_0\mathrm{H}'(\mathrm{Q}_0)\delta_b\}U''(T_b)$$
$$+U''\left(\mathrm{W}_0-\mathrm{Q}_0\right)=0$$

$$\Rightarrow \rho H'(Q_0)\delta_a\left\{\mathrm{U}'(\mathrm{T_a})+\mathrm{Q}_0(\,1+\mathrm{H}(\mathrm{Q}_0)\delta_a\mathrm{U}''(\mathrm{T_a})\,)\right\}+$$
$$\rho(1+H(Q_0)\delta_a)^2 * U''(T_a)+(1-\rho)H'(Q_0)\delta_b\{U'(T_b)+Q_0(1+H(Q_0)\delta_b U''(T_b)\}+(1-\rho)*$$
$$(1+H(Q_0)\delta_b)^2 U''(T_b)+U''(W_0-Q_0)=0$$

$$\mathrm{U}'(\mathrm{T_a})+\mathrm{Q}_0(\,1+\mathrm{H}(\mathrm{Q}_0)\delta_a\mathrm{U}''(\mathrm{T_a})\,)=\mathrm{U}'(\mathrm{T_a})+(\mathrm{T_a}\text{-}\mathrm{W}_1)\mathrm{U}''(\mathrm{T_a})$$

$$=T_a^{-\alpha}-\alpha(T_a-W_1)T_a^{-\alpha-1}=T_a^{-\alpha-1}(T_a-\alpha(T_a-W_1))$$

$$\mathrm{U}'(\mathrm{T_a})+\mathrm{Q}_0(\,1+\mathrm{H}(\mathrm{Q}_0)\delta_a\mathrm{U}''(\mathrm{T_a})\,)=T_a^{-\alpha-1}((1-\alpha)T_a+\alpha W_1)$$

$$U'(T_b)+\mathrm{Q}_0(\,1+\mathrm{H}(\mathrm{Q}_0)\delta_b U''(T_b)=T_b^{-\alpha-1}((1-\alpha)T_b+\alpha W_1)$$

$$\Rightarrow \rho H'(Q_0)\delta_a\{T_\alpha^{-\alpha-1}((1-\alpha)T_a+\alpha W_1\}+(1-\rho)H'(Q_0)\delta_a\left\{T_b^{-\alpha-1}((1-\alpha)T_b+\alpha W\right\}=$$
$$-\rho(1+H(Q_0)\delta_a)^2U''(T_a)-(1-\rho)(1+H(Q_0)\delta_b)^2U''(T_b)-U''(W_0-Q_0)$$

$$\Rightarrow H'(Q_0)=\frac{-\rho(1+J(Q_0)\delta_a)^2U''(T_a)-(1-\rho)(1+H(Q_0)\delta_b)^2U''(T_b)-U''(W_0-Q_0)}{\rho\delta_0\{T_\alpha^{-\alpha-1}((1-\alpha)T_a+\alpha W\}+(1-\rho)\delta_b\{T_b^{-\alpha-1}((1-\alpha)T_b+\alpha W\}}$$

We have:

$$U''(x)\prec 0$$

$$\Rightarrow -\rho(1+H(Q_0)\delta_a)^2U''(T_a)-(1-\rho)(1+H(Q_0)\delta_b)^2U''(T_b)-U''(W_0-Q_0)\succ 0$$

Let's put:

$$N=\rho\delta_a\left\{T_a^{-\alpha-1}((1-\alpha)T_a+\alpha W\right\}+(1-\rho)\delta_b\left\{T_b^{-\alpha-1}((1-\alpha)T_b+\alpha W\right\}$$

$$\delta_a\succ 0,\ \delta_b\prec 0$$

$$G(T)=T^{-\alpha-1}((1-\alpha)T+\alpha W$$

$$=(1-\alpha)T^{-\alpha}+\alpha W_1T^{-\alpha-1}$$

$$G'(T) = -\alpha(1-\alpha)T^{-\alpha-1} - \alpha(\alpha+1)W_1T^{-\alpha-2}$$

$$= -\alpha T^{-\alpha-2} - \left[(1-\alpha)T + (\alpha+1)W_1\right]$$

$$\Rightarrow G'(T) \prec 0$$

$$\begin{cases} G'(T) \prec 0 \\ \delta_b \prec \delta_a \Rightarrow T_b \le T_a \end{cases} \Rightarrow G(T_b) \ge G(T_a)$$

$$\Rightarrow \delta_b G(T_b) \le \delta_b G(T_a)$$

$$\text{N}=\rho\delta_a G(T_a) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow \text{N} \le \rho\delta_a G(T_b) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow \text{N} \le G(T_b)\left[\rho\delta_a + (1-\rho)\delta_b\right]$$

If $\rho\delta_a + (1-\rho)\delta_b \prec 0 \Rightarrow \text{N} \prec 0$

$$\Rightarrow \text{if } \rho \prec \frac{-\delta_b}{\delta_a - \delta_b} \Rightarrow \text{N} \prec 0$$

$$\Rightarrow H'(Q_0) \prec 0$$

If $\rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b} \Rightarrow \text{N} \succ 0$

So, θ is an increasing function for Q_0, if: $\rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b}$

and θ is a decreasing function for Q_0, if: $\rho \prec \frac{-\delta_b}{\delta_a - \delta_b}$

Appendix 3

Case 2: Study change in θ in terms of W_0:

Assuming that Q_0, W_1, δ_a, δ_b, α and ρ are constants.

$$\rho(1+\theta\delta_a)\, U'(W_1+Q_0(1+\theta\delta_a)) + (1-\rho)(1+\theta\delta_b)\, U'(W_1+Q_0(1+\theta\delta_b)) - U'(W_0-Q_0)=0$$

$$\rho(1+H(W_0)\delta_a)\, U'(W_1+Q_0(1+H(W_0)\delta_a)) + (1-\rho)(1+H(W_0)\delta_b)*$$
$$U'(W_1+Q_0(1+H(W_0)\delta_b)) - U'(W_0-Q_0)=0$$

$$T_a=(W_1+Q_0(1+H(W_0)\delta_a))$$

$$T_b=(W_1+Q_0(1+H(W_0)\delta_b))$$

$$\Rightarrow \rho H'(W_0)\delta_a U'(T_a) + \rho(1+H(W_0)\delta_a)Q_0 H'(W_0)\delta_a U''(T_a)$$
$$+(1-\rho)H'(W_0)\delta_b U'(T_b) + (1-\rho)(1+H(W_0)\delta_b)Q_0 H'(W_0)\delta_b U''(T_b) = U''(W_0-Q_0)$$

$$\Rightarrow H'(W_0)*\{\rho\delta_a U'(T_a) + \rho(1+H(W_0)\delta_a)Q_0\delta_a U''(T_a)$$
$$+(1-\rho)\delta_b U'(T_b) + (1-\rho)(1+H(W_0)\delta_0)Q_0\delta_b U''(T_b)\} = U''(W_0-Q_0)$$

$$\Rightarrow H'(W_0)\left\{\rho\delta_a\left[U'(T_a) + (T_a-W_1)U''(T_a)\right] + (1-\rho)\,\delta_b\left[U'(T_b) + (T_b-W_1)U''(T_b)\right]\right\} = U''(W_0-Q_0)$$

$$\Rightarrow H'(W_0)\left\{\rho\delta_a T_a^{-\alpha-1}\left[(1-\alpha)T_a + \alpha W_1\right] + (1-\rho)\,\delta_b T_b^{-\alpha-1}\left[(1-\alpha)T_b + \alpha W_1\right]\right\} = U''(W_0-Q_0)$$

$$\Rightarrow H'(W_0) = \frac{U''(W_0 - Q_0)}{\rho\delta_a T_a^{-\alpha-1}\left[(1-\alpha)T_a + \alpha W_1\right] + (1-\rho)\,\delta_b T_b^{-\alpha-1}\left[(1-\alpha)T_b + \alpha W_1\right]}$$

We have:

$$U''(W_0 - Q_0) \prec 0$$

$$N = \rho\delta_a \left\{T_a^{-\alpha-1}((1-\alpha)T_a + \alpha W\right\} + (1-\rho)\delta_b \left\{T_b^{-\alpha-1}((1-\alpha)T_b + \alpha W\right\}$$

$$\delta_a \succ 0, \delta_b \prec 0$$

$$G(T) = T_a^{-\alpha-1}((1-\alpha)T_a + \alpha W$$

$$= (1-\alpha)T^{-\alpha} + \alpha W_1 T^{-\alpha-1}$$

$$G'(T) = -\alpha(1-\alpha)T^{-\alpha-1} - \alpha(\alpha+1)W_1 T^{-\alpha-2}$$

$$= -\alpha T^{-\alpha-2} - \left[(1-\alpha)T + (\alpha+1)W_1\right]$$

$$\Rightarrow G'(T) \prec 0$$

$$\begin{cases} G'(T) \prec 0 \\ \delta_b \prec \delta_a \Rightarrow T_b \leq T_a \end{cases} \Rightarrow G(T_b) \geq G(T_a)$$

$$\Rightarrow \delta_b G(T_b) \geq \delta_b G(T_a)$$

$$N = \rho\delta_a G(T_a) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow N \leq \rho\delta_a G(T_b) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow N \leq G(T_b)\left[\rho\delta_a + (1-\rho)\delta_b\right]$$

If $\rho \prec \dfrac{-\delta_b}{\delta_a - \delta_b} \Rightarrow N \prec 0$

$$\begin{cases} U''(W_0 - Q_0) \prec 0 \\ N \prec 0 \end{cases} \Rightarrow H'(W_0) \succ 0$$

If $\rho \succ\succ \dfrac{-\delta_b}{\delta_a - \delta_b} \Rightarrow N \succ 0$

$$\begin{cases} U''(W_0 - Q_0) \prec 0 \\ N \succ 0 \end{cases} \Rightarrow H'(W_0) \prec 0$$

So, θ is an decreasing function for W_0, if: $\rho \succ\succ \dfrac{-\delta_b}{\delta_a - \delta_b}$

and θ is a increasing function for W_0, if: $\rho \prec \dfrac{-\delta_b}{\delta_a - \delta_b}$

Appendix 4

Case 3: Study change in θ in terms of β:

$Q_0 = W_{0\,\beta}$ Where $0 \prec \beta \prec 1$ and W_0, W_1, δ_a, δ_b, α and ρ are constants.

$$\rho(1+\theta\delta_a)U'(W_1+Q_0(1+\theta\delta_a))+(1-\rho)(1+\theta\delta_b)U'(W_1+Q_0(1+\theta\delta_b))-U'(W_0-Q_0)=0$$

$$\Rightarrow \rho(1+\theta\delta_a)U'(W_1 + W_0\beta(1+\theta\delta_a)) + (1-\rho)(1+\theta\delta_b)U'(W_1 + W_0\beta(1+\theta\delta_b)) - U'((1-\beta)W_0) = 0$$

$$T_a = (W_1 + W_0\beta(1+\theta\,\delta_a))$$

$$T_b = (W_1 + W_0\beta\,(1+\theta\,\delta_b))$$

$$= \rho.\frac{d\theta}{d\beta}\delta_a U'(T_a) + \rho(1+\theta\delta_a)U''(T_a)\left\{W_0\ (1+\theta\delta_a) + \beta W_0\delta_a\frac{d\theta}{d\beta}\right\} +$$

$$(1-\rho)\frac{d\theta}{d\beta}\delta_b U'(T_b) + (1-\rho)(1+\theta\delta_b)U''(T_b)\left\{W_0(1+\theta\delta_b) + \beta W_0\delta_b\frac{d\theta}{d\beta}\right\} +$$

$$= -W_0 U''((1-\beta)W_0)$$

$$\Rightarrow \rho.\frac{d\theta}{d\beta}\delta_a\{U'(T_a) + \beta W_0(1+\theta\delta_a)U''(T_a)\} + \rho W_0(1+\theta\delta_a)^2 U''(T_a) + (1-\rho)*\frac{d\theta}{d\beta}\delta_a$$

$$\left\{U'(T_b) + \beta W_0(\ 1+\ \theta\delta_b)U''(T_b)\right\} + (1-\rho)W_0(1+\theta\delta_b)^2 U''(T_b) = -W_0 U''((1-\beta)W_0)$$

$$\Rightarrow \rho\frac{d\theta}{d\beta}\delta_a\left\{U'(T_a) + (T_a - W_1\ )U''(T_a)\right\} + (1-\rho)\frac{d\theta}{d\beta}\delta_b\left\{U'(T_b) + (T_b - W_1)U''(T_b)\right\} =$$

$$-\rho W_0(1+\theta\delta_a)^2 U''(T_a) - (1-\rho)W_0(1+\theta\delta_b)^2 U''(T_b) - W_0 U''((1-\beta)W_0)$$

$$\Rightarrow \frac{d\theta}{d\beta}\left\{\rho\delta_a T_a^{-\alpha-1}\left[(1-\alpha)T_a + \alpha W_1\right] + (1\text{-}\rho\ )\ \delta_b T_b^{-\alpha-1}\left[(1-\alpha)T_b + \alpha W_1\right]\right\} =$$

$$-\rho W_0(1+\theta\delta_a)^2 U''(T_a) - (1-\rho)W_0(1-\theta\delta_b)^2 U''(T_b) - W_0 U''((1-\beta)W_0)$$

$$\Rightarrow \frac{d\theta}{d\beta} = \frac{-\ \rho W_0\ (\ 1+\ \theta\delta_a)^2 U''(T_a)\ -\ (1\text{-}\rho\ )W_0\ (\ 1+\ \theta\delta_b)^2\ -\ W_0 U''((1\text{-}\beta)W_0)}{\rho\delta_a T_a^{-\alpha-1}\left[(1-\alpha)T_a + \alpha W_1\right] + (1\text{-}\rho\ )\ \delta_b T_b^{-\alpha-1}\left[(1-\alpha)T_b + \alpha W_1\right]}$$

We know that:

$$-\rho W_0\left(1+\theta\delta_a\right)^2 U''\left(T_a\right) - \left(1-\rho\right)W_0\left(1+\theta\delta_b\right)_2 U''\left(T_b\right) - W_0 U''\left(\left(1-\beta\right)W_0\right) \succ 0$$

Let's put:

$$N = \rho\delta_a\left\{T_a^{-\alpha-1}((1-\alpha)T_a + \alpha W\right\} + (1-\rho)\delta_b\left\{T_b^{-\alpha-1}((1-\alpha)T_b + \alpha W\right\}$$

$$\delta_a \succ 0, \delta_b \prec 0$$

$$G(T) = T_a^{-\alpha-1}((1-\alpha)T_a + \alpha W$$

$$= (1-\alpha)T^{-\alpha} + \alpha W_1 T^{-\alpha-1}$$

$$G'(T) = -\alpha(1-\alpha)T^{-\alpha-1} - \alpha(\alpha+1)W_1 T^{-\alpha-2}$$

$$= -\alpha T^{-\alpha-2} - \left[\left(1-\alpha\right)T + \left(\alpha+1\right)W_1\right]$$

$$\Rightarrow G'\left(T\right) \prec 0$$

$$\begin{cases} G'\left(T\right) \prec 0 \\ \delta_b \prec \delta_a \Rightarrow T_b \le T_a \end{cases} \Rightarrow G(T_b) \ge G(T_a)$$

$$\Rightarrow \delta_b G(T_b) \ge \delta_b G(T_a)$$

$$N = \rho\delta_a G(T_a) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow N \le \rho\delta_a G(T_b) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow N \le G(T_b)\left[\rho\delta_a + (1-\rho)\delta_b\right]$$

If $\rho\delta_a + (1-\rho)\delta_b \prec 0 \Rightarrow N \prec 0$

$$\Rightarrow \text{If } \rho \prec \frac{-\delta_b}{\delta_a - \delta_b} \Rightarrow N \prec 0$$

$$\Rightarrow \frac{d\theta}{d\beta} \prec 0$$

If $\rho \succ\succ \dfrac{-\delta_b}{\delta_a - \delta_b} \Rightarrow N \succ 0$

$$\Rightarrow \frac{d\theta}{d\beta} \succ 0$$

So, θ is an increasing function for β, if: $\rho \succ\succ \dfrac{-\delta_b}{\delta_a - \delta_b}$

And θ is a decreasing function for β, if: $\rho \prec \dfrac{-\delta_b}{\delta_a - \delta_b}$

Appendix 5

Case 4: Study change in θ in terms of ρ:

W_0, W_1, δ_a, δ_b, α and Q_0 are constants.

$$\rho(1+\theta\delta_a)\, U'(W_1+Q_0(1+\theta\delta_a)) + (1-\rho)(1+\theta\,\delta_b)\, U'(W_1+Q_0(1+\theta\,\delta_b)) - U'(W_0-Q_0)=0$$

$$\Rightarrow (1+\delta_a)\, U'(W_1+Q_0(1+\delta_a)) - (1+\delta_b)\, U'(W_1+Q_0(1+\delta_b)) + \rho\frac{d\theta}{d\rho}\delta_a U'(W_1+Q_0(1+\theta\delta_a))$$

$$+\rho(1+\theta\delta_a)Q_0\delta_a\frac{d\theta}{dp}U''(W_1+Q_0(1+\theta\delta_a)) + (1-\rho)\frac{d\theta}{dp}\delta_b U'(W_1+Q_0(1+\theta\delta_b))$$

$$+(1-\rho)(1+\theta\delta_b)Q_0\delta_b\frac{d\theta}{dp} * U''(W_1+Q_0(1+\theta\delta_b)) = 0$$

$$\Rightarrow (1+\theta\delta_a)U'(T_a) - (1+\theta\delta_b)U'(T_b) + \rho\frac{d\theta}{dp}\delta_a U'(T_a) + \rho(1+\theta\delta_a)Q_0\delta_a\frac{d\theta}{dp} * U''((T_a)$$

$$+(1-\rho)\frac{d\theta}{dp}\delta_b U'(T_b) + (1-\rho)(1+\theta\delta_b)Q_0\delta_b\frac{d\theta}{dp}U''(T_b) = 0$$

$$\Rightarrow \frac{d\theta}{dp}\left\{\rho\delta_a U'(T_a) + \rho(1+\theta\delta_a)Q_0\delta_a U''(T_a) + (1-\rho)\delta_b U'(T_b) + (1-\rho)(1+\theta\delta_b)Q_0\delta_b U''(T_b)\right\}$$
$$= (1+\theta\delta_a)U'(T_a) - (1+\theta\delta_b)U'(T_b)$$

$$\Rightarrow \frac{d\theta}{dp}\left\{\rho\delta_a\left[U'(T_a) + (T_a - W_1)U''(T_a)\right] + (1-\rho)\delta_b\left[U'(T_b) + (T_b - W_1)U''(T_b)\right]\right\} =$$
$$(1+\theta\delta_a)U'(T_a) - (1+\theta\delta_b)U'(T_b)$$

The signal should be studied:

$$(1+\theta\delta_a)\, U'(T_a) - (1+\theta\delta_b)\, U'(T_b)$$

$$(1+\theta\delta_a)U'(T_a) - (1+\theta\delta_b)U'(T_b) = \frac{1}{Q_0}(T_b - W_1)U'(T_b) - \frac{1}{Q_0}(T_a - W_1)U'(T_a)$$

$$= \frac{1}{Q_0}\left[(T_b - W_1)U'(T_b) - (T_a - W_1)U'(T_a)\right]$$

$$\phi(T) = (T - W_1)U'(T)$$

$$\phi'(T) = U'(T) + (T - W_1)U''(T)$$

$$\varphi'(T) = T^{-\alpha} - \alpha(T - W_1)T^{-\alpha-1} = T^{-\alpha-1}(T - \alpha(T - W_1))$$

$$\varphi'(\mathrm{T}) = \mathrm{T}^{-\alpha-1}\left((1-\alpha)\mathrm{T} + \alpha \mathrm{W}_1\right) \succ 0$$

Function φis an increasing function.

So, the signal of $\varphi(T_b)-\varphi(T_a)$: Is the same as the signal of T_b-T_a, and is equal to the signal of $\delta_a-\delta_b$.

Since case a is the encouraging state (case of profit), with probability ρ and case b is the discouraging state (case of loss), with probability (1−ρ), so, $\delta_b - \delta_a \succ 0$.

And,

$$\frac{1}{Q_0}\left[(T_b - W_1)U'(T_b) - (T_a - W_1)U'(T_a\right] \prec 0$$

$$\mathrm{N} = \rho\delta_a\left\{T_a^{-\alpha-1}((1-\alpha)T_a + \alpha W\right\} + (1-\rho)\delta_b\left\{T_b^{-\alpha-1}((1-\alpha)T_b + \alpha W\right\}$$

$$\delta_a \succ 0, \delta_b \prec 0$$

$$\mathrm{G}\left(\mathrm{T}\right) = T_a^{-\alpha-1}((1-\alpha)T_a + \alpha W$$

$$= (1-\alpha)T^{-\alpha} + \alpha W_1 T^{-\alpha-1}$$

$$G'(T) = -\alpha(1-\alpha)T^{-\alpha-1} - \alpha(\alpha+1)W_1 T^{-\alpha-2}$$

$$= -\alpha T^{-\alpha-2} - \left[(1-\alpha)T + (\alpha+1)W_1\right]$$

$$\Rightarrow G'\left(T\right) \prec 0$$

$$\begin{cases} \mathrm{G}'(\mathrm{T}) \prec 0 \\ \delta_b \prec \delta_a \Rightarrow T_b \leq T_a \end{cases} \Rightarrow G(T_b) \geq G(T_a)$$

$$\Rightarrow \delta_b G(T_b) \geq \delta_b G(T_a)$$

$$\mathrm{N} = \rho\delta_a G(T_a) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow \mathrm{N} \leq \rho\delta_a G(T_b) + (1-\rho)\delta_b G(T_b)$$

$$\Rightarrow \mathrm{N} \leq G(T_b)\left[\rho\delta_a + (1-\rho)\delta_b\right]$$

$$\text{If } \rho\delta_a + (1-\rho)\delta_b \prec 0 \Rightarrow \prec 0$$

$$\Rightarrow \text{If } \rho \prec \frac{-\delta_b}{\delta_a - \delta_b} \Rightarrow \mathrm{N} \prec 0$$

$$\Rightarrow \frac{d\theta}{d\rho} \succ 0$$

$$\text{If } \rho \succ\succ \frac{-\delta_b}{\delta_a - \delta_b} \Rightarrow \mathrm{N} \succ 0$$

$$\Rightarrow \frac{d\theta}{d\rho} \prec 0$$