International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2017, 7(2), 384-396.

Estimation of Volatility and Correlation with Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models: An Application to Moroccan Stock Markets

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ABSTRACT

Volatility and correlation are important metrics of risk evaluation for financial markets worldwide. The latter have shown that these tools are varying over time, thus, they require an appropriate estimation models to adequately capture their dynamics. Multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models were developed for this purpose and have known a great success. The purpose of this article is to examine the performance of multivariate GARCH models to estimate variance covariance matrices in application to 10 years of daily stock prices in Moroccan stock markets. The estimation is done through the most widely used multivariate GARCH models, dynamic conditional correlation (DCC) and Baba, Engle, Kraft and Kroner (BEKK) models. A comparison of estimated results is done using multiple statistical tests and with application to volatility forecast and value at risk (VaR) calculation. The results show that BEKK model performs better than DCC in modeling variance covariance matrices and that both models failed to adequately estimate VaR.

Keywords: Volatility, Correlation, Multivariate Generalized Autoregressive Conditional Heteroskedasticity, Diagonal Baba; Engle; Kraft and Kroner, Dynamic Conditional Correlation, Stock Markets, Morocco
JEL Classifications: C3, E44, G1

1. INTRODUCTION

Asset pricing, risk assessment, and portfolio management are undoubtedly the most important activities in financial institutions. Institutions are concerned by measuring and forecasting changes in their portfolio values according to different market conditions. The tool that practitioners and researchers use to quantify these changes is volatility. Volatility is defined as variation in an asset prices over a period of time. Correlation is also an important tool that measures co-movements between multiple assets.

Financial markets worldwide have shown that volatility is varying over time. A number of researchers studied this phenomenon, for example: Officer (1973) showed that volatility changes are related macroeconomic variables, Black (1976) and Christie (1982) argued that financial leverage partly explains it, Merton (1980) among others[1] related macroenomic volatility to interest rates.

Volatility is not observable and then should be estimated. In 1982, Engle introduced the autoregressive conditional heteroskedasticity (ARCH) model. The AR part is due to the fact that these models are autoregressive models in squared returns. The conditional part comes from the fact that next period volatility is conditional on information set until this period. Heteroscedasticity means non constant volatility. As there were no other methods available before, Engle's ARCH model contributed significantly to financial econometrics by allowing conditional variances to change over time. The primary descriptive tool was the rolling standard deviation.

In 1986, Bollerslev developed a generalization of the ARCH model. He built a more general concept of Engle's ARCH model by including q lags of the conditional variance (denoted generalized ARCH [GARCH] [P,Q]). Later on, many extensions were added to the ARCH family models, Nelson (1991) and his exponential GARCH (EGARCH) model (P,O,Q) which models the natural

1 Pindyck (1984), Bollerslev et al. (1988).

logarithm of the variance instead of modeling the variance directly, Runkle (1993) and his GJR-GARCH model which is an extension of the standard GARCH(P,Q), it includes asymmetric terms in the conditional variance of equities. Other models also exist in literature, we cite TARCH model (Rabemananjara and Zakoian [1993]) and APARCH of Ding and Granger (1996).

Although Univariate GARCH family models were a great invention in the financial econometrics, they are unable to estimate correlations when multiple assets are involved. For example, the main problem in portfolio selection, which is defining the optimal weights of the underlying asset returns, depends on correlations estimation. Another example is pricing of derivatives contracts based on multiple underlying assets that requires correlations estimation. Actually, it is commonly accepted that assets volatilities move together, consequently, it is important to capture adequately the interdependence's between different assets in a portfolio. Multivariate GARCH models were developed to respond to these needs. However, these models face many challenges to adequately capture variances and covariance's dynamics.

From one hand, the number of parameters of a model may explode when the number of assets increases, this causes complexities problems in computation and interpretation of parameters, also, reducing the number of parameters should not impact the quality of the model in capturing variances and covariance's dynamics. From the other hand, and as stated in Merton (1972), Roll (1977), and Jobson and Korkie (1989), among others, multivariate GARCH models should insure the positive-definiteness of the resulted variance covariance's matrix. This requirement may appear to be satisfied, as by definition, the variance of random variables is positive. But, there are many possibilities were covariance matrix can be non-positive definite, for example in case assets exhibit non constant volatility or in case of the use of insufficient observation in estimation. For the positive definiteness can be achieved in general in two ways. The first one is constructing a model in a way that the resulted matrix is definite positive, the second one is imposing condition in a way that the computed variance covariance matrix is positive definite, the second alternative is hard to apply in practice.

Over the past two decades, a considerable literature has been developed around multivariate GARCH models. The first one was introduced by Bollerslev et al. (1988), the VEC model. It had the advantage that its coefficients can be directly interpreted. But this model is very general and difficult to implement in practice, it does not guarantee the positive definiteness of the variance covariance's matrix, also it suffers from high dimensionality of parameters to estimate. Bollerslev et al. (1988) proposed diagonal VEC model which is a simplified version of the VEC model. It reduced the number of parameters to estimate by assuming that model coefficients are diagonal matrices, it guarantees also the positive definiteness of the variance covariance's matrix, but because of element of the covariance matrix depends on past observations of itself, it does not capture interactions between different variances and covariance's.

Engle and Kroner (1995) proposed the Baba, Engle, Kraft and Kroner (BEKK)[2] model which can be viewed as a restricted version of the VEC model. It has the advantage that variance-covariance matrix is definite positive by construction, but the interpretation of model parameters is not easy, also, like the VEC model, it suffers from high dimensionality problem. Many other simplifications of the full BEKK model were added to the literature, we cite the Diagonal and Scalar BEKK models. These two models reduced the number of parameters to estimate in comparison to the VEC model.

To overcome the problem of high number of parameters to estimate in the previously cited models, Engle et al. (1990) introduced another approach of MGARCH models. These models are called factor GARCH models. It is assumed that the observations are generated by underlying factors that are conditionally heteroskedastic and possess a GARCH-type structure. The number of factors to be modeled can be much smaller than the number of assets which make these models feasible even for high number of assets. Alexander (2001) developed the orthogonal-GARCH model. This model is computationally simple. It is based on only the first few components of the entire risk factors. This model reduced the number of parameters to estimate but like the BEKK model, parameters are difficult to interpret.

Another approach in MGARCH is modeling the conditional variances and correlations instead of modeling the conditional covariance matrix directly. The conditional covariance matrix is decomposed into conditional standard deviations and a correlation matrix. Bollerslev (1990) introduced the first category of these models, namely the constant conditional correlation (CCC) model. Conditional correlation matrix is assumed to be time invariant in this model which makes it computationally attractive. It insures the positive definiteness of the variance-covariance matrix and reduces the number of parameters to estimate. But the assumption that conditional correlation matrix is constant is unrealistic in reality.

Engle (2002) suggested a generalization of the CCC model by letting the conditional correlation matrix to be time varying. This model is called dynamic conditional correlation model (DCC). There has been a growing interest to this model since its introduction and it has a central role in dynamic correlation modeling. Firstly, the number of parameters to estimate is smaller comparing to BEKK model, also, it decreases the complexity of the computation in case of a high number of assets, because the estimation is done into two steps. Also, one main advantage of this model is that it guarantees the positive definiteness of variance covariance matrix at each point in time.

The purpose of this article is to empirically study, based on the theory of multivariate GARCH models, co movements between assets in Moroccan stock markets. We selected stocks that have the longest and continuous available historical prices. We choose also two of the most widely used multivariate GARCH models, diagonal BEKK and DCC. Models order selection is done through the appropriate statistical tests. Once the models are estimated, we study the quality of the estimation and we use the obtained results in calculating the value at risk (VaR) and volatility forecast. This will allow us to compare between the estimated models in capturing variance covariance dynamics.

2 BEKK named after model authors: Baba, Engle, Kraft, and Kroner.

This paper is organized as follows; next section presents a theoretical framework of Univariate GARCH, diagonal BEKK and DCC models, the third section presents data used and the empirical results before concluding in the last section.

2. THEORETICAL FRAMEWORK

2.1. Univariate GARCH Models

Define σ_t as the standard deviation, S_t the price and r_t the excess return of an asset on day t. (ϵ_t) is an i.i.d sequence. The excess return on t is defined as:

$$r = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

Engle (1982) introduced the ARCH(p) model. The variance is estimated based on the past p observations, less weights is given to older observations. The model equation is established as below:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \ r_{t-i}^2 \tag{2}$$

The simplest ARCH model is ARCH(1). Equation becomes:

$$\sigma_t^2 = \omega + \alpha_1 r_{t-1}^2 \tag{3}$$

To insure that σ is positive, ω and α should both be >0. Also, for stationary purposes, α should be <1. In this model, large values of r_{t-1} imply high values of σ_t. Large values of r_{t-1} mean high volatility at t−1. Consequently, in an ARCH (1) model, high volatility cluster may cause more volatility clustering. Bollesrev (1986) suggested a generalization of the ARCH(P) model that features this behavior, the GARCH(P,Q) model, the current level of volatility depends not only of P lags of excess returns, but also from Q lags of variances. The equation of GARCH(P,Q) is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{4}$$

$\alpha_1, \alpha_2, \ldots, \alpha_p, \beta_1 \ldots, \beta_q$ are positive, ω > 0 and: $\sum_{i=1}^{\max(p,q)} \alpha_i \beta_i < 1$ for stationary.

GARCH model lags p and q can be selected using the Akaike information criterion (AIC) and Bayesian information criterion (BIC). AIC and BIC allow us to choose GARCH model that best fit our data. AIC and BIC formulas will be detailed later in this article.

GARCH coefficients can be estimated using the maximum likelihood. Another way to check the goodness of fit of a GARCH model is checking the autocorrelation of standardized residuals using the autocorrelation function (ACF). If the ACF shows a little autocorrelation of the standardized residuals, then the model is well fitted. This means that most of the autocorrelation was explained by the model.

2.2. Multivariate GARCH

When we move from a scalar r to a d dimensional one, the variance σ become a d × d covariance matrix process ∑. The multivariate GARCH model defines r_t as:

$$r_t = H_t^{\frac{1}{2}} \eta_t \tag{5}$$

Where, r_t is a N × 1 vector of excess returns of N assets with $E(r_t) = 0$ and η_t is an i.i.d vector error with $E=[\eta_t \eta_t^{'}]$. H_t is the conditional covariance matrix of r_t (N × N matrix).

The objective of multivariate GARCH process is to estimate Ht. Many approaches have been developed to model it; we will use two of them in the present article. The first approach estimates the covariance matrix directly, it includes VEC and BEKK models. The second one models the conditional variances and correlations instead of directly modeling H_t, this class of models includes CCC and DCC.

Bollerslev et al. (1988) introduced the VEC (P,Q) model, it is a generalization of the Univariate GARCH model, its equation is defined as below:

$$\text{vech}\left(H_t\right) = c + \sum_{j=1}^{p} A_j \text{vech}\left(\eta_{t-j} \ \eta_{t-j}^{'}\right) + \sum_{j=1}^{q} B_j \ \text{vech}\left(H_{t-j}\right) \tag{6}$$

Where, vech(.) (Vech is an abbreviation for vector-half) is an operator that creates a column vector whose elements are the stacked columns of the lower triangular elements of matrix, c is an N(N + 1)/2 × 1 vector and A_j and B_j are N(N + 1)/2 × N(N + 1)/2 matrices of parameters. But the VEC model presents many disadvantages, for example it does not guaranty the positive definiteness of H_t, also, the number of parameters to estimate is very large, it is equal to: $(p + q)(N(N + 1)/2)^2+(N + 1)/2$. Bollerslev et al. (1988) presented the diagonal VEC model, it is a simplified version of the initial VEC model that guaranty the positive definiteness of H_t with (p + q + 1)N(N + 1)/2 parameters. It assumes that A_j and B_j are diagonal matrices. But this version of the VEC model is still computationally demanding.

Engle and Kroner (1995) introduced a restricted version of the VEC model, it is called BEKK model. This model equation is written as below:

$$H_t = CC^{'} + \sum_{j=1}^{p}\sum_{k=1}^{K} A_{kj}^{'} \eta_{t-j} \eta_{t-j}^{'} A_{kj} + \sum_{j=1}^{q}\sum_{k=1}^{K} B_{kj}^{'} H_{t-j} B_{kj} \tag{7}$$

Where, A_{kj}, B_{kj} are N × N parameter matrices and C is N × N lower triangular matrix. This model economizes the number of parameters to estimate and allow cross dynamics of conditional covariance's. If we set B = AD, where D is a diagonal matrix, equation 7 can be interpreted as modeling conditional variances and covariance's. The diagonal BEKK model is a simplified version of BEKK where matrices A and B are considered as diagonal. Equation B = AD is satisfied in this model. BEKK model insure the positive definiteness of H_t.

The second approach that we are interested at in the present article are the ones that models conditional covariance's matrix indirectly, it has the form bellow:

$$H_t = D_t \Gamma_t D_t \tag{8}$$

Where, $D_t = \mathrm{diag}(h_1 t^{(1/2)},...,h_N t^{(1/2)})$ is diagonal matrix of conditional variances and Γ_t is the conditional correlation matrix of r_t, $\Gamma_t = (\rho_{ij,t})$ is positive, definite and symmetric with $\rho_{ii} = 1\ \forall$ i, each h_{jt} is modeled by an Univariate GARCH process.

Bollerslev (1990) proposed the CCC model where Γ_t is assumed to be constant. This model simplifies the estimation. However, assuming that Γ_t is constant is not realistic in practice.

Engle (2002) introduced the DCC model by letting Γ_t to be time varying. It is defined as:

$$\Gamma_t = \mathrm{diag}\left(Q_t^{-\frac{1}{2}}\right) Q_t \ \mathrm{diag}\left(Q_t^{-\frac{1}{2}}\right) \tag{9}$$

Where, $Q_t = (1\text{-}\alpha\text{-}\ \beta)\ \bar{Q} + \alpha\ u_{t-1}\ u_{t-1} + \beta\ Q_{t-1}$ is $N \times N$ symmetric positive definite matrix. u_t is the standardized residuals matrix. $\bar{Q}$ is the $N \times N$ unconditional variance matrix of u_t and α and β are non-negative scalar parameters with α and $\beta < 1$.

3. EMPIRICAL RESULTS

3.1. Data Description

Data we employed in this study are daily observations on stock returns. We used daily data to ensure large number of observations to adequately fit models. We choose stocks that have the longest and continuous available historical prices in Casablanca Stock exchange market. Data series start from date 22/02/2004 to 20/10/2015 providing a total of 2749 observations for each series. Six equities have been selected, they are labeled: IAM, CTM, LESU, MNG, SOND and HOL. Our data were retrieved from Thomson Reuters databases and all calculations were done using Matlab software. Table 1 presents descriptive statistics of our series.

Figure 1 plots daily returns. It can be seen from this figure that our series volatilities don't keep constant over time, also high returns tend to be followed by high returns and low returns tend to be close with low returns, this is the volatility clustering property.

Kurtosis[3] calculated for our series presented in Table 1 are >3, indicating that they have fat tailed distributions. The skewness[4] are different from zero meaning that they are asymmetric distributions. The computed statistics of Jarque-Berra normality test[5] are all greater that the critical value (in our case it is = 5.9716) which rejects the hypothesis of normality of our daily returns.

3 Kurtosis gives a measure of the thickness in the tails of a probability density function. For a normal distribution the kurtosis is 3. A fat-tailed or thick-tailed distribution has a value for kurtosis that exceeds 3. This is called leptokurtosis.

4 Skewness gives a measure of how symmetric the observations are about the mean. For a normal distribution the skewness is 0. A distribution skewed to the right has positive skewness and a distribution skewed to the left has negative skewness.

5 Jarque-Berra is a test of normality, we test the null hypothesis: H0: Normal distribution, skewness is zero and excess kurtosis is zero; against the alternative hypothesis: H1: Non-normal distribution.

3.2. Model Order Selection

For the order selection of our models, we use the AIC and the BIC[6] values calculated in Table 2. Model with the lowest values is preferred, the formulas of these two criteria are:

$$AIC = -2 \times LLF + 2 \times m$$

$$BIC = -2 \times LLF + 2 \times \ln(n) \tag{10}$$

Where, LLF is the log likelihood function, m is the number of parameters estimated in the model and n is the number of observations.

As indicated in Table 2, models BEKK(2,1) and DCC(1,1) are the best that suite for our data. The stars (*) in the tables indicates the lowest values, both criteria's AIC and BIC confirmed our models orders choice.

3.3. Multivariate GARCH Models Estimation and Diagnostic

MGARCH models estimation is done using the maximum likelihood method. The estimated parameters A, B and C[7] of diagonal BEKK model as illustrated in equation 7 are presented in Appendix A.

We estimate also DCC (1,1) model parameters. But we perform first test of dynamic correlation[8] to confirm that DCC model is preferred than CCC. P-value obtained from this test is 4.2770×10^{-11} <0.05, which rejects the null hypothesis that the correlation is constant. Therefore, DCC is more suitable than CCC for our series. Appendix B presents DCC (1,1) estimated parameters for our data sample. We note that all coefficients estimated for DCC (1,1) are statistically significant, but not all for the BEKK (2,1). Matrices A and B and diagonal coefficients of matrix C are all statistically significant. Other non-diagonal element of matrix C are not statistically significant. Estimated correlations from the two models are plotted in Appendix C.

As mentioned above, models are estimated using maximum likelihood techniques which assume that residuals have normal distributions. Thus, to check the fitted model robustness, we check if residuals are white noise process. Figures in Appendix D plots the standardized residuals from the estimated models, they are calculated using the formula: $X_t = H_t^{\frac{1}{2}}$. The first impression we get from standardized residuals figures is that they are white noise.

We perform also Ljung-Box Q-test for residuals autocorrelation to examine if the standard residuals are random and independent over time. Table 3 presents results of the Ljung-Box Q-test for residuals autocorrelation. Test statistics results reject the null

6 Burnham, K.P., Anderson, D.R. (2004), Multimodel inference: Understanding AIC and BIC in model selection. Sociological Methods and Research, 33, 261-304.

7 M = CC.

8 Engle and Sheppard, 2001.

Figure 1: Daily returns series plot

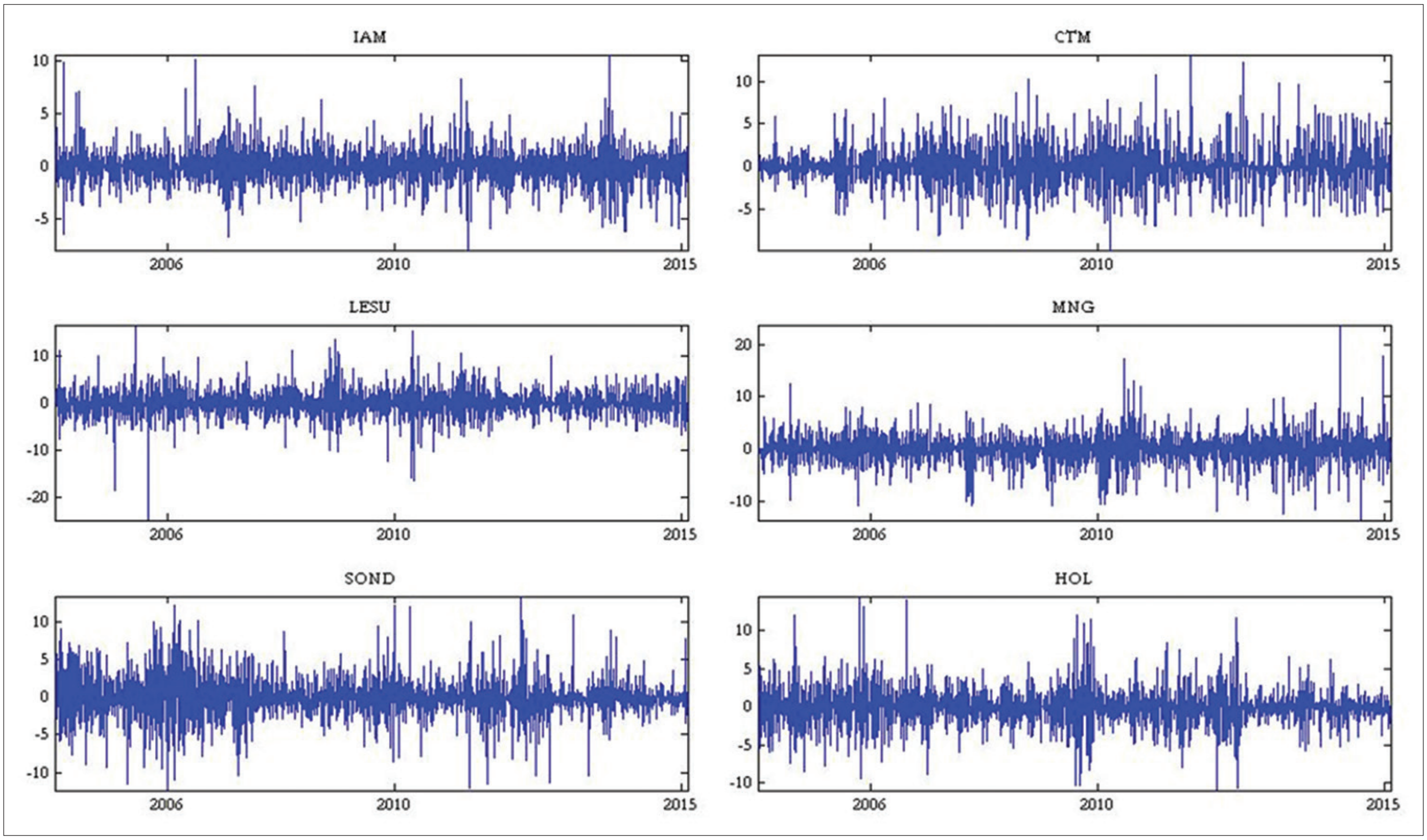


Table 1: Descriptive statistics of daily returns

Statistic	IAM	LESU	CTM	MNG	SOND	HOL
Mean (×10−17)	−1214.7	35.12	12.48	16.51	−6.18	−29.56
Standard error	0.0298	0.0411	0.0503	0.0524	0.049	0.0428
Median	0.0088	−0.0091	0.0165	0.0327	−0.01	0.0345
Mode	0.0088	−0.0091	0.0165	0.0327	−0.01	0.0345
Standard deviation	1.564	2.1542	2.6378	2.7455	2.5669	2.2451
Sample variance	2.4462	4.6405	6.958	7.5377	6.5888	5.0403
Kurtosis	4.7566	3.4628	7.5508	5.7903	3.6587	4.8238
Skewness	0.3226	0.3325	−0.2358	0.1531	0.1256	0.3277
Minimum	−8.0061	−9.7679	−24.8296	−13.6192	−12.4004	−10.9522
Maximum	10.6383	13.0937	16.5225	23.7386	13.4513	14.5111
JB statistic (×103)	2.6264	1.4168	6.5262	3.8328	1.5325	2.7013

Table 2: Model order selection

Model	AIC	BIC
BEKK		
BEKK (1,1)	−80251.31	−80056.00
BEKK (1,2)	−80225.91	−79995.09
BEKK (2,1)	−80448.27*	−80217.44*
BEKK (2,2)	−80294.76	−80028.42
DCC		
DCC (1,1)	−81175.41*	−80968.26*
DCC (1,2)	−81173.41	−80960.34
DCC (2,1)	−81173.87	−80960.79
DCC (2,2)	−81171.86	−80952.87

DCC: Dynamic conditional correlation, BEKK: Baba, Engle, Kraft and Kroner, AIC: Akaike information criterion, BIC: Bayesian information criterion

hypothesis that there is no autocorrelation. They are all greater than the critical. This rejects the conclusion we get from analyzing standardized residuals plots.

Another examination of the adequacy of our model estimation in capturing volatility and correlations dynamics is comparing the estimated volatility to the true volatility. While volatility is not observable, we need to choose an approximation to it. The most common proxies of squared volatility are the squared returns and realized volatility. Because realized volatility is based on intraday data that are not available for our assets, we will use in our comparison squared returns as a proxy for volatility. We compute volatilities for an equally weighted portfolio of ours assets, they are plotted in Figure 2. We notice that the estimated volatilities follow squared returns tendency.

4. MODELS COMPARISON

In addition of diagnostics done in the previous paragraph, we are going to use mean absolute error (MAE) of estimated and

Figure 2: Dynamic conditional correlation and Baba, Engle, Kraft and Kroner estimated volatility versus empirical volatility portfolio

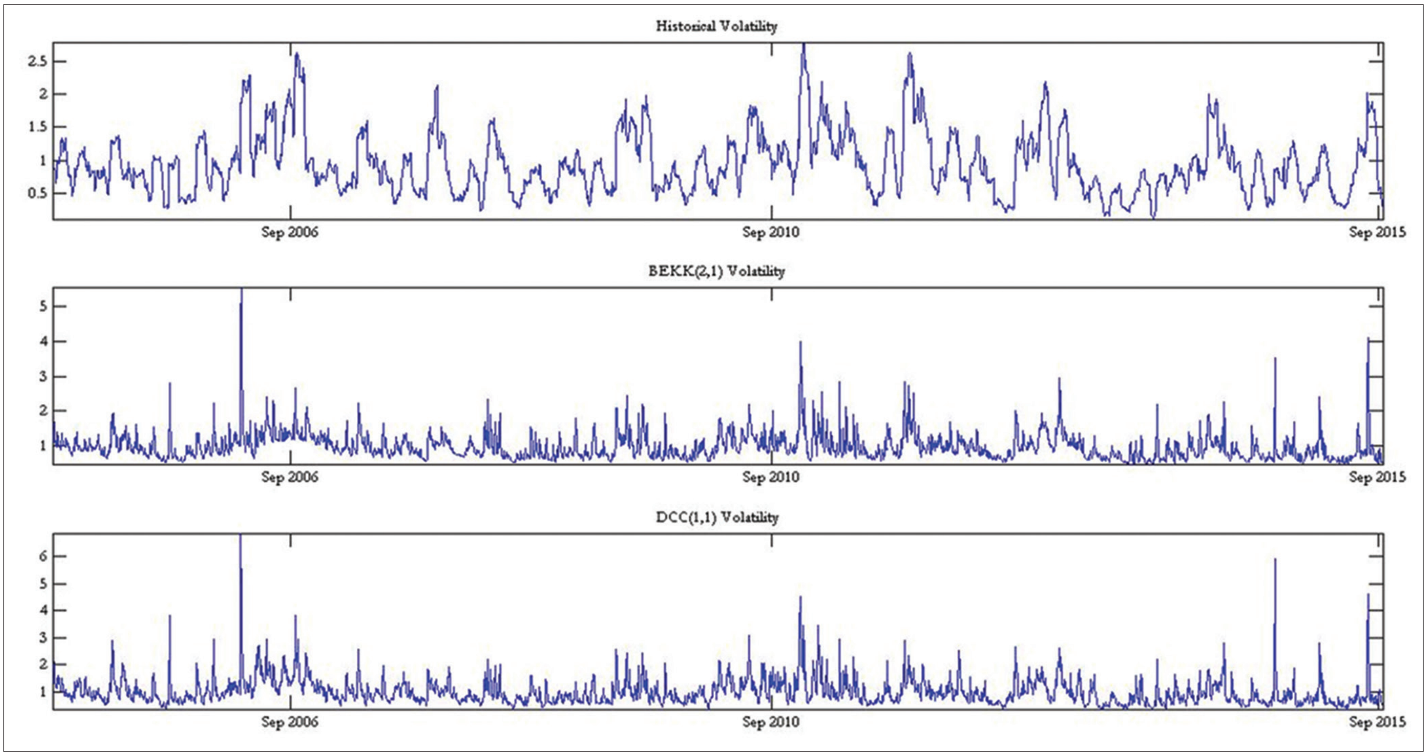


Table 3: T-statistic Ljung-Box Q-test for of autocorrelation of standardized residuals

Model	Lag	IAM	LESU	CTM	MNG	SOND	HOL	Critical value
DCC (1,1)	5	71.38	82.97	135.48	36.33	34.26	73.74	11.07
DCC (1,1)	10	80.27	91.82	138.07	40.89	39.89	80.24	18.31
DCC (1,1)	15	82.72	95.84	139.12	42.81	45.87	85.13	25
DCC (1,1)	20	85.84	106.21	144.24	45.15	48.1	96.46	31.41
BEKK (2,1)	5	66.93	85.8	148.46	25.98	27.93	70.62	11.07
BEKK (2,1)	10	74.91	95.04	151.1	31.08	33.03	77.47	18.31
BEKK (2,1)	15	77.33	97.46	152.07	32.37	38.64	81.9	25
BEKK (2,1)	20	79.9	108.53	158.19	34.74	41.23	94.9	31.41

DCC: Dynamic conditional correlation, BEKK: Baba, Engle, Kraft and Kroner

Table 4: Models mean absolute errors model

MAE diagonal BEKK (×10−2)	2.7842
DCC (×10−2)	3.3567

DCC: Dynamic conditional correlation, BEKK: Baba, Engle, Kraft and Kroner, MAE: Mean absolute error

squared returns (volatility proxy) to compare models, the MAE is computed as bellow:

$$\text{MAE} = \frac{1}{N}\sum_{k=1}^{N}\left|\sigma_k - \widehat{\sigma_k}\right| \quad (11)$$

Where, N is the number of observations, $\hat{\sigma}$ is empirical (squared returns) and σ is estimated volatility at day k. Table 4 presents the MAE of each model:

MAE measures how close the estimated volatility from the empirical volatility. According to calculated values for both models, BEKK model performs better than DCC. This result is in accordance with the one we get when testing autocorrelation of standardized residuals of each model. The Ljung-Box Q-test statistics of BEKK(2,1) is less than DCC(1,1) in almost all cases. This is an indication that the part of autocorrelation explained by BEKK(2,1) model is more important than the one explained by DCC(1,1).

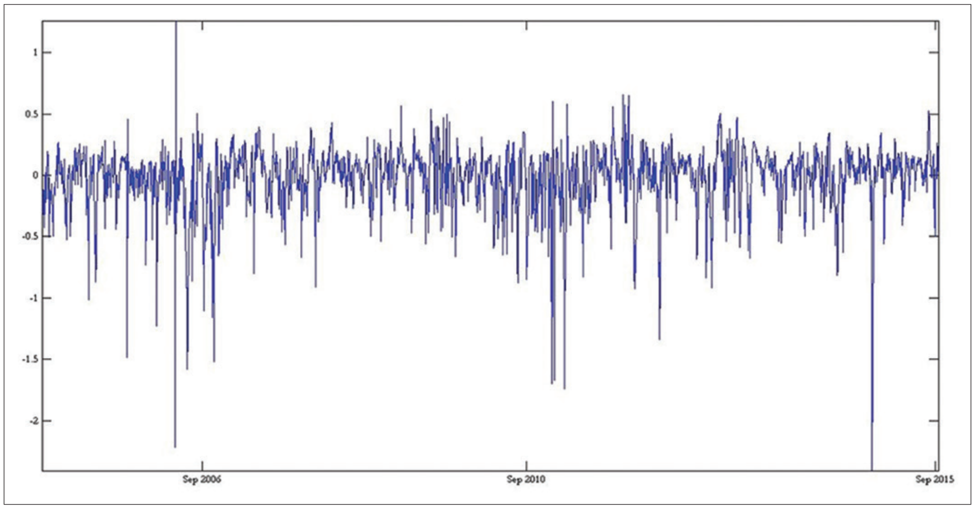
Another way to compare our models is checking the difference between the estimated volatilities. Figure 3 displays these differences. The calculated differences are small with a mean equal to 3.78×10^{-6}. The positive value of the mean indicates that volatility estimated by DCC model is higher than BEKK model[9].

To check if the two models well capture volatility clustering, we perform ARCH LM test[10]. Appendix E Table presents results of this test before and after estimation. Table E1 presents test results before the estimation. They show a high presence of volatility clustering in our series. Table E1 contains results of ARCH LM test of models residuals. Test statistics indicates that the null hypothesis of non-existence of ARCH effect (volatility clustering) in BEKK

9 Because we substructed BEKK estimated volatilies from DCC ones.

10 It assesses the null hypothesis that a series exhibits no conditional heteroscedasticity (ARCH effect), Engle, R. (1988), Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 96, 893-920.

Figure 3: Difference between the estimated volatilities



models cannot be rejected for all lags. However, statistics of the DCC(1,1) model indicate that it successfully captures volatility clustering in our series as the null hypothesis is accepted for all them and for all lags.

4.1, Forecasting with BEKK and DCC Models\Label

After we estimated BEKK and DCC models, we can perform volatility forecasting. Below the conditional covariance equation of the BEKK model:

$$\hat{H}_{t+1}=\hat{C}\,\hat{C}'+\hat{A}\,E_t\left(\grave{o}_t\grave{o}_t'\right)\,\hat{A}'+\hat{B}\,H_t\hat{B}' \tag{12}$$

Forecast with DCC model is done through two steps procedure. As stated in equation 8, the conditional covariance is written as below:

$$H_{t+1}=D_{t+1}\Gamma_{t+1}D_{t+1} \tag{13}$$

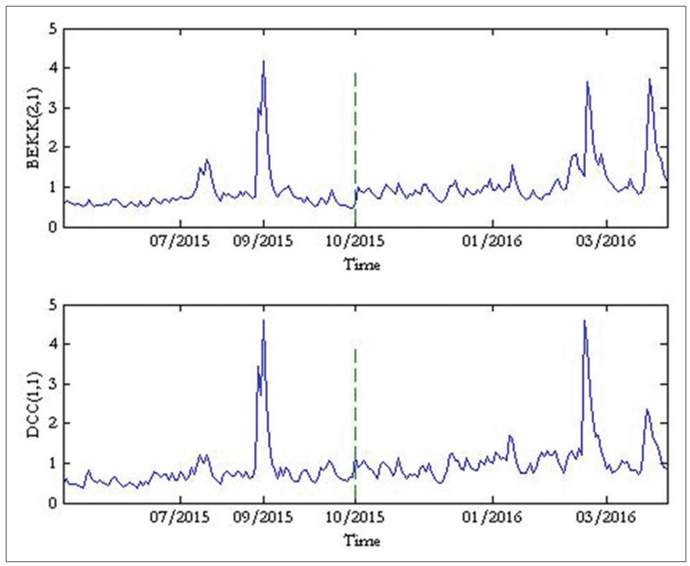
The first step in the forecasting procedure with DCC model consists of predicting diagonal element of matrix D_{t+1} using a GARCH(1,1) process. The second step consists of estimating matrix Γ_{t+1} using equation 9. Once both steps are completed, the conditional covariance of DCC model can be deduced.

We will perform forecast for our assets for the period from 21/10/2015 to 31/03/2016, the forecast is done ahead 1 day through this period. Figure 4 plots the foretasted volatility results for our equally weighted portfolio assets. The vertical dotted line in this figure separates estimation and forecast period. According to this figure, forecasted results show that they follow the same tendency as the estimated ones, also, the forecasts with both models appear to be stable. Similarly to section 3.4, we use the MAE to evaluate the quality of the forecasts. Table 5 presents the computed values of MAE for our forecasted results with squared returns used as volatility proxy. Computed MAE values show that DCC performs better than BEKK model: Computed MAE for DCC is lower than BEKK model for all assets.

4.2. Application to VaR

VaR measures the potential loss in value of a portfolio over a defined period at a given confidence interval. VaR is a capital

Figure 4: Forecasted volatilities with dynamic conditional correlation and Baba, Engle, Kraft and Kroner models



regularity requirement to financial institutions, it is considered as a measurement of risk they are exposed too. The VaR of a given portfolio is defined as below:

$$VaR(\alpha)=\alpha\sqrt{X'\sum X} \tag{14}$$

Where, α is the confidence interval, σ is the variance covariance matrix and X is a column vector of asset values.

Covariance matrices already estimated for both models will be used in equation 14, the obtained results will be used as a performance measure to evaluate models DCC and BEKK in calculating VaR. The comparison is done through The dynamic quantile test introduced by Engle and Manganelli (2001)[11].

We define the hit variable:

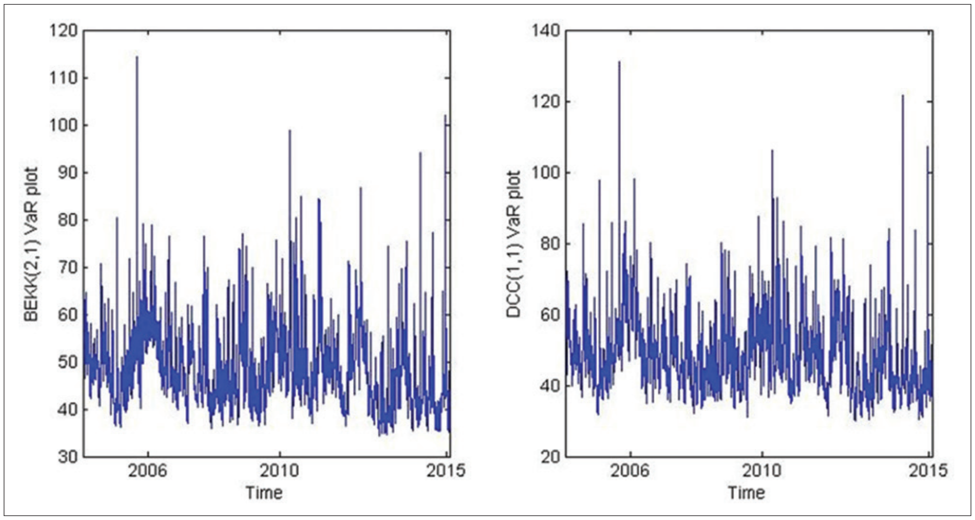
11 Engle and Manganelli, (2001), Value-at-risk models in finance. Working Paper 75, Working Paper Series, European Central Bank.

Table 5: Models mean absolute errors for evaluating forecast

Model	IAM	LESU	CTM	MNG	SOND	HOL
DCC (×10−2)	1.005	2.5100	2.6426	1.6493	3.2112	4.7359
Diagonal BEKK (×10−2)	1.2548	2.5881	3.1159	1.7949	4.3989	5.2302

DCC: Dynamic conditional correlation, BEKK: Baba, Engle, Kraft and Kroner

Figure 5: Dynamic conditional correlation and Baba, Engle, Kraft and Kroner models calculated value at risk plots



$$Hit(\alpha)=I_{\alpha}(\alpha)-\alpha \quad (15)$$

$I_t(\alpha)$ is the binary variable associated to past observations of VaR violation at time t:

$$I_t(\alpha)=\begin{cases} 1, \text{ if } r_t<VaR_{t|t-1}(\alpha) \\ 0, \text{ otherwise} \end{cases} \quad (16)$$

$VaR_{t|t-1(\alpha)}$ is the conditional VaR

Each time VaR exceeds the asset value, the Hit variable takes the value of 1-α and α otherwise. From VaR definition, the $Hit_t(\alpha)$ variable should be unpredictable based on its own lagged values or any other variable, in particular, past VaR and assets returns. The dynamic quantile test consists of testing the null hypothesis that all coefficients and the intercept, of a regression of this variable on its past values, VaR, and a set of other variables, are all null. In our study, we regress the hit variable on its past values (5 lags are taken), VaR, portfolio returns and squared portfolio returns. Our regression equation is as below:

$$Hit(\alpha)=\gamma+\sum_{k=1}^{5}\beta_k HIT_{t-k}+\theta VaR_t+\sigma\, r_t+\lambda r_t^2+\grave{o}_t \quad (17)$$

Using an equally weighted portfolio for our stocks with a total value of 1000 MAD, we calculate VaR over a 1 day horizon and at a 95% confidence level. Results are presented in Figure 5. Both calculated VaRs have the same tendency, but DCC (1,1) model tends to estimate relatively higher values in comparison with BEKK (2,1) model. The dynamic quantile test regression results are presented in Table E3. According to obtained P-values, two variables (the intercept and squared returns) are not statistically significant, thus, the hypothesis of joint nullity of parameters of the regression is rejected.

Engle and Manganelli deduced a simple test, under the null hypothesis, The dynamic quantile test is equivalent to a conditional efficiency test. We denote by $\Psi = (\gamma, \beta_1,\ldots, \beta_5, \theta, \sigma, \lambda)$ the vector of 2K+1 parameters and by Z the matrix of explanatory variables of equation 18. Wald statistic associated to conditional efficiency test, noted DQ_{cc} satisfies:

$$DQ_{cc}=\frac{\widehat{\psi' Z' \hat{\psi}}}{\alpha(1-\alpha)}\ \chi^2(2K+1) \quad (18)$$

Test results reject the hypothesis of joint nullity of parameters of equation 18. The Wald statistics calculated are 10003.87 and 10005.76 for BEKK(2,1) and DCC(1,1) models respectively. These two values are highly greater than the critical value of chi-square distribution which is equal to 19.9190 at 9° of freedom and 1-α = 95% probability level. These results are in accordance with regression results presented in Table E3. We conclude from the dynamic quantile test done that both models failed to adequately calculate VaR in the case of our selected stocks.

5. CONCLUSION

Volatility is a key element in risk and asset management. It is a measure of assets risk based on the variance of returns. Volatility is not observable and then should be estimated. The most popular methods in estimating volatility are historical volatility, implied volatility and auto-regressive heterokedastic models, for example GARCH family models. The first model in GARCH family was introduced by Engle in 1982, the ARCH model, it was considered as a great invention in financial econometrics. Later on, many extensions of ARCH were developed, e.g., GARCH, EGARCH, etc. GARCH family models are useful only in estimating volatility for Univariate dimension, they are enable to estimate volatility and correlation when multiple assets are involved. Multivariate GARCH models were developed for this purpose.

The aim of this article was to examine correlation between stocks chosen from Casablanca stocks exchange markets using multivariate GARCH models. We selected six stocks that have the longest and continuous daily prices. Descriptive statistics of our series showed that volatility is not constant over time, also, we noticed that our data suffers from volatility clustering, this was confirmed by ARCH LM tests performed. We used in our study the most widely used multivariate GARCH models, BEKK and DCC. These two models have attracted a considerable interest in the financial econometric literature. Model order selection tests we performed showed that DCC(1,1) and BEKK(2,1) are the most suited to our series. The choice of DCC instead of CCC model was based also on test of dynamic correlation which showed that correlation is not constant over time.

Estimated models comparison showed that BEKK (2,1) performs better than DCC (1,1) in the case of our series. First the MAE in comparison to empirical volatility of BEKK is less than DCC, secondly, the Ljung-Box Q-test indicated that the part of volatility explained by BEKK is more important than DCC. However, we found out using ARCH LM test, that DCC captures well volatility clustering than do BEKK model. Another examination of estimated models is comparing estimated volatility to the empirical one. We used squared returns as a proxy for the empirical volatility. The computed values of the MAE showed that the fitting performance of BEKK is higher than DCC.

After we estimated BEKK and DCC for our series, we performed volatility forecasting. The forecasts with both models have the same tendency as estimated ones; also, they appear to be stable. The comparison between forecasted and empirical volatility showed that DCC performs better than BEKK. The easy computation is another advantage of DCC model, we noticed that it is less demanding in time than needs BEKK.

The estimated variance and covariance matrices using BEKK and DCC models were applied also to VaR calculation. VaR is one of the most popular tools used by financial institutions, it measures the maximum loss of a portfolio at a given confidence level and a time horizon. DCC model estimated a relatively high VaR values than BEKK, but with same tendency. The quality of the estimation was tested using the dynamic quantile test of Engle and Manganelli. This test results showed that both models failed to adequately estimate VaR for our series.

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APPENDICES

Appendix A: BEKK (2,1) coefficients estimation

Coefficient	Estimation	Standard error	t-statistic
M (1,1)	0.000203	0.0000124	16.413
M (1,2)	0.00000219	0.00000303	0.72333
M (1,3)	0.00000138	0.00000263	0.524411
M (1,4)	−0.00000145	0.00000373	−0.389404
M (1,5)	−0.00000748	0.00000556	−1.346126
M (1,6)	0.000000726	0.00000334	0.217124
M (2,2)	0.0000154	0.00000168	9.147694
M (2,3)	−0.000000529	0.000000896	−0.590486
M (2,4)	−0.00000107	0.000000985	−1.089826
M (2,5)	0.00000389	0.00000288	1.347909
M (2,6)	−0.0000000321	0.000000701	−0.045746
M (3,3)	0.0000288	0.00000244	11.81638
M (3,4)	0.00000112	0.00000106	1.057964
M (3,5)	−0.000000104	0.00000253	−0.041084
M (3,6)	−0.000000372	0.000000732	−0.508196
M (4,4)	0.0000304	0.00000287	10.60451
M (4,5)	0.00000131	0.00000322	0.406006
M (4,6)	−0.000000539	0.000000879	−0.613408
M (5,5)	0.000161	0.0000146	10.98759
M (5,6)	0.00000451	0.00000298	1.512518
M (6,6)	0.00000357	0.000000444	8.037292
A1 (1,1)	0.353479	0.020463	17.27423
A1 (2,2)	0.295759	0.014473	20.4352
A1 (3,3)	0.278994	0.014584	19.1305
A1 (4,4)	0.327969	0.012393	26.46482
A1 (5,5)	0.482548	0.015469	31.19477
A1 (6,6)	0.017377	0.015566	1.116374
A2 (1,1)	−0.455349	0.023959	−19.00575
A2 (2,2)	−0.2132	0.021748	−9.803076
A2 (3,3)	−0.100728	0.029604	−3.402446
A2 (4,4)	0.132033	0.027061	4.879112
A2 (5,5)	0.281524	0.033555	8.390009
A2 (6,6)	−0.155057	0.005977	−25.9433
B1 (1,1)	0.646748	0.019445	33.26066
B1 (2,2)	0.920644	0.005184	177.5853
B1 (3,3)	0.892339	0.008073	110.528
B1 (4,4)	0.90744820	0.005967	152.0782
B1 (5,5)	0.71512	0.021343	33.50623
B1 (6,6)	0.984682	0.001094	900.3076

BEKK: Baba, Engle, Kraft and Kroner

Appendix B: DCC (1,1) coefficients estimation

Parameter	Estimation	Standard error	t-value
ω_1	135.28013	86.58288	1.562435091
α_1	0.2712445	0.3367281	0.805529743
β_1	0.762976403	0.007835544	97.37376282
ω_2	165.1068612	0.1495558	1103.981666
α_2	0.329009	0.1326704	2.479897551
β_2	0.77881184	0.02112435	36.86796706
ω_3	0.6372866	0.0610729	10.43485081
α_3	0.41494041	0.03610632	11.49218226
β_3	0.757185753	0.007019868	107.8632466
ω_4	75.39039	108.76261	0.69316459
α_4	0.1070037	162.6292249	0.000657961
β_4	0.78191449	0.01259818	62.0656706
ω_5	0.00178703	0.14218614	0.012568243
α_5	0.02998864	0.28244741	0.106174243
β_5	0.97726531	0.04096206	23.85781648
ω_6	0.001356091	0.069170771	0.019604972
α_6	0.173388	0.4464916	0.388334293
β_6	0.88502061	0.02477999	35.71513185
DCCα	0.02011386	0.000254597	79.00283036
DCCβ	0.03955214	0.32286853	0.122502308

DCC: Dynamic conditional correlation

Appendix C

Figure C1: Diagonal Baba, Engle, Kraft and Kroner model estimated correlations plot

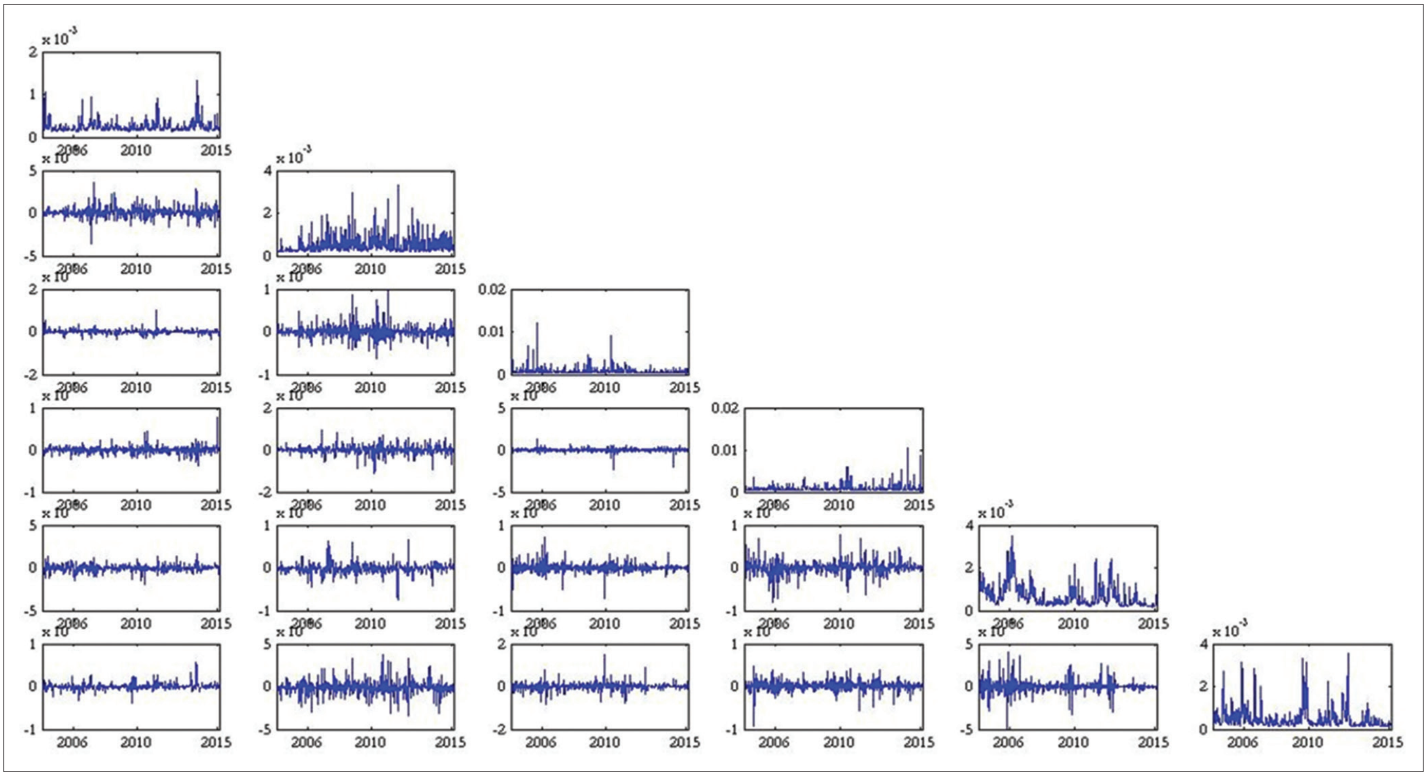
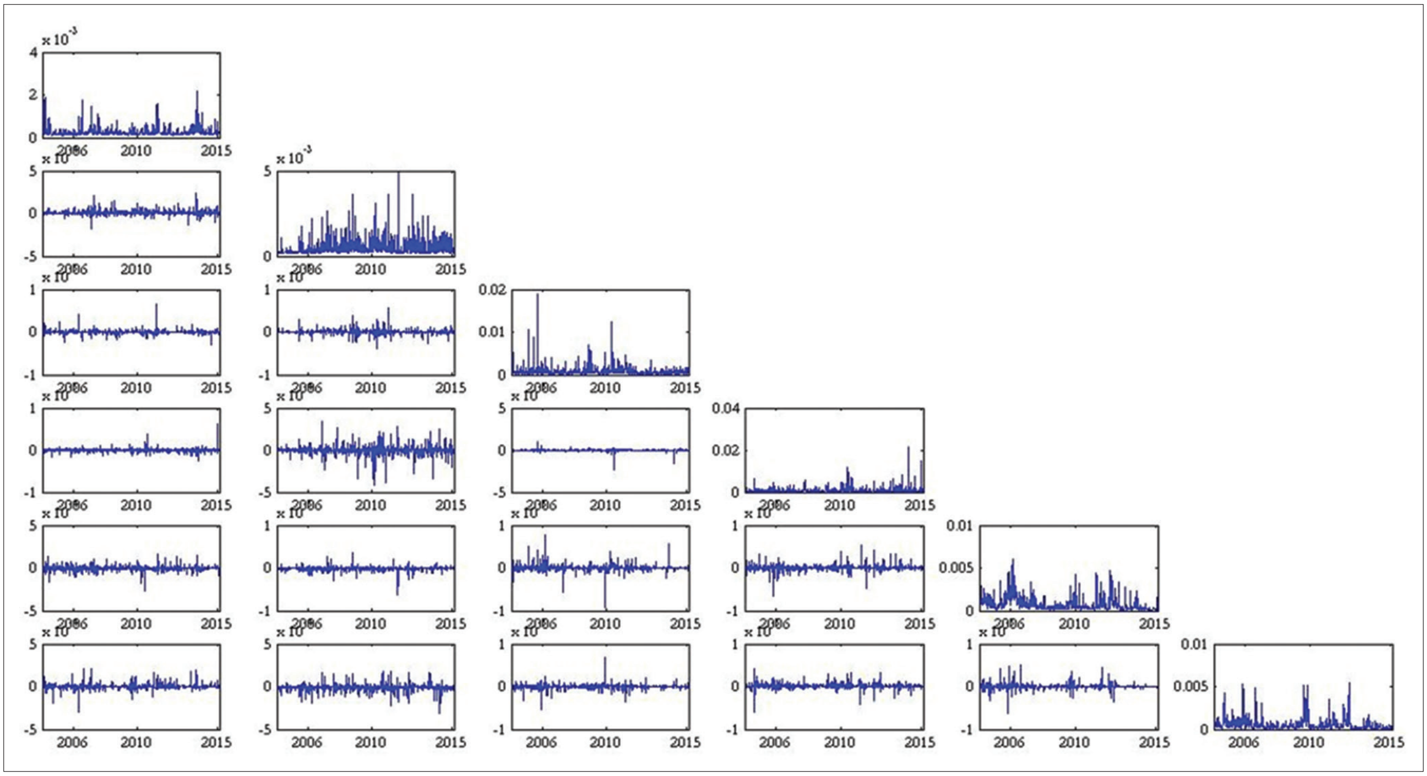


Figure C2: Dynamic conditional correlation model estimated correlations plot



Appendix D

Figure D1: Baba, Engle, Kraft and Kroner model standardized residuals plot

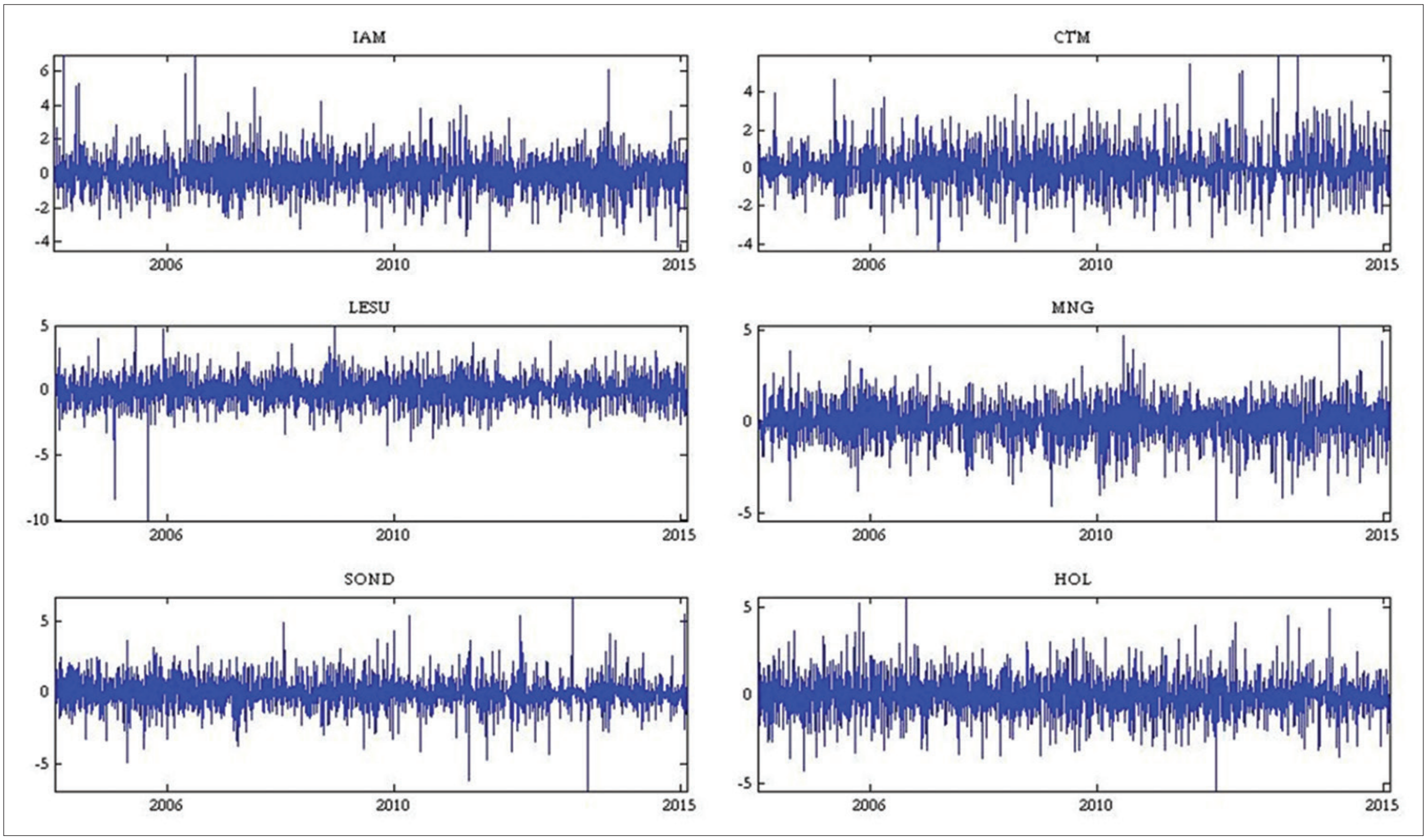
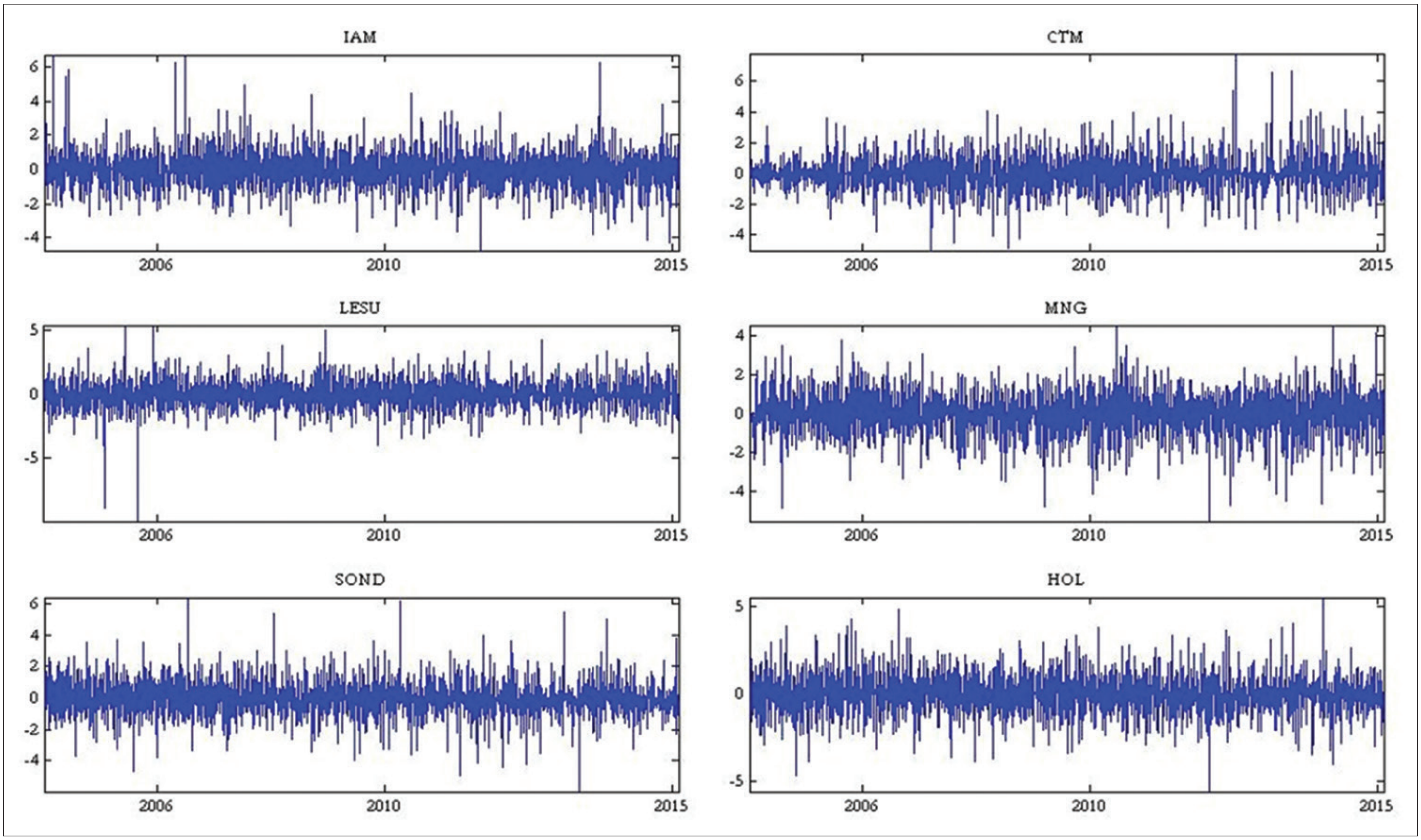


Figure D2: Dynamic conditional correlation model standardized residuals plot



Appendix E

Table E1: ARCH LM test for daily returns

Series	IAM	LESU	CTM	MNG	SOND	HOL
P value	0	0	0	0	0	0
Test statistic	70.4189	132.6917	64.1082	89.5894	72.7168	113.9469
Critical value	3.8415	3.8415	3.8415	3.8415	3.8415	3.8415

ARCH: Autoregressive conditional heteroskedasticity

Table E2: ARCH LM tests for standardized residuals

Model	Lag	Variable	IAM	LESU	CTM	MNG	SOND	HOL
DCC (1,1)	1	P value	0.5903	0.2470	0.4319	0.8208	0.4163	0.3343
		Test statistic	0.2899	1.3399	0.6177	0.0513	0.6606	0.9323
		Critical value	3.8415	3.8415	3.8415	3.8415	3.8415	3.8415
	5	P value	0.5074	0.4419	0.7847	0.7379	0.2634	0.3167
		Test statistic	4.2978	4.7913	2.4455	2.7539	6.4668	5.8941
		Critical value	11.0705	11.0705	11.0705	11.0705	11.0705	11.0705
	10	P value	0.4450	0.6387	0.8964	0.9044	0.4341	0.3057
		Test statistic	9.9485	7.8987	4.9213	4.7949	10.0736	11.6994
		Critical value	18.3070	18.3070	18.3070	18.3070	18.3070	18.3070
	20	P-value	0.0924	0.0248	0.9980	0.6261	0.1185	0.1288
		Test statistic	28.7685	34.2082	6.4938	17.4116	27.6275	27.2355
		Critical value	31.4104	31.4104	31.4104	31.4104	31.4104	31.4104
BEKK (2,1)	1	P value	0.0103	0	0.3192	0.0201	0	0
		Test statistic	6.5880	74.6317	0.9921	5.4027	20.4242	33.0170
		Critical value	3.8415	3.8415	3.8415	3.8415	3.8415	3.8415
	5	P value	0.0483	0.0000	0.9304	0.0993	0.0000	0.0000
		Test statistic	11.1624	76.6855	1.3435	9.2558	33.8767	35.7401
		Critical value	11.0705	11.0705	11.0705	11.0705	11.0705	11.0705
	10	P value	0.0994	0.0000	0.9880	0.1226	0.0001	0.0000
		Test statistic	16.0075	80.6279	2.6777	15.2671	36.0209	40.4832
		Critical value	18.3070	18.3070	18.3070	18.3070	18.3070	18.3070
	20	P value	0.0994	0.0000	0.9880	0.1226	0.0001	0.0000
		Test statistic	16.0075	80.6279	2.6777	15.2671	36.0209	40.4832
		Critical value	18.3070	18.3070	18.3070	18.3070	18.3070	18.3070

ARCH: Autoregressive conditional heteroskedasticity, DCC: Dynamic conditional correlation, BEKK: Baba, Engle, Kraft and Kroner

Table E3: Hit regression results

		BEKK (2.1)		
Variable	**Estimate**	**Standard error**	**t-Stat**	**P-value**
(Intercept)	0.515864396	0.038631123	13.35359553	1.89×10-39
H itt−1	−0.000961118	0.012400926	−0.077503762	0.93822847
H itt−2	−0.013410133	0.012349631	−1.085873133	0.277630857
H itt−3	0.00592022	0.012355105	0.479171987	0.631854604
H itt−4	−0.018955998	0.012358024	−1.533902056	0.125169454
H itt−5	0.010050019	0.012298799	0.817154508	0.413911337
VaRt	−0.000582008	0.000707428	−0.822710689	0.410744284
rt	−4.9×10-9	3.78×10-9	−1.295739961	0.19517447
rt2	−0.000396445	0.00000637	−62.23433128	0
DCC (1.1)				
(Intercept)	0.501959029	0.03302687	15.19850449	3.83×10-50
H itt−1	−0.000799852	0.012404143	−0.064482643	0.948590642
H itt−2	−0.013250749	0.01234836	−1.073077657	0.283330979
H itt−3	0.006080367	0.012354503	0.492157991	0.622647174
H itt−4	−0.018782073	0.012356715	−1.519989192	0.128629324
H itt−5	0.01024675	0.012296703	0.833292402	0.40475263
VaRt	−0.000303418	0.000583455	−0.520037083	0.603079833
rt	−0.00000000497	0.00000000381	−1.304049237	0.192326573
rt2	−0.000396501	0.00000637	−62.23204228	0

DCC: Dynamic conditional correlation, BEKK: Baba, Engle, Kraft and Kroner, VaR: Value at risk