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Fractal Market Hypothesis and Markov Regime Switching Model: A Possible Synthesis and Integration

Mishelle Doorasamy[1]*, Prince Kwasi Sarpong[2]

[1]School of Accounting, Economics and Finance, University of Kwa-Zulu Natal, Westville campus, South Africa, [2]School of Accounting, Economics and Finance, University of Kwa-Zulu Natal, Westville campus, South Africa.
*Email: doorasamym@ukzn.ac.za

ABSTRACT

Peters (1994) proposed the fractal market hypothesis (FMH) as an alternative to the efficient market hypothesis (EMH), following his criticism of the EMH. In this study, we analyse whether the fractal nature of a financial market determines its riskiness and degree of persistence as measured by its Hurst exponent. To do so, we utilize the Markov Switching Model to derive a persistence index (PI) to measure the level of persistence of selected indices on the Johannesburg stock exchange (JSE) and four other international stock markets. We conclude that markets with high Hurst exponents, show stronger persistence and less risk relative to markets with lower Hurst exponents.

Keywords: Fractal Market Hypothesis, Markov Switching Model, Efficient Market Hypothesis
JEL Classifications: G150, G140

1. INTRODUCTION

Peters (1996) developed the fractal market hypothesis (FMH) as an alternative theory to the efficient market hypothesis (EMH), and seeks to explain the daily randomness of market returns and the turbulence that comes with market crashes and crises. The FMH falls within the framework of chaos theory and describes financial markets through fractal geometry. Fractals are geometric shapes that can be broken into parts and still reproduce the shape of the whole. The FMH argues that financial markets are fractals with non-linear dynamic systems which have positive feedbacks and therefore "what happened yesterday influences what happens today" (Peters, 1996. p. 9). Edward Lorenz, one of the pioneers of chaos theory describes chaos as "…when the present determines the future, but the approximate present does not approximately determine the future" (Hand 2014. p. 45). Therefore, even with the existence of positive feedback in a given time series, with the implication that its future behaviour is influenced by initial conditions with no random elements involved, small differences in these initial conditions yield widely divergent outcomes thus making long-term prediction of their behaviour generally impossible (Boeing, 2016).

In developing the FMH, Peters (1989) applied the rescaled range analysis, which is used to derive the Hurst exponent (H). The Hurst exponent measures long-term memory in time series and relates to the autocorrelations in time series. It also explains the rate at which such autocorrelations decrease when the lags increase. Harold Edwin Hurst developed this exponent in hydrology while determining the optimum dam size of river Nile for the volatile drought and rain situations observed over a long period (Hurst, 1951; Lloyd, 1966).

The Hurst exponent is also known as the "index of dependence" and quantifies the relative propensity of a time series to regress to the mean or to cluster in a direction (Parmar and Bhardwaj, 2013). It is used to measure three types of trends time series (persistence, mean reversion and randomness). According to Peters (1991), a time series with a high Hurst exponent denotes less noise and more persistence with a more distinct trend relative to a time series with a lower value. Time series with high Hurst exponents are also less risky (Peters, 1991). A Hurst exponent that falls within 0.5-1 depicts a time series with a positive long-term autocorrelation, therefore there is a high probability that a high value in the series will be followed by another high value and the values also tend to be high for longer into the future and vice versa. A Hurst exponent between 0 and 0.5 depicts a time series with long-term switching or reversion to the mean therefore a high value will probably be followed by a low value, and this tendency will persist for a long time into the future. A Hurst exponent equal to 0.5 may indicate an uncorrelated time series. It may also indicate that autocorrelations at small lags may be negative or positive, but the absolute values of the autocorrelations decay quickly to zero (Onali and Goddard, 2011).

Another non-linear model that evaluates the probability of switching is the Markov Switching Model (MSM) which assumes that the underlying process that leads to the nonlinear dynamics in a given series is latent (Chan et al., 2017). The MSM is based on the seminal work of Hamilton (1989) and allows periodic shifts in the parameters that describe the dynamics and volatility of a system. Conceptually, this model is appealing because, with time, the variable of interest, for example the time series of a market index, is deemed to possess a certain probability of abruptly switching among a number of states or regimes (for example, bull and bear market).

In this study, we apply the MSM to provide evidence on the assertion of Peters (1989) on the ability of the Hurst exponent to describe the persistence or mean reverting nature of financial time series. The similarities between the rescaled range analysis and the MSM are that they are both non-linear models and they attempt to estimate the probability of a given series switching from one state to another. The MSM provides actual estimated switching probabilities to confirm the relative propensity of a time series to regress to the mean or to cluster in a direction. Given these similarity, it should be possible to synthesize the two methods under the following hypothesis:

H_1: A time series with a higher H will exhibit a probability of remaining in the same regime as the preceding regime under the MSM.

H_2: A time series with a lower H will exhibit a higher probability of switching regimes under the MSM.

H_3: A time series with a lower H and a high probability of switching is more volatile than a time series with a higher H and a higher probability of remaining in the same regime as the preceding regime.

2. LITERATURE REVIEW

The frequency at which financial crises occur in recent decades have ignited a debate among proponents of the EMH and the FMH. On one hand, proponents of the EMH argue that financial crises are highly improbable, they are random events and consequently, do not provide any explanation for the occurrence of these crises. Critics of the EMH on the other hand argue that financial crises occur more frequently than suggested by the EMH.

The EMH is based on the assumption that (1) investors are homogeneous and have a one-period investment horizon with constant expected returns (Vasicek and McQuown, 1972), (2) investors are rational, and (3) there is no friction in financial markets. The FMH however argues that there are different investors with differing investment horizons, which range from the very short-term, such as day-traders, to the very long-term. Secondly, the FMH posits that investors interpret information differently and therefore take opposing sides of trades that occur in financial markets. The different interpretation of information and subsequent trading activities based on the differing interpretation of information is responsible for the liquidity and smooth functioning of financial markets. Finally, the FMH argues that the occurrence of financial crises is as a result of the dominance of one investment horizon which is due to investors interpreting information in the same manner. During such periods, investors in all the different investment horizons act in the same manner. For example, during political crises or a panic in financial markets, both long and short-term investors rush dispose of their assets to hold safer securities. This create the situation where even long-term investors switch to a short-term horizon. The market then becomes dominated by the short-term horizon thereby creating liquidity problems as there are fewer investors willing to take the opposite side of a trade. This liquidity dry-ups are responsible for the occurrence of financial crises.

The argument of the FMH on liquidity offers an interesting description of stability in financial markets as well as the market trends anomaly in EMH (Li et al., 2017). Kristoufek (2013) provides empirical evidence on the dominance of one investment horizon and subsequent liquidity dry-up during financial crises. The study applied the continuous wavelet transform analysis to obtain wavelet power spectra which provides the information about the distribution of variance across scales and how it evolves in time. Kristoufek (2013) concludes that short-term investment horizons dominated financial markets during the most turbulent periods during the global financial crisis confirming the assertion of the FMH. Dar et al. (2017) following Kristoufek (2013), also provide evidence that financial markets around the world display a dominance of higher frequencies in the periods of financial crises.

Li et al. (2017) created a laboratory market to study the relationship between investors heterogeneity regarding investment horizon, liquidity and the stability of the financial market using an agent-based approach based on the assertions of the FMH. Their simulation results showed that the market tend to be more stable with increasingly divergent investors who are more likely to take up the orders of the other side thereby maintaining a narrow trade gap. Their study further concludes that markets with highly heterogeneous investors are more efficient, less volatile and less prone to crash. On 15 July 2015, the financial stability board, an international body tasked with monitoring and making recommendations about the global financial system, agreed to exempt the asset management industry from regulations on systemic risk suggesting that attention rather be focused on market liquidity (Walter, 2015).

Another technique used in the FMH is the rescaled range analysis. The rescaled range analysis is used to derive the Hurst exponent also referred to as the “index of dependence” and quantifies the relative propensity of a time series to regress to the mean or to cluster in a direction (Parmar and Bhardwaj, 2013). The Hurst exponent has been used to describe the fractal nature of financial markets around the world (Ikeda, 2017). A Hurst exponent between 0 and 0.5 depicts a time series with long-term switching or reversion to the mean thus a high value will probably be followed by a low value, and this tendency will persist for a long time into the future (Onali and Goddard, 2011).

The Hurst exponent technique has been used to detect potential turning points in the stock markets. On the Dow Jones Industrial Index, Grech and Mazur (2004) investigated the crashes of 1929 and 1987 and concluded that the Hurst exponent technique can provide critical signals on impending extreme events. Czarnecki et al. (2008) and Grech and Pamuła (2008) studied the critical events of the main stock index of Poland (WIG20) and also concluded that the local Hurst exponent is an important technique for detecting impending crashes. Morales et al. (2012) extended the use of time-dependent Hurst exponent on a portfolio of stocks in the United States and concluded that the Hurst exponent values can be associated with different phases of the market.

In financial markets, the state transition processes in the form bull–bear market swings, have significant practical relevance (Wang, 2008). One model that can be used to determine the probability of a regime switch is the MSM. In econometrics, the MSM of Hamilton (1989) is one of the most important models mainly because it can allow for changes both in variance and mean, it can allow for multiple breaks and can detect outliers in time series.

If applied properly, the MSM, is able to explain and illustrate economic fluctuations around boom–recession and even other complex multi-phase cycles (Wang, 2008). The MSM has been used to analyse bull and bear markets in various financial markets (Bejaoui and Karaa, 2016; Chi et al., 2016; Yu et al., 2017; Frøystad and Johansen, 2017). Bejaoui and Karaa (2016) for example sought to provide a better understanding of the bull and bear markets with an extension of the multi-state MSM of Maheu and McCurdy (2000). The study applied a four-state-regime model defined as boom, bull, crash and bear states to define the bear and bull markets on trend-based schemes and established an indicator of market state which can detect inflexion points in a market cycle.

3. DATA AND METHODOLOGY

3.1. Data

The study obtained daily and monthly data on four indices on the Johannesburg stock exchange (JSE) namely, the FTSE/JSE All Share, FTSE/JSE Top 40, FTSE/JSE Mid Cap and FTSE/JSE small cap index from the database of McGregor BFA from 1 June 1995 to 31 August 2017. On the international markets, daily and monthly data are obtained from Yahoo Finance for the Shanghai Stock Exchange (SSE Composite) of China, the São Paulo Stock, Mercantile & Futures Exchange's IBOVESPA (Brazil), IPC Mexico and the Dow Jones Industrial Average from 1 June 1995 to 31 August 2017.

3.2. The Rescaled Range Analysis (The Hurst Exponent)

The rescaled range analysis can be applied to estimate the fractal nature of a time series. In developing the FMH, Peters (1994) applied a modified rescaled range (R/S) procedure, pioneered by Hurst (1951). Peters (1994) and Howe et al. (1997) review the steps for computing the R/S analysis. First, the selected index series are converted into logarithmic returns, S_t, at time period t of the JSE index series.

In line with Peters (1994), we divide the time period into A contiguous sub-periods with length n, such that A×n = N, where N is the length of the series N_t. We label each sub-period I_a where a = 1,2,3,…A and label each element in I_a is as $N_{k,a}$ where k = 1,2,3,…,n. The average value e_a for each I_a of length n is given as.

$$e_a = \left(\frac{1}{n}\right) \times \sum_{k=1}^{n} N_{k,a} \tag{1}$$

The range R_{I_a} is expressed as the maximum minus the minimum value $X_{k,a}$, in every sub-period I_a:

$$R_{I_a} = \max\left(X_{k,a}\right) - \min\left(X_{k,a}\right), \text{where} 1 \le k \le n, 1 \le a \le A \tag{2}$$

Where,

$$X_{k,a} = \sum_{i=1}^{k} \left(N_{i,a} - e_a\right), k=1,2,3,\ldots n, \tag{3}$$

Is the series of accumulated deviations from the mean for each sub-period. To normalise the range, each range R_{I_a} is divided by the sample standard deviation S_{I_a} corresponding to it. The standard deviation is expressed as:

$$S_{I_a} = \left[\left(\frac{1}{n}\right) \times \sum_{k=1}^{n} \left(N_{k,a} - e_a\right)^2\right]^{0.5} \tag{4}$$

The mean R/S values for length n is expressed as:

$$\left(R/S\right)_n = \left(\frac{1}{A}\right) \times \sum_{a=1}^{A} (R_{I_a} / S_{I_a}) \tag{5}$$

An OLS regression with log(n) as the independent variable and log(R/S) as the dependent variable and log(n) b. The slope coefficient of the regression represents the Hurst exponent, H. An H of 0.07 means there is a 70% probability that if the preceding move in a series was negative, then the next move will also be negative.

The autocorrelation within the time series is computed as:

$$CN = 2^{(2h-1)} - 1 \tag{6}$$

The CN represents the percentage of variations in a time series which can be explained by historical information (Peters, 1994). A CN = 0 denotes randomness in a time series and suggests a weak-form efficient market, where historical information cannot be used to outperform the market.

3.3. Fractal Dimension (FD)

The FD is a statistical measure that provides an indication of how a fractal appears to completely fill space, when one zooms in to finer scales (Rangarajan and Sant, 2004).

$$FD = 2\text{-}H \tag{7}$$

The FD can also be estimated from the Haussdorf dimension (D_H), a metric space given as

$$D_H = \lim_{\varepsilon \to 0} \frac{\ln\left[N(\varepsilon)\right]}{\ln \varepsilon} \tag{8}$$

Where N(ε) is the number of open balls of a radius ε required to cover the whole set. An open ball with radius ε and centre P in a metric space with metric d is given as a set of all points x with d(P, x) <ε.

If 1.5<FD<2, the time series is more jagged, and reverts to the mean more often than a random walk would. If 1<FD<1.5, the time series is exhibits long memory process and persistent.

3.4. Markov Regime Switching Model

One assumption under the lognormal regime-switching model, is that the process of stock return falls within one of K states or regimes. If ρ_t represents the applicable regime in the interval [t, t+1) (in days), $\rho_t = 1, 2,\ldots, K$, and S_t is the total index of return at time t; then

$$\log \frac{t+1}{} \mid \rho_t \sim N(\mu_{\rho t},\sigma_{\rho t}) \tag{9}$$

The transition matrix P represents the probabilities of shifting regimes, that is,

$$p_{ij} = \Pr[\rho_{t+1}=j \mid \rho_t=i]\ i=1,2, j=1,2 \tag{10}$$

Therefore, for the conditionally independent two-regime lognormal model, we estimate six parameters, $\Theta = \{\mu_1, \mu_2, \sigma_1, \sigma_2, p_{1,2}, p_{2,1}\}$

3.5. Maximum Likelihood Estimation

If $Y_t=\log\left(\frac{S_{t+1}}{S_t}\right)$ is the log return in month t+1, then the likelihood for observations $y = (y_1, y_2,\ldots y_n)$ is given as

$$L(\Theta)=f(y_1 \mid \Theta)f(y_2 \mid \Theta,y_1)f(y_3 \mid \Theta, y_1, y_2)\ldots f(y_n \mid \Theta, y_1,\ldots y_{n-1}) \tag{11}$$

Where f is the probability distribution function (pdf) for y. Therefore, the contribution to the log-likelihood of the t^{th} observation is given as

$$\log f(y_t \mid y_{t-1}),y_{t-2},\ldots,y_1,\Theta) \tag{12}$$

Following Hamilton and Susmel (1994), we estimate this recursively, by computing for each t:

$$f(\rho_t, \rho_{t-1}), y_t|y_{t-1},\ldots,y_1,\Theta)=p(\rho_{t-1}, \mid y_{t-1}, \ldots, y_1, \Theta) \times p(\rho_t \mid \rho_{t-1},\Theta)f(y_t|\rho_t, \Theta) \tag{13}$$

The equation on the right-hand side, $p(\rho_t|\rho_{t-1},\Theta)$ is the probability of transition between the regimes $f(y_t|\rho_t,\Theta) = \phi\left(\frac{y_t - \mu_{\rho t}}{\sigma_{\rho t}}\right)$

Where ϕ represents the standard normal probability density function, and the probability function $p(\rho_{t-1},|y_{t-1},\ldots,y_1,\Theta)$ is derived from the prior recursion and is equal to

$$(f(\rho_{t-1}, \rho_{t-2}=1,y_{t-1})|y_{t-2}, y_{t-3},\ldots,y_1,\Theta)+f(\rho_{t-1}), \rho_{t-2}=2,y_{t-1}|y_{t-2}, y_{t-3},\ldots,y_1,\Theta))\div f(y_{t-1}|y_{t-2},y_{t-3}\ldots,y_1,\Theta) \tag{14}$$

We then estimate $f(y_{t-1}|y_{t-2},y_{t-3}\ldots,y_1,\Theta)$ as the total of the four possible values of Equation (1): For $\rho_t = 1, 2$ and $\rho_{t-1} = 1, 2$.

To begin the recursion, we require a value (given Θ) for ρ (ρ_0), which can be found from the regime-switching Markov chain's invariant distribution. The invariant distribution, $\pi = (\pi_1, \pi_2)$ represents the unconditional probability distribution of the process. Every transition returns the same distribution under the invariant distribution p; that is

$\pi P=\pi$, giving $\pi_1\ p_1,\ 1+\pi_2\ p_{2,1} = \pi_1$ and $\pi_1\ p_{1,2}+\pi_2\ p_{2,2} = \pi_2$. Thus, $p_{1,1} + p_{1,2} = 1.0$, and so, $\pi_1 = p_{2,1}/(p_{1,2}+p_{2,1})$, and $\pi_2 = 1-\pi_1 = p_{1,2}/(p_{1,2}+p_{2,1})$.

We can then proceed with the recursion by estimating for a given parameter set :

$$\begin{aligned} f(\rho_1 = 1, y_1\Theta) &= E\phi\left(\frac{y_1 - \mu_1}{\sigma_1}\right), f(\rho_1=2, y_1|\Theta) \\ &= \pi_2\Theta\left(\frac{y_1 - \mu_2}{\sigma_2}\right), f(y_1| \Theta) \\ &= f(\rho_1=1, y_1| \Theta)+f(\rho_1=2, y_1| \Theta) \end{aligned} \tag{15}$$

We then estimate for use in the following recursion, the two values of

$$p(\rho_1|y_1, \Theta) = \frac{f(\rho_1,y_1|\Theta)}{f(y_1|\Theta)} \tag{16}$$

Standard search methods can be applied to maximize the likelihood function over the six parameters.

3.6. Persistence index (PI)

To rank the level of persistence, we device a simple ordinal persistence measure, a PI given as:

$$PI = p_{1,1}+p_{2,2,} \tag{17}$$

The higher the PI the higher the probability of an index remaining in a bull (bear) market given that the preceding regime was a bull (bear) market.

4. EMPIRICAL RESULTS

If the time series of the selected indices are normally distributed, then properties of normal distribution can be applied for the data analysis process. We therefore test for normality to determine whether the selected indices follow a normal distribution. Table 1a shows the descriptive statistics for selected indices on the JSE in South Africa. The Small Cap index had the lowest standard deviation among the selected indices on the JSE. This implies that the small cap index is less volatile than the all share, top 40, and mid cap indices in South Africa. Skewness is −1.2116, −0.9870, −0.9707, and −1.0713 for the all share, top 40, mid cap and small cap index respectively. Kurtosis is 9.7889, 7.9764, 11.6414 and 8.2110 for the all share, top 40, mid cap and small cap index respectively. Table 1b shows the descriptive statistics for the selected international indices. The standard deviation of the Dow Jones is lower than that of the SSE, IBOVESPA and IPC implying that the Dow Jones is less volatile than the selected international indices. Skewness is −0.1761, −1.0309, −1.0284 and −0.7847 for the SSE composite, IBOVESPA, IPC Mexico and Dow Jones index respectively. Kurtosis is 4.4027, 7.4685, 7.4716 and 4.7090 for the SSE composite, IBOVESPA, IPC Mexico and Dow Jones index respectively.

For a given time series to be deemed to follow a normal distribution, skewness and kurtosis should be equal to 0 and 3 respectively. We can therefore conclude that the time series of the selected indices do not follow normal distribution.

Our conclusion can be also confirmed again through the Lilliefors, and Anderson Darling tests. There are two hypotheses, H_0 and H_1, where H_0 states that the time series are normally distributed and H_1 states that the time series are not normally distributed. In Table 2a and b presents the results of our test of normal distribution.

Given that the concomitant probabilities for all the indices are lower than 5% for the Lilliefors and Anderson-Darling tests, we reject the H_0 at the 5% level for the selected indices on the JSE and the selected international indices.

Table 3a and b shows the results of our rescaled-range analyses. The all share, top 40 and mid cap indices have H of 0.4858, 0.4929 and 0.5096 respectively for the South African indices and the H of the SSE, IBOVESPA and IPC are 0.5281, 0.5018 and 0.4911 respectively.

The small cap index on the other hand has an H of 0.66 while the Dow Jones has an H of 0.62 signifying persistence in the time series therefore there is a high probability that a data point in future will be like a data point that preceded it. Furthermore, a CN of 0.26 for the small cap index implies 26% of variations in this index are dependent on historical information whereas CN −0.02, −0.01 and 0.01 for the all share, top 40 and mid cap indices respectively imply <3% of variations in the time series are dependent on historical information. A CN of 0.19 for the Dow Jones implies that 19% of variations on the Dow Jones are dependent on historical information. CN is <0.04 for the remaining international indices.

The FD of the South African small cap index and the Dow Jones (1.3 and 1.4 respectively) are >1 but <1.5 signifying the existence of long memory process in the series whereas the remaining indices had FD ≥1.5 but <2 signifying jagged time series and mean reversion that is more often that a random walk process would.

This study considered a two-regime market in line with Hamilton (1989). The results from our MSM from Table 4a and b are all

Table 1a: Descriptive statistics (South Africa)

Statistics	All share	Top 40	MID CAP	Small CAP
Mean±SD	0.9206±5.3811	0.8909±5.6589	0.0487±0.8900	0.9071±4.8489
Skewness	−1.2116	−0.9870	−0.9707	−1.0713
Kurtosis	9.7889	7.9764	11.6414	8.2110

SD: Standard deviation

Table 1b: Descriptive statistics (international)

Statistics	SSE composite	Ibovespa	IPC Mexico	DOW Jones
Mean±SD	0.6596±8.2165	1.0713±8.7234	1.1843±6.6304	0.6429±4.2008
Skewness	−0.1761	−1.0309	−1.0284	−0.7847
Kurtosis	4.4027	7.4685	7.4716	4.7090

SD: Standard deviation

Table 2a: Normality test (South Africa)

Test	All share		TOP 40		MID cap		Small cap	
	Value	P	Value	P	Value	P	Value	P
Lilliefors	0.0675	0.0052	0.0678	0.0049	0.0707	0.0000	0.0621	0.0149
Anderson-darling	1.4608	0.0009	1.2931	0.0023	61.596	0.0000	1.5146	0.0007

Table 2b: Normality test (international)

Test	SSE composite		Ibovespa		IPC Mexico		Dow Jones	
	Value	P	Value	P	Value	P	Value	P
Lilliefors	0.0696	0.0029	0.0738	0.0011	0.0783	0.0004	0.0712	0.0021
Anderson-darling	1.5271	0.0006	1.5631	0.0005	2.1938	0.0000	1.9938	0.0000

significant except for the SSE composite. This confirms the existence of a two-regime market on the selected indices with the exception of the Chinese market. For a possible synthesis of the FMH and the MSM, the transition probabilities for the selected indices should corroborate the results from the rescaled range analysis.

Table 3a: Rescaled range analysis (South Africa)

Statistics	All share	Top 40	Mid cap	Small cap
Hurst exponent (H)	0.4858	0.4929	0.5096	0.6638
CN	−0.0195	−0.0098	0.0134	0.2549
FD	1.5142	1.5142	1.4904	1.3362

FD: Fractal dimension

Table 3b: Rescaled range analysis (international)

Statistics	SSE composite	Ibovespa	IPC Mexico	Dow Jones
Hurst exponent (H)	0.5281	0.5018	0.4911	0.6244
CN	0.0397	0.0025	-0.0123	0.1882
FD	1.4719	1.4982	1.5089	1.3756

FD: Fractal dimension

Table 4a: Markov switching model (South Africa)

Statistics	All share	Top 40	Mid cap	Small cap
Regime 1				
Coefficient	1.055753	1.022669	1.149021	1.773104
Standard error	0.303860	0.324145	0.292845	0.375441
P	0.0005	0.0016	0.0000	0.0000
Regime 2				
Coefficient	−34.88516	−33.90635	−34.51565	−7.221562
Standard error	4.958695	5.388579	4.798005	1.624854
P	0.0000	0.0000	0.0000	0.0000

Table 4b: Markov switching model (international)

Statistics	SSE composite	Ibovespa	IPC Mexico	Dow Jones
Regime 1				
Coefficient	−0.780542	1.585074	1.492735	1.152305
Standard error	0.741134	0.493456	0.367206	0.276438
P	0.2923	0.0013	0.0000	0.0000
Regime 2				
Coefficient	6.954535	−31.00649	−28.73190	−9.574930
Standard error	1.823390	5.766273	4.801327	2.226720
P	0.0001	0.0000	0.0000	0.0000

An index with H >0.5 depicts persistence therefore it should exhibit a relatively higher probability that a high (low) value in the series will be followed by another high (low) value and this will persist longer into the future. An index with H >0.5 should therefore have a high probability of remaining in a bull (bear) market regime given that the preceding regime was a bull (bear) market. The results from Table 5a shows that the small cap index which had the highest H (0.66) also has the highest PI (1.44). Although the SSE composite showed the highest PI (1.81), the results from the MSM was not significant. Of the significant results for the international market, the Dow Jones had the highest PI (1.20) and also had the highest H (0.62).

Panel A shows the smoothed regime probabilities and the closing price of the selected indices for this study. Smoothed probabilities show the estimated probabilities of each regime occurring at each point in time.

The findings from our study confirm that a time series with a higher H will exhibit a higher probability of remaining in the same regime as the preceding regime under the MSM and therefore display a relatively high PI. Secondly, a time series with a lower H will exhibit a higher probability of switching under the MSM and display a lower PI. Finally, a time series with a lower H and a high probability of switching is riskier than a time series with a higher H and a higher probability of remaining in the same regime as the preceding regime. The MSM model therefore corroborates the FMH.

In line with the FMH, the small cap index and the Dow Jones Industrial, which has the highest H, were also the least risky among the selected indices and had the highest PI. Our results from the MSM confirms the assertion of Peters (1991) that a time series with a high Hurst exponent signifies more persistence with a more distinct trend and also less risky.

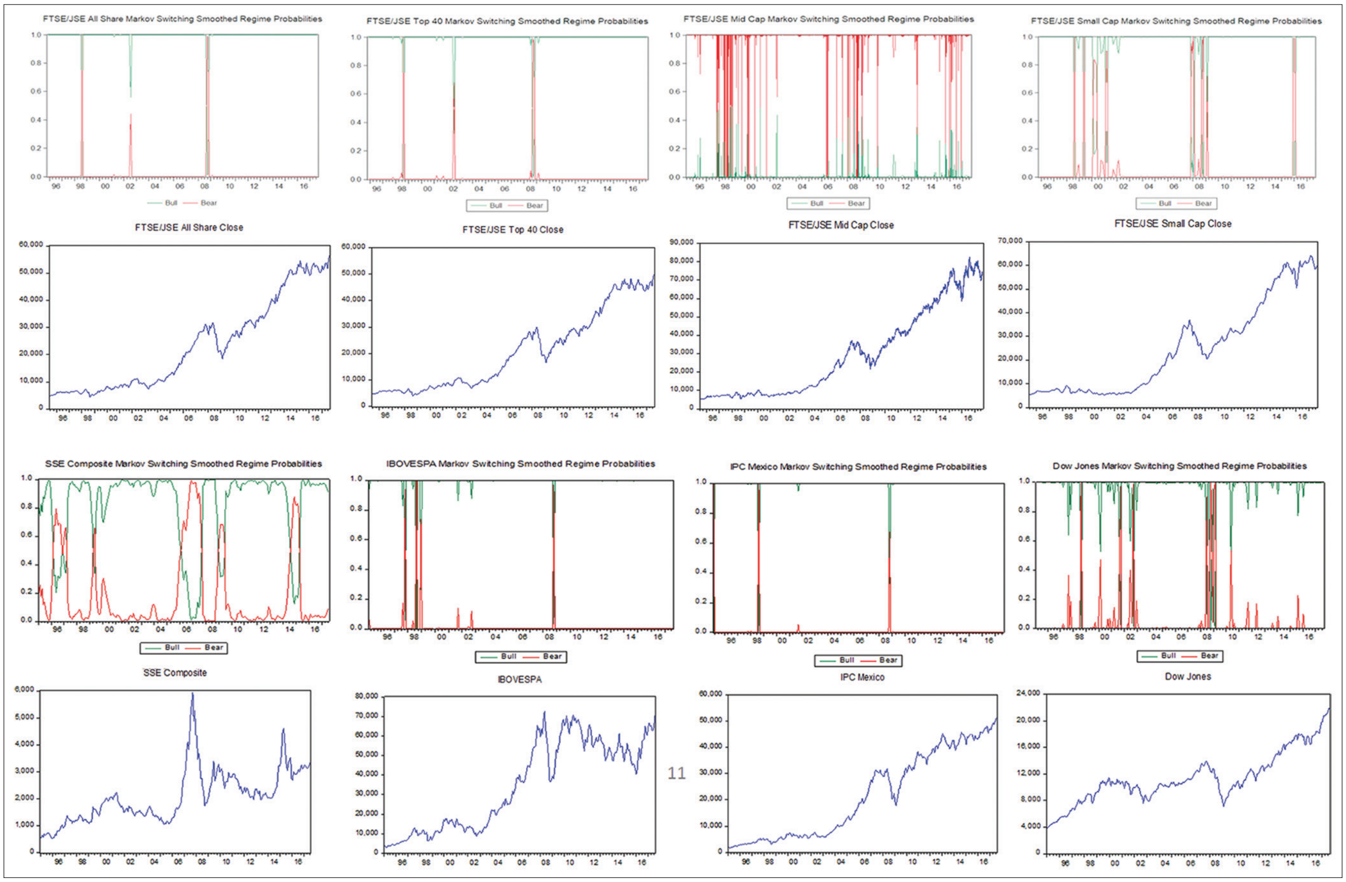
Table 5a: Transition probability (South Africa)

Regime	All share		Top 40		Mid cap		Small cap	
Constant transition probability								
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Regime 1	0.996225	0.003775	0.996215	0.003785	0.996223	0.003777	0.948469	0.051531
Regime 2	1.000000	6.32E-09	0.999880	0.000120	1.000000	1.75E-08	0.486941	0.513059
PI	0.996225006		0.996335		0.996223018		1.461528	

Table 5b: Transition probability (international)

Regime	SSE composite		Ibovespa		IPC Mexico		Dow Jones	
Constant transition probability								
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Regime 1	0.965483	0.034517	0.983985	0.016015	0.989725	0.010275	0.962412	0.037588
Regime 2	0.152142	0.847858	1.000000	5.66E-09	0.999915	8.49E-05	0.755440	0.244560
PI								
	1.813341		0.983985006		0.9898099		1.206972	

Panel A: Smoothed regime probabilities and closing price



5. CONCLUSION

Regardless of the numerous criticisms, the EMH remains the dominant hypothesis that explain financial markets because among other factors, it has a plethora of models that are built on its assumptions and provide corroborating empirical evidence to support the hypothesis. For the FMH to be considered a credible alternative to the EMH, it must be supported by new and existing models that provide empirical evidence to support the assertions of the hypothesis. In this study, we provide a synthesis of the FHM and the MSM to conclude that the small cap index in South Africa is less risky and shows more persistence than the all share, top 40 and mid cap indices. On the selected international markets, the Dow Jones is less risky and shows more persistence that the SSE of China, IBOVESPA of Brazil and IPC of Mexico. The MSM therefore corroborates FMH regarding the behavior of financial time series.

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