



## **Order Flow and Exchange Rate Dynamics in Continuous Time: New Evidence from Martingale Regression**

**Zi-Yi Guo\*\***

Corporate Model Risk Management Group, Wells Fargo Bank, N.A., 301 S College St, Charlotte, NC 28202, USA.

\*Email: [zachguo0824@gmail.com](mailto:zachguo0824@gmail.com)

### **ABSTRACT**

The so-called “foreign exchange rate determination puzzle” has been a hard topic in international finance for several decades. The puzzle illustrates the weak explanatory power of macroeconomic-based models of the nominal exchange rate fluctuations. We investigate the foreign exchange rate determination puzzle in a continuous-time framework. Following the market microstructure literature, a simple model of the determination of foreign exchange rates is developed, and the model concludes a result which is essentially a continuous-time version of the equation in Evans and Lyons (2002a). For estimation, we take an advantage of a newly-developed econometric tool based on a time change from calendar to volatility time. With this new estimation methodology, our results indicate that the effect of order flow on exchange rate is significantly improved compared with the traditional econometric tools.

**Keywords:** Order Flow, Time-change Sampling, Martingale Estimator

**JEL Classifications:** C22, G15, G17

# All views are those of the author and do not necessarily reflect those of the organization to which the author is affiliated.

### **1. INTRODUCTION**

One of the daunting problems in international macroeconomics is the weak explanatory power of existing theories of the nominal exchange rates. Since Meese and Rogoff (1983) empirically demonstrate that a random walk model performs as well as existing theories in explaining the exchange rate change in the short run, little progress has been made (Lewis, 1995, for a survey). Lyons (2001) calls the weak explanatory power of macroeconomic fundamentals as the “exchange rate determination puzzle.”

Following the pioneer work by Evans and Lyons (2002), recent empirical evidence from market microstructure approach has shown that most short-run exchange rate volatility can be explained by a new variable - order flow (Vitale, 2007 for a survey). The order flow is defined as the net of buyer-initiated and seller-initiated orders for the asset trading in the market, a measure of net buying pressure. The reason why order flow

drives the nominal exchange rates is because order flow conveys heterogeneous information, either of the future macroeconomic fundamentals or of unobserved liquidity demands, hedging demands, or speculative demands and so on. In the traditional macro approach, with homogeneous information the mapping from the information to equilibrium exchange rates is immediate, so order flow does not convey any information about the market clearing prices. While within the market microstructure framework individuals own private information, the private information is conveyed by order flow during the trading process, which in turn affects market-clearing prices.

Most empirical studies on the relationship between nominal exchange rates and order flow in the finance literature have relied on high-frequency data, such as daily, hourly and 5-min (e.g. Evans and Lyons, 2002a; 2002b; 2008; Evans, 2002; 2010; Froot and Ramadorai, 2005; Rime et al., 2010, Babbs and Guo, 2016; Guo, 2016), and the traditional econometric tools (such as ordinary least square). However, it is well known that the direct use of high

frequency data in traditional econometric estimations has several drawbacks. First, the conditional mean processes of interest in many economic and financial models are dominated by the error processes, since the error processes have higher magnitude; second, the distributions of the errors in many models are changing overtime and far from being the normal distributions. For instance, one of well-known features of the financial data is the peakedness and fat-tails phenomenon; finally, the variables in conditional mean processes might be correlated with the errors if the orthogonality condition is approximated by the Euler scheme, which creates severe identification issues. In this paper, we empirically study the impact of order flow on the nominal exchange rates by using high-frequency data in a continuous time framework. We take an advantage of a newly developed econometric methodology by Park (2010). The methodology relies on random sampling using a time change from calendar to volatility time instead of a fixed-interval sampling. The sampling chronometer runs at a rate inversely proportional to the volatility. After using this chronometer, the error processes become a standard Brownian motion and samples could be treated as being i.i.d. normal<sup>1</sup>. With this new methodology, our estimation reflects that the impact of order flow on exchange rates increases significantly compared with the traditional econometric estimation in Evans and Lyons (2002a). The normality tests of regression residuals confirm the validity of this new methodology and after time change regression residuals are normally distributed.

The paper will be organized as follow. Next section we will present a simple model of exchange rate determination in continuous time. In section three the main econometric tool will be introduced. Then we will describe our data and our estimation results. We conclude in the last section.

## 2. A SIMPLE MODEL

Evans and Lyons (2002a) define order flow as “the net of buyer-initiated and seller-initiated orders.” While each transaction involves a buyer and a seller, the initiator of the transaction determines the sign of the transaction. Different initiators (either the buyers or sellers) convey different private information, either of the expectation of future fundamentals or of the hedging trades. In the traditional macro approach, with homogeneous information the mapping from that information to equilibrium exchange rate is immediate, so order flow does not convey any information about the market clearing prices. While within the market microstructure framework, individuals possess private information, and the private information is conveyed by order flow during the trading process, which in turn affects market-clearing prices.

To formalize the idea that market-clearing prices are determined by both homogeneous information and private information, we assume individual demand  $b_{Ft}^i$  for foreign currency is linearly determined by three different components: The public information  $I_t^p$ , the private information  $I_t^i$  and the exchange rates (asset prices)  $s_t$ . Bacchetta and van Wincoop (2006) demonstrate this assumption in a discrete model, and the discretized version can

also be found in other market microstructure literature (e.g., Kyle, 1985). The assumption can be rewritten as<sup>2</sup>:

$$db_{Ft}^i = \alpha_1 dI_t^p + \alpha_2 ds_t + \alpha_3 dI_t^i + \sigma_t d\omega_t + \sigma_t^i d\omega_t^i, \tag{1}$$

Where,  $\omega_t$  denotes the volatility term which the market shares, and  $\omega_t^i$  is the idiosyncratic volatility term. Both of them are assumed to be Brownian motion and orthogonal to each other. The idiosyncratic volatility term is independent across  $i$  and cancels each other on average. The demand consists of two components: Market orders (order flow) and limit orders. In the paper, we treat the foreign exchange market as an explicit auction market<sup>3</sup>. In the market, the limit orders are on the passive side, and also provide liquidity to the market. Market orders are defined as the initiator orders and be confronted with the passive outstanding limit orders. We assume limit orders only depend on public information, while market orders exclusively depend on private information. Since market orders only depend on private information, we derive,

$$dx_t^i = \alpha_3 dI_t^i - E(\alpha_3 dI_t^i | I_t^p) + \sigma_t^i d\omega_t^i \tag{2}$$

Where  $x_t^i$  denotes individual  $i$ 's cumulative market orders until time  $t$ . We do not include  $ds_t$  in the public information set, because market orders are placed before the exchange rate is revealed. Since limit orders only depend on public information, we have market limit order  $x_t^p$  equal to,

$$dx_t^p = \alpha_1 I_t^p dt + \alpha_2 ds_t + E(\alpha_3 I_t^i dt | I_t^p) + \sigma_t dt \omega_t \tag{3}$$

Further, we assume  $E(\alpha_3 dI_t^i | dI_t^p) = \alpha_4 dI_t^p$  and define order flow as  $dx_t = \int x_t^i di$ . Combining equation (1-3) and the market clearing condition in equilibrium  $\int b_{Ft}^i di = 0$ , we have,

$$dx_t^p = -(\alpha_1 + \alpha_4) I_t^p dt - \alpha_2 ds_t - \sigma_t d\omega_t \tag{4}$$

From above analysis, we can see that if individual's foreign exchange demand is linearly determined by private information, asset prices, and public information, and at the same time each individual shares a fixed portion of public information, the foreign exchange rate will be determined as follows,

$$ds_t = -\frac{\alpha_1 + \alpha_4}{\alpha_2} I_t^p dt - \frac{1}{\alpha_2} ds_t - \frac{\sigma_t}{\alpha_2} d\omega_t \tag{5}$$

The above equation basically means that foreign exchange rate is jointly determined by public information and private information. The later one is summarized by the order flow  $x_t$ . Evans and Lyons

1 This idea has also been used by Yu and Phillips (2001), Jeong et al. (2015), Chang et al. (2016) and so on.

2 The original equation in Bacchetta and van Wincoop is  $db_{Ft}^i = \alpha_1 dI_t^p + \alpha_2 ds_t + \alpha_3 dI_t^i$ . We take difference of the equation and assume  $dI_t^p$  and  $\alpha_3 dI_t^i$  follow the geometric Brownian process and we could derive the continuous-time version of the equation as above.

3 Bacchetta and van Wincoop (2006) also make this assumption. Lyons (2001) has a detailed discussion of the nature of the foreign exchange market.

(2002a) use a portfolio shifts model and prove that exchange rate change is a linear function of the public-information increment and the unobservable portfolio shift term, which is essentially a discretized version of our result.

### 3. ESTIMATION METHODOLOGY

As common in the order flow literature (e.g., Evans and Lyons, 2002a; Rime et al., 2010), we use interest rate differentials to approximate the component of public information. The model then can be rewritten as,

$$ds_t = -\frac{\alpha_1 + \alpha_4}{\alpha_2} I_t^p dt - \frac{1}{\alpha_2} ds_t - \frac{\sigma_t}{\alpha_2} d\omega_t + \sigma_t' d\omega_t' \tag{6}$$

Where,  $r_t$  denotes the interest rate differential at time  $t$  between the home country and the foreign country.  $\sigma_t' d\omega_t'$  is the measurement error term.  $\omega_t$  is an independent Brownian motion of  $\omega_t$ . Further, the model is simplified as,

$$ds_t = -\frac{\alpha_1 + \alpha_4}{\alpha_2} I_t^p dt - \frac{1}{\alpha_2} ds_t + \sigma_{0t}' d\omega_{0t}', \text{ where } \sigma_{0t} = \sqrt{\frac{\sigma_t^2}{\alpha_2^2} + \sigma_t'^2} \tag{7}$$

$\omega_{0t}$  is a standard Brownian motion. To estimate, of course one has to reply on the discretized data. However, as well known for the direct use of high-frequency data in a continuous-time model, there are several drawbacks. First, the mean process is dominated by the contaminative volatility process, since the magnitude of the volatility term is much larger than the conditional mean term when the sampling interval is small enough; second, the distribution of errors usually is far away from being normal, such as the peakedness and fat-tail phenomena in financial data, and also highly heterogeneous across time; finally, the magnitude of the volatility term is usually correlated with the conditional mean, and we might face a serious identification issue. In the paper, we take an advantage of a new developed econometric methodology by Park (2010) and elegantly avoid these problems.

#### 3.1. Time Change Sampling

Instead of fixed sampling which might have time-varying and high-magnitude volatility term, we use a random sampling by using a time change from calendar to volatility time. The main idea is based on the so-called Dambis, Dubins-Schwarz (DDS), Revuz and Yor (1994) theorem.

**Lemma 1** (DDS theorem) suppose  $U_t$  is a continuous martingale, we define a time change, i.e., a non-decreasing collection of stopping times,  $T_t$  by,

$$T_t = \inf_{s>0} \{ [U]_s > t \} \tag{8}$$

Where  $[U]_s$  is the quadratic variation of  $U_s$  up to time  $s$ . The, we have,

$$U_{T_t} = V_t \text{ or } U_t = V_{[U]_t} \tag{9}$$

Where,  $V_t$  is the standard Brownian motion.

The DDS theorem says that essentially any martingale is a Brownian motion with differences only in their quadratic variations, and all continuous martingales become Brownian motion if their sample paths are read using a clock running at the speed set inversely to the rate of increase in their quadratic variations.

In the paper, we apply DDS theorem to our model. Thus the stop time is calculated as,

$$T_t = \inf_{s>0} \left\{ \left[ \int_0^s \sigma_{0t} d\omega_{0t} \right]_s > t \right\} \tag{10}$$

Since  $\omega_{0t}$  is a Brownian motion, we have  $\left[ \int_0^s \sigma_{0t} d\omega_{0t} \right]_s = \int_0^s \sigma_{0t}^2 dt$ . Then, our main estimation equation can be written as,

$$ds_{T_t} = \eta_1 dr_{T_t} + \eta_2 dx_{T_t} + dV_t \tag{11}$$

Where,  $\eta_1 = -(\alpha_1 + \alpha_4)/\alpha_2$ ,  $\eta_2 = -1/\alpha_2$ , and  $V_t$  is a Brownian motion. As one can see, after time change, the volatility term of our estimation becomes the standard Brownian motion.

#### 3.2. Martingale Estimator

Since we assume all the volatilities are summarized by the term  $\sigma_{0t} d\omega_t$  and  $\int_0^t (\eta_1 r_{T_t} dT_t + \eta_2 dx_{T_t})$ , is of bounded variation, we have,

$$\left[ \int_0^s \sigma_{0t} d\omega_{0t} \right]_s = [s_t]_s \tag{12}$$

To implement our estimation, suppose we have  $n$ -observations at time  $t \in \{t_0, t_1, \dots, t_n\}$ . Further,  $[s_t]_s$  is estimated according to,

$$\sum_{i=1}^n (s_{t_i} - s_{t_{i-1}})^2 \tag{13}$$

Where,  $0 = t_0 < t_1 < \dots < t_n = s$ . The time change  $T_t$  in our estimation is obtained by finding the  $t_i$  which has a minimum distance from the estimated quadratic variations to  $t$ . In this context, as long as  $\max_{i,j} |t_i - t_{j-1}| \rightarrow 0$  fast enough, we will have a consistent estimator of  $T_t$ . The reader can refer to Park (2010) for details.

For simplicity, we consider the case that fixed increment quadratic variation for time change sampling. Let  $\Delta > 0$ , equation (11) can be rewritten as,

$$z_i = \Delta^{-1/2} [s_{T_{i\Delta}} - s_{T_{(i-1)\Delta}} - \eta_1 (r_{T_{i\Delta}} - r_{T_{(i-1)\Delta}}) - \eta_2 (x_{T_{i\Delta}} - x_{T_{(i-1)\Delta}})] \tag{14}$$

For  $i = 1, \dots, n$ ,  $\{z_i\}_{i=1, \dots, n}$  are distributed as i.i.d. standard normal.  $T_{i\Delta}$  is defined as,

$$T_{i\Delta} = \arg \min_{t_k > T_{(i-1)\Delta}} \left| \sum_{j=T_{(i-1)\Delta}+1}^k (s_{t_j} - s_{t_{j-1}})^2 - \Delta \right| \tag{15}$$

To estimate the parameter  $\theta = (\eta_1, \eta_2) \in \Theta$ , we introduce the Martingale Estimator as developed by Park (2010) and define  $z_{i,d}(\theta) = (z_i(\theta), \dots, z_{i-d+1}(\theta))'$ , for  $i \geq d$ . Assuming that  $(z_i(\theta))$  is strictly stationary, we denote for each  $\theta \in \Theta$ ,  $\Pi(\cdot, \theta)$  and  $\Pi_N(\cdot, \theta)$  the joint distribution function and empirical distribution function of  $(z_{i,d})$  respectively. The empirical distribution function follows as,

$$\begin{aligned} \Pi_N(z, \theta) &= \frac{1}{N-d+1} \sum_{i=d}^N \mathbb{1}\{z_{i,d}(\theta) \leq z\} \\ &= \frac{1}{N-d+1} \sum_{i=d}^N \mathbb{1}\{z_i(\theta) \leq z_1\} \dots \mathbb{1}\{z_{i-d+1}(\theta) \leq z_d\} \end{aligned} \tag{16}$$

For  $z = (z_j) \in R^d$ . When  $\theta = \theta_0$  the model is correctly specified, then the joint distribution function follows the multivariate standard normal distribution  $\Pi(z, \theta_0) = \Phi(z_1) \dots \Phi(z_d)$ , where  $z$  is the same as in empirical distribution function and  $\Phi(\cdot)$  is the standard normal distribution.

The d-dimensional martingale estimator (MGE) of the parameter  $\theta$  is defined as,

$$\hat{\theta}_N = \arg \min_{\theta \in \Theta} \int_R [\Pi_N(z, \theta) - \Pi_N(z, \theta_0)]^2 \varpi(dz) \tag{17}$$

Where,  $\varpi$  is some weight measure, and  $N$  is the number of observations selected after the time change. With some regular conditions, we have,

$$\sqrt{N}(\hat{\theta}_N - \theta_0) \rightarrow_d N(0, \ddot{Q}(\theta_0)^{-1} P \ddot{Q}(\theta_0)^{-1}) \tag{18}$$

Where,  $P = \iint \dot{\Pi}(x, \theta_0) \Sigma(x, y) \dot{\Pi}(x, \theta_0)' \varpi(dx) \varpi(dy)$  and  $\ddot{Q}(\theta_0) = \iint \dot{\Pi}(z, \theta_0) * \dot{\Pi}(z, \theta_0)' \varpi(dz)$ . Here  $\Sigma(x, y)$  is the covariance kernel of the Gaussian process (see Park 2010 for details).

The Martingale Estimator essentially is a minimum distance estimator, which minimizes the distance between the empirical distribution of  $z_{i,d}(\theta)$  and the true distribution of  $z_{i,d}(\theta_0)$ . The main drawback of the estimator proposed above is its computational burden, since it involves numerical integration. However, here the optimization function can be readily evaluated using simple algebraic computational procedures for each  $\theta \in \Theta$  if we choose the weight  $\varpi$  appropriately. Precisely, if  $\varpi$  is the measure given by the distribution function  $\Pi(\cdot, \theta_0)$ , for a fixed  $\theta \in \Theta$  let  $(z_i)$  be the observed values of  $z_i(\theta)$  arranged in the ascending order, i.e.,  $\{z_{(1)} < \dots < z_{(N)}\}$ , and suppose  $w_i = \Phi(z_{(i)})$ , where  $\Phi$  is the standard normal distribution function. The

one-dimensional ( $d = 1$ ) Martingale estimator (MGE) of  $\theta$  can be introduced as,

$$\theta_N = \operatorname{argmin} \min_{\theta \in N} \left\{ \frac{1}{N} \sum_{i=1}^N \left( \frac{2i-1}{2N} - \omega_i \right)^2 + \frac{1}{12N^2} \right\} \tag{19}$$

Park (2010) proves its asymptotic consistency and provides other MGEs with high dimensions. For simplicity, we only consider the 1D MGE.

## 4. EMPIRICAL RESULTS

To estimate the coefficients  $\eta_1$  and  $\eta_2$ , we need three variables in each observation: One is order flow, one should measure the public information, and another one is the exchange rate. As mentioned before, we use interest rates differentials to approximate the public information. The highest frequency of interest rates we can collect is daily. We assume the interest rates do not change intraday and use the daily interest rates to approximate the instantaneous interest rates within that day. Since in our sample the Deutsche daily interest rate has only changed 6 times and the US daily interest rate has changed a little bit more, 15 times, our treatment should not have a substantial impact to the estimation results.

### 4.1. Data

The dataset of order flow is the same as in Evans (2002)<sup>4</sup>, and readers can refer to that paper for details. The original dataset contains time-stamped, tick-by-tick observations on actual transactions on the Reuters D2000-1 system for the largest spot market (DM/\$) over a 4-month period, May 1 - August 31, 1996. At that time, that system is the most widely used direct electronic dealing system. According to Reuters, over 90% of the world's direct interdealer transactions took place through the system. Although trading can be made on the system 24 hours a day, 7 days a week, the dataset excludes weekends (too few observations in the weekends) and a feed interruption caused by a power failure and has 79 full trading days in the sample and 255,497 trades. That interdealer order flow is positive (negative) is defined as a dealer initiating a bilateral conversation purchases (sells) foreign exchange at the ask (bid) quote. The data does not have information of the size of individual transactions and we use the number of transactions as a proxy variable<sup>5</sup>. Evans accumulates the order flow (in thousands) in every five-minute interval and uses the last purchase price and sale price in that interval as the purchase price and sale price for that interval. Eventually, the dataset has 13,434 observations. In this paper, we use the average of the purchase price and the sale price as the exchange rate in the corresponding time interval.

To estimate the yield curves of term structure, we collect the daily data of interest rates. We use the daily overnight interest

4 Evans and Lyons (2002c, 2008) also use this dataset.

5 Jones et al. (1994) show that trade size contains no information beyond that in the number of transactions.

rates for the dollar and the deutsche mark, and both of them are from Datastream.

### 4.2. Estimation

The theoretical estimation does not have any requirement for  $\Delta$  and the current literature also have not demonstrated the optimal value for it. However, in practice we choose the optimal  $\Delta$  based on two considerations. First,  $\Delta$  should not be too small. If it is too small, we do not have enough samples to effectively estimate the quadratic variations, and therefore the estimation of time change might have serious bias. On the other hand, if we choose  $\Delta$  in a large value, after time change sampling we could not have enough observations for our martingale estimation. Based on these two considerations, we choose  $\Delta = 0.4188$ , and the time change sampling gives us  $N = 80$  observations. We also change the value of  $\Delta$  as a robust check.

As we can see in Table 1, when  $\Delta = 0.4188$  and  $N = 80$ , with fixed-time interval sampling, the coefficient of 1.949 in the estimation equation implies that 1000 more dollar purchase than sales increases the deutsche mark price of a dollar by 1.949%, which is very consistent of the results obtained in Evans and Lyons (2002a). However, with time-change random sampling, the coefficient increases to 5.470, which not only supports the view that order flow conveys information and correlates with foreign exchange rates, but also improves the prediction by a large amount. Given an average trade size in our sample of

\$3.9 million, our MGE indicates \$1 billion of net dollar purchases increases the deutsche mark price by 1.403% ( $=5.470/3.9$ ) instead of by 0.500% ( $=1.949/3.9$ ) in traditional econometric estimation. We change the value of  $\Delta$  and  $N$ , our estimation still shows very similar results.

The reason why it is better to use time-change sampling than fixed-time sampling is because the nature of high-frequency data. Figure 1 presents the distribution of the estimated errors. The solid line is the true distribution of our estimation, and the dotted line is the normal distribution which has the same mean and variance as the true distribution. Panel A is the estimated error density sampled at fixed-time intervals, and Panel B is the estimated error density sampled at time change. The figure shows that if we sample at fixed-time intervals, the estimated error is far away from being normal, while if we sample at time change, the estimated error is very close to be normal. Our normality test supports our interpretation, as shown in Table 2. With time change sampling, we cannot reject the null hypothesis at any level. With time-change sampling, we cannot reject the null hypothesis at 1%, 5%, or 10% level.

## 5. CONCLUSION

This paper makes use of a new novel econometric methodology to estimate the correlation between foreign exchange and order flow, a key variable to explain the short-run exchange rate fluctuations. This correlation has been demonstrated in a lot of empirical literature. However, most of the literature have relied on high-frequency data, and largely ignored the short of this kind of data. To tackle this problem, we collect our samples with a time change instead of the traditional fixed-time interval approach. Our empirical results support the argument that order flow has a strong impact on foreign exchange rate, and the impact may be even stronger than conventional estimations. The key idea of this new econometric methodology is that for any continuous martingale, as long as we read it at its quadratic variation time, the process will become a Brownian motion.

The reason why order flow correlates with foreign exchange rates is because order flow conveys private information, either the private information of macroeconomic fundamentals or heterogeneous hedging demands. In our simple model, we only assume the private information is a component of the determination of foreign exchange rates, but do not provide a rationale with our assumption. Second, as well known to use continuous time model one has to rely on a long-time period data. Although our data spans a relative long period compared a lot of other empirical literature of order flow on exchange rate dynamics, it is still not long enough. Finally, we do not consider jumps in our estimation, which is another important phenomenon in high-frequency data.

Figure 1: Regression residuals with fixed-time sampling and time-change sampling

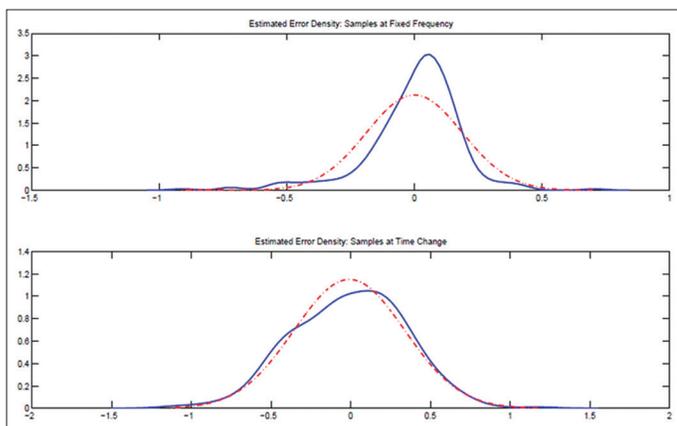


Table 1: Main results of estimation

Coefficients	N=80		N=40	
	Fixed-time	Time-change	Fixed-time	Time-change
$\eta_1$	0.224	0.258	0.127	0.080
$\eta_2$	1.9498**	5.470**	1.704**	5.325**

\*\*indicates statistical significance at 5% level

Table 2: Normality test for the estimated residuals

Sampling methods	Kolmogorov–Smirnov	Shapiro–Wilk	Cramer-von Mises	Anderson-Darling
Fixed-time	0.009	0.000	0.006	0.003
Time-change	0.434	0.125	0.354	0.398

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