



The Glosten-Jagannathan-Runkle-Generalized Autoregressive Conditional Heteroscedastic approach to investigating the foreign exchange forward premium volatility

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ABSTRACT

This paper empirically investigates the volatility dynamics of the EUR/USD forward premium via generalized autoregressive conditional heteroscedastic (GARCH-M) (1,1) and Glosten-Jagannathan-Runkle (GJR)-GARCH (1,1) and GJR-GARCH (1,1)-M models. Our empirical analysis is based on daily data related to the EUR/USD forward premiums. Our daily analysis reveals several results. Firstly, we confirm that the 9 month and 1 year forward premiums are explained in large part by their conditional variances. Secondly, according to the theoretical predictions of the asymmetric framework, we show that the conditional variances equations exhibit an asymmetry in the dynamics of the conditional variance only for the 9 months and 12 months horizons. Thirdly, for the 6 month, 9 month and 12 month forward premiums; the GJR-GARCH in mean effect is totally absent.

Keywords: Conditional Volatility, Glosten-Jagannathan-Runkle-Generalized Autoregressive Conditional Heteroscedastic, Generalized Autoregressive Conditional Heteroscedasticity, Volatility Persistence

JEL Classifications: C58, G14, G13, G15

1. INTRODUCTION

The uncovered interest parity puzzle which is known under the appearance of “the forward premium puzzle” has become nowadays well documented and it is considered as a result that has led a second generation of research work attempting to explain its existence. Indeed, various explanations have been made to explicit this anomaly, but none of them proved to be fully satisfactory. In addition, a line of research has affirmed the presence of a “Peso problem,” or even released the assumption of “rational expectations” in order to reach some reconciliation between the theory and the puzzle. Only a few other explanations of the forward premium puzzle could attribute the exchange risk premium to the interest rate differentials (Carlson and Osler, 2003; including the work of Obstfeld and Rogoff, 1998; and Hagiwara, 1999; Mark and Wu, 1998; Meredith and Ma, 2002; Driskill and McCafferty, 1982).

Thereby, since the early work of Fama (1984) to more recent, such as those of Zhuang (2015), several studies have shown the failure of the forward exchange rate to serve as unbiased predictor

of future spot exchange rate. In the literature, the robustness of this puzzle has been tested in multiple aspects, including different time periods (Zhou and Kutan, 2005), different countries (Bansal and Dahlquist, 2000), different maturities (Chinn and Meredith, 2004), different exchange rate regimes (Flood and Rose, 1996), or even the effect of weekdays (Ding, 2012). In this sense, the recent literature has found a validation of the long memory behavior (Baillie and Bollerslev, 1994) or of unit root (Kellard et al., 2001) in the forward premium, suggesting a rejection of the Forward Rate Unbiased Hypothesis. Similarly, Maynard and Phillips (2001) suggest that literature should subsequently examine the reasons why the forward premium could show such time series properties. Besides, in the context of economic models, a partial list of attempts have been proposed to explain the forward premium puzzle including consumption asset pricing theories of Bansal et al. (1995), Verdelhan (2005) and Lustig and Verdelhan (2016), the term structure models of Backus et al. (2001) and Bansal (1997), the risk premium based on equilibrium models or asset valuation models including several works from Frankel and Engel (1984), Hodrick (1987), Bekaert and Hodrick

(1993), Bekaert (1996) to Verdelhan (2010), Shaliastovich and Bansal (2010) and Menkhoff et al. (2012). Other recent works are based on the context of treatment of incomplete information (Bacchetta and van Wincoop, 2009); the differences in developed markets versus emerging markets (Bansal and Dahlquist, 2000 and Frankel and Poonawala, 2010); and eventually on profitability and economic value of the currency speculation (Burnside et al., 2011 and Della Corte et al., 2009). These studies conclude that, to get an enhanced explanation to the forward premium puzzle is in itself a formidable challenge for any economic model of rational expectations. Despite a substantial number of studies invoked the ability of general equilibrium models, primarily related to the model of Lucas (1982), to explain the forward premium puzzle (Hodrick, 1989; Macklem, 1991; Canova and Marrinan, 1993; Bekaert, 1994), they failed to explain the substantial variation that takes place in the magnitude of expected excess returns.

Following these developments, we propose, in this paper, to conduct a comparative analysis of the volatility of the forward premium separately in a univariate symmetric framework and a bivariate asymmetric framework. To do this, the first approach is taken using the generalized autoregressive conditional heteroskedasticity in mean (generalized autoregressive conditional heteroscedastic [GARCH]-in Mean [1,1]) model in which the conditional variance is supposed to explain the foreign exchange forward premium. Since the linear framework described above occult possible asymmetric shocks that can characterize (the conditional variance equation) of the forward premium, then we proceed to a second approach on the part of the Glosten-Jagannathan-Runkle (GJR-GARCH) model of Glosten et al. (1993).

The remainder of this paper is organized as follows: Section 2 presents the methodology employed in this study. The data and the unit root tests are presented in Section 3. Section 4 discusses the empirical findings and the last section concludes.

2. METHODOLOGY

To analyze the forward exchange premium, we specify the difference between the spot exchange rate and the forward exchange rate $f_t^{t+1} - s_t$ as the forward premium, we denote by:

s_t : The natural logarithm of the spot exchange rate at time t

f_t^{t+1} : The natural logarithm of the forward exchange rate at time t

$E_t(\cdot)$: The expectations operator conditional on the information available at that date

ε_t : A random term with zero mean.

Following these developments, we propose, in this paper, to study the dynamics of the EUR/USD forward premium and its main features via a symmetric linear and univariate, and an asymmetric and bivariate ARCH/GARCH modeling. This choice is based on the works that argue that ARCH/GARCH models provide a better forecasting of low horizon variability characterizing the foreign exchange risk premium. First, we estimate a GARCH-in

mean model in which the conditional variance is supposed to explain the forward exchange premium. However, the quadratic specification in the conditional variance equation that characterizes the GARCH-M model conceals the asymmetric shocks. Given this, we will look at a variety of other nonlinear extensions that have been proposed, including the GJR-GARCH model of Glosten et al. (1993).

Hereafter, we propose to estimate the GARCH-M model (p, q) defined as follows:

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t \quad (1)$$

$$h_t = a_0 + a_1 h_{t-1} + b_1 \varepsilon_{t-1}^2 + \eta_t \quad (2)$$

$$(\varepsilon_t / \eta_t) \sim N(0, h_t) \quad (3)$$

Equation (1) is the equation of the conditional mean.

Equation (2) is the equation of the conditional variance.

Equation (3) is the assumption of conditional normality of errors.

h_t : Is the conditional variance of forward premium series which is assumed to follow a GARCH (1,1) process.

h_{t-1} : Represents the forecasting of the variance at the last period and the coefficient a_1 associated therewith represents the GARCH parameter.

ε_{t-1}^2 : Represents the squared delayed residuals informing us about the volatility or the instantaneous variability, and the coefficient b_1 associated therewith is referred to the ARCH parameter.

We note that Equation (1) is the pivotal equation of GARCH-M model in which the forward exchange premium is a function of its conditional variance. In this specification in mean, the conditional variance is introduced into the mean equation and the choice of such a model depends on its ability to capture stylized facts of forward exchange premiums (at low or high frequency).

The GARCH-M model is among the linear models based on a quadratic specification of disturbances on the conditional variance. They assume that the magnitude and not the sign of the shock that determines the volatility. Therefore, positive and negative shocks of the same size have the same impact on the conditional variance. In other words, they are symmetrical process. However, the asymmetric efficiency of shocks on the volatility, i.e. the conditional variance reacts differently to shocks of the same magnitude as the sign of the latter is very realistic for financial and monetary series. Symmetric ARCH models have the disadvantage of not taking into account the stylized fact possible in the series studied.

In what follows, we model the EUR/USD forward premium using the GARCH (1,1)-M model. Therefore, we move from GARCH-M model which is within the framework of linear ARCH/GARCH

models to the application of asymmetric ARCH/GARCH models¹ such as GJR-GARCH and GJR-GARCH-M models.

Under the univariate ARCH models, the model of Glosten et al. (1993), known by the abbreviation GJR, describes the asymmetry by distinguishing between two types of shocks that may affect the price of an asset that is a positive return not anticipated and an unanticipated negative return.

Another approach to capture the effect of asymmetric disturbances on the conditional variance is introduced by Glosten et al. (1993). The GJR - GARCH formulation is in fact a GARCH model with the addition of a dummy variable which is multiplied by the square of the error term of time spent in the conditional variance equation. It is a threshold model where the indicator function, that is the dummy variable, is equal to one if the residual of the previous period is negative and it is zero otherwise. In this way, the conditional variance follows two different processes depending on the sign of the error terms.

Consider $\varepsilon_t = Z_t \sqrt{h_t}$, the equation for the conditional variance of a GJR-GARCH process is:

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j} + \gamma \varepsilon_{t-1}^2 I_{t-1} \quad (4)$$

Where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, 0 if not

With the conditions $\alpha_0 > 0, \alpha_i, \beta_j \geq 0$ and $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j + 0.5\gamma < 1$

The GJR-GARCH (p, q) model captures the asymmetric effect of the disturbances on the conditional variance.

For further empirical investigation in this contribution, we propose to move to the modeling GJR-GARCH-in Mean, in order to integrate the conditional variance in the variance equation of the GJR-GARCH model.

We specify that the estimations of the GJR-GARCH model for the EUR/USD 1 month, 3 month, 6 month, 9 month and 12 month forward premiums are made based on the algorithm BHHH (1974).

3. DATA

The empirical study investigating the foreign exchange forward premium volatility is based on daily data related to the EUR/USD parity over the period running from 08 January 08, 1999 to January 08, 2016. The data collected are daily frequency and are obtained from Data stream. Our time series of the Euro/U.S. Dollar have a set of 4436 observations corresponding to the spot exchange rates and the and the 1 month, 3 month, 6 month, 9 month and 1 year forward exchange rates and are expressed in logarithmic form to avoid the Siegel's paradox (Baillie and McMahon, 1989).

¹ Among these models, we include the EGARCH model, GJR-GARCH, and APARCH VSGARCH, Tarch and TGARCH, QGARCH, LSTGARCH and ANSTGARCH.

In order to apply the GARCH -in -Mean specification for the EUR/USD 3, 6 and 12 month forward premium series, we should firstly check certain conditions that will allow us to confirm the use of such a heteroscedastic model in which there is inevitably a volatility effect. Indeed, the ARCH-type models can model chronics that have an instantaneous volatility depending on the past, and it will then be possible to develop a dynamic forecasting of the exchange risk premium in terms of mean and variance.

In the light of Table 1, the descriptive statistics show that high standard deviation value, considered as a volatility measure, is attributed to 1 year forward premium. Moreover, the skewness coefficients are positive for all the distributions of EUR/USD forward premiums (whatever the 1, 3, 6 and 9 month horizon) showing asymmetric and thicker right series. On the other side, the skewness coefficients and their respective averages have opposite signs which induces that there are extreme values for 1, 3, 6 and 9 month forward premiums. This is an evidence of phases of sudden depreciation and appreciation experienced by the EUR/USD parity throughout the period studied. Henceforth, this is not the case for the 1 year horizon.

Regarding the kurtosis coefficients, they are higher than the reference value of the normal distribution equal to 3. Then, the distribution of the 1,3, 6, 9 and 12 month forward premium series is leptokurtic and has a thicker tail than that of the normal distribution.

To test the normality of the EUR/USD forward premium series, we refer to the asymptotic Jarque-Bera (1980) test. These normality tests have helped us to prove some heteroscedasticity materialized by leptokurtic distributions. Considering the JB statistic is much higher than the critical value given by the Chideux table with two degrees of freedom equal to 5.99 at the 5% level significance, we confirm that the null hypothesis of normality is strongly rejected and thereby it is indeed volatile variables.

To test the existence of non-linearity which can be largely explained by the presence of ARCH effect, we examine the Q statistic which it is distributed asymptotically as a Chideux (at 12 and 24 degrees of freedom). We note clearly, from this table, all Q Ljung-Box statistics of forward premiums are above $\chi^2(20)$ read in the table at 5% level significance and with a value of 31.41. Also, they clearly indicate, by their critical zero probabilities, series of forward premiums unrepresentative of white noise. They also indicate that these series demonstrate significantly the phenomenon of volatility clustering and therefore of heteroscedasticity.

Thereafter, we report that the implementation of GARCH models and their extensions requires first the specification of the order of integration of the different series of forward premiums. To do this, we use the Augmented Dickey Fuller the unit root tests of Dickey and Fuller test (noted ADF) (1979, 1981), Elliot, Rothenberg and Stock (denoted ADF-GLS) (1996) and Kwiatkowski et al. test (denoted KPSS) (1992). At this level, it should be noted that these tests are carried out under the following three assumptions in the autoregressive equations related to various tests:

4. EMPIRICAL RESULTS

- Absence of a constant
- Presence of a constant
- Presence of a constant and a trend.

The results of stationarity tests of the Euro/U.S. Dollar 1 month, 3 month, 6 month, 9 month, 1 year forward premiums are reported in Table 2.

Given the results of unit root tests, we note that the EUR/USD forward premium series at 1 month, 3 months, 6 months, 9 months and 1 year horizons are processes (difference stationary), using the terminology of Nelson and Plosser (1982), in which there is a shock persistence property that does not exist in trend stationary processes. Then, we reject the hypothesis H_1 of stationary series of forward premiums whatever the level of significance of 1% and 5% et we conclude that they are integrated of order 1 since their first differences of the series show a stationarity which is maintained whatever the model considered.

In order to empirically study the volatility effect of the EUR/USD forward premium, we first estimate GARCH (1,1)-M model (Equations 1-3) given that 1 month, 3 month, 6 month, 9 month and 1 year forward premiums are neither normal distributions nor white noise processes, as required by the heteroscedastic ARCH specification. Results obtained from parameters estimations of this model are reported in Table 3.

The GARCH-M (1,1) estimation results show that the coefficient β of the mean equation is negative and statistically significant only for the 1 month, 3 month and 6 month horizons, and the constant of the variance equation. Besides, the ARCH and the GARCH terms are statistically significant at 1% significance level This implies the time-varying of conditional volatility of forward premiums and that the conditional variance is strongly explained by the one period lagged series. Also, the sum of the ARCH and

Table 1: Descriptive statistics

Statistics	Forward premium (1 month)	Forward premium (3 months)	Forward premium (6 months)	Forward premium (9 months)	Forward premium (12 months)
Number of observations	4435	4435	4435	4435	4435
Mean	-1.64 ^{e-07}	-4.82 ^{e-07}	-8.80 ^{e-07}	-1.23 ^{e-06}	-1.50 ^{e-06}
Median	-8.30 ^{e-07}	8.45 ^{e-08}	8.98 ^{e-07}	0.0000	0.0000
Standard deviation	0.0,00,221	0.0,00,247	0.0,00,297	0.0,00,369	0.0,00,453
Skewness (Sk)	0.2,93,815	0.5,64,399	0.573509	0.375257	-0.0,05,395
Kurtosis (Ku)	754.7206	496.8674	249.7942	118.9065	61.01,538
Jarque-Bera (JB)	1.04 ^{e+08}	45.0,71,853	11.2,55,420	24,82,653	6,21,968.9
P	0.0,000	0.0,000	0.0000	0.0000	0.0,000
Q(12)	775.78	506.15	140.18	57.859	45.925
Q(24)	784.69	510.16	155.64	82.947	67.146

Statistics provided by Eviews 5.0

Table 2: The unit root tests

Unit root tests	ADF test		ADF-GLS Test		KPSS Test	
	H_0 : Unit root		H_0 : Unit root		H_0 : Stationarity	
	In level	In 1 st difference	In level	In 1 st difference	In level	In 1 st difference
EUR/USD 1-month forward premium						
Test statistic	-2.3778*** (5) [1]	-71.6177 (1) [1]	-2.2251*** (2) [2]	-35.6221 (1) [2]	0.5389** [2]	0.1148 [2]
Critical value	-2.5,65,484	-2.5,65,484	-2.5,65,484	-2.5,65,484	0.4630	0.4630
EUR/USD 3-month forward premium						
Test statistic	-2.0477*** (1) [1]	-61.9299 (1) [1]	-1.1440*** (1) [2]	-54.5515 (1) [2]	0.5593** [2]	0.1681 [2]
Critical value	-2.5,65,485	-2.5,65,484	-2.5,65,484	-2.5,65,484	0.4630	0.4630
EUR/USD 6-month Forward premium						
Test statistic	-1.6494*** (1) [1]	-52.5279 (1) [1]	-0.8114*** (1) [2]	-49.7681 (1) [2]	0.5932** [2]	0.2533 [2]
Critical value	-2.5,65,484	-2.5,65,484	-2.5,65,484	-2.5,65,484	0.4630	0.4630
EUR/USD 9 month-Forward premium						
Test statistic	-1.5769*** (1) [1]	-49.3086 (1) [1]	-0.7314*** (1) [2]	-45.0577 (1) [2]	0.6320** [2]	0.3274 [2]
Critical value	-2.5,65,484	-2.5,65,484	-2.5,65,484	-2.5,65,484	0.4630	0.4630
EUR/USD 12-month Forward premium						
Test statistic	-1.5849*** (1) [1]	-47.4612 (1) [1]	-0.7186*** (1) [2]	-28.2960 (1) [2]	0.6744** [2]	0.3327 [2]
Critical value	-2.5,65,484	-2.5,65,484	-2.5,65,484	-2.5,65,484	0.4630	0.4630

We note that the ADF and ADF-GLS tests were conducted in the presence of levels of delay from 1 to 40 in the first differences of the series of the variables studied. Concerning the KPSS test, it was conducted in the window Newey-West (respectively that of Bartlett). Values in parentheses denote the number of lags used. **Significant at 5% significance level. ***Significant at 1% significance level. ADF: Augmented Dickey Fuller, KPSS: Kwiatkowski, Phillips, Schmidt, and Shin. Values in brackets indicate the type of model used for knowing the ADF test: The model [1]: Without constant. The model [2]: With constant. The model [3]: Constant and trend

GARCH parameters are of approximately (0.1554, 0.7086, 0.8485) respectively for the horizons of 1 month, 3 months and 6 months not being very close to unity indicate the absence of persistence of shocks induced by the volatility. On the other side, for the 9 month and 1 year horizons which they are consequently explained in large part by their conditional variances, i.e., by their volatilities.

Furthermore, it is not the case for the 9 month forward and the 1 year forward premium for which neither the coefficient b nor the constant are not significant. Contrariwise, for these horizons, the forecasts of the conditional variance converge very slowly to the regular state and there is a remarkably a high degree of volatility persistence. The results indicate that these series demonstrate significantly the phenomenon of volatility clustering which is ultimately linked to the notion of heteroscedasticity.

Accordingly, GARCH(1,1)-M model is considered as a good specification only for the 9 month and 1 year forward premiums.

Thereafter, in order to model the conditional volatility's dynamic and to measure the persistence of volatility shocks of the EUR/USD forward premium, we estimate respectively the GJR-GARCH (1,1), the AR (1)-GJR-GARCH (1,1) and the AR (1)-GJR-GARCH-M (1,1) models.

Primarily, Table 4 recapitulates estimations results of the GJR-GARCH (1,1) model.

Based on the estimation results of the forward premiums, we can affirm that the estimated model has good statistical properties. In

fact, the estimated coefficients of Arch and GARCH parameters are statistically significant and have the same sign (whatever the 3, 6, 9 and 12 month horizon) in the presence of a GARCH term demonstrating a great significance. Besides, the shocks imposed on the conditional variance are quite persistent over time which can reveal the presence of regime shifts in the process explaining the variance of the fact that the sum of the Arch and Garch parameters is very close to unity. On the other hand, the coefficient D related to the non-linear (or asymmetric) component of the conditional variance and referring to the leverage is positive and statistically significant for the 9 month and 12 month EUR/USD forward premiums. Hence, the conditional variances equations exhibit an asymmetry in the dynamics of the conditional variance.

Secondly, Table 5 recapitulates estimations results of the AR(1)-GJR-GARCH (1,1) model.

In the light of these results, the AR (1)- GJR - GARCH (1,1) model with the presence of a normal distribution seems to be the most adequate to retrace the high persistence of shocks imposed over time on the conditional variance and the strong presence of the autoregressive effect of order 1 for all maturities, in addition to the asymmetry effect highlighted by the coefficient D characterizing the forward premium. Therefore, an increase of the conditional variance is associated with an increase of the conditional premium.

Subsequently, aiming to analyze any asymmetric impact on the conditional variance of forward premiums, we integrate in-mean effect in the AR (1)-GJR-GARCH specification. In fact, the GJR-GARCH(1,1)-M stands for Glosten et al. (1993) Generalized

Table 3: Estimation of GARCH (1,1)-M model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$

$$h_t = a_0 + a_1 h_{t-1} + b_1 \varepsilon_{t-1}^2 + \eta_t$$

$$(\varepsilon_t / t) \sim N(0, h_t)$$

The series studied	1-month forward premium	3-month forward premium	6-month forward premium	9-month forward premium	12-month forward premium
Constant (M)	0.0,00,020* (7.93,689)	0.0,00,018* (4.39,754)	0.0,00,015* (3.27,899)	0.0,00,001 (0.22,922)	0.0000 (0.00,434)
β	-799.84739* (-16.34,811)	-416.81,544* (-8.12,819)	-173.30,524* (-3.08,569)	-26.38,814 (-0.87,764)	-18.20061 (-0.54,738)
Arch	0.1,93,555* (33.09,814)	0.25,943* (20.49,032)	0.22,473* (41.11,325)	0.4,71,187* (81.21,468)	0.30,088* (69.89,032)
GARCH	-0.0,38,173* (-2.30,945)	0.44,927* (21.46,145)	0.62,383* (60.16,096)	0.72,701* (169.17,229)	0.77,392* (176.01,319)

Estimate made by Rats 7.0 software. Values in parentheses are t-student statistics. The exponent (*) indicates that the coefficient is statistically significant. The values in parentheses are tstudent statistics. ARCH: Autoregressive conditional heteroskedasticity, GARCH: Generalized autoregressive conditional heteroskedasticity

Table 4: Estimation of the GJR-GARCH (1,1) model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j} + \gamma \varepsilon_{t-1}^2 I_{t-1}$$

The series studied	1 month forward premium	3 month forward premium	6 month forward premium	9 month forward premium	12 month forward premium
Mean	-0.0,00,001* (-10.79,522)	3.04,62 ^{e-06} (0)	7.17,29 ^{e-06} * (2.28,894)	-1.25,29 ^{e-06} (-0.43815)	-2.3897 ^{e-06} (-0.56010)
Constant (v)	0.00,000* (44.977)	1.21,62 ^{e-08} * (30.81,677)	1.2,21,02 ^{e-08} * (11.29,433)	1.53,79 ^{e-09} * (5.08,021)	3.1328 ^{e-09} * (4.99249)
Arch	1.0,54,677* (12.58,489)	0.2520* (21.49,609)	0.26,328* (8.67,310)	0.3627* (9.70,865)	0.2319* (10.50,986)
GARCH	-0.0,03,493 (-1.73,105)	0.5577* (97.87,673)	0.60,809* (21.78,531)	0.7244* (59.68,712)	0.7919* (64.88,423)
D	-0.9,50,659* (-15.19,406)	-0.0803* (-9.82,736)	0.05680 (1.31,084)	0.2484* (4.42,604)	0.0703* (2.49,157)

Estimate made by rats 7.0 software. The digital resolution was achieved via the algorithm BHHH (Berndt et al., 1974). D is the skewness. The values in parentheses are the t-student. The exponent * indicates that the coefficient is statistically significant. GARCH: Generalized autoregressive conditional heteroscedastic

Autoregressive Conditional Heteroscedasticity of order 1,1 with a Mean term that models the conditional risk premium. A number of studies (Nelson, 1991; Glosten et al., 1993; Rabemananjara and Zakoian, 1993) show that good news (measured by positive return shocks) and bad news (measured by negative return shocks) have an asymmetric impact on the conditional variance of stock returns.

The Table 6 presents estimated AR (1)-GJR-GARCH-M (1,1) results for EUR/USD forward premiums series.

The estimated parameters of AR(1)-GJR-GARCH-M (1,1) show it does not seem to be a good specification given that the β coefficient of the conditional mean is negative and statistically not significant for most horizons. Moreover, there is a high persistence of shocks of the conditional variance as the sum of the Arch and Garch parameters is close to unity (for 9 month and 12 month forward

premiums). Similarly, the asymmetry is remarkably present for these horizons of the fact that the D coefficient is positive and statistically significant. In addition, the autoregressive effect is also strongly required. Nevertheless, for the 6 month, 9 month and 12 month forward premiums, the β coefficients are not significantly different from zero. Thus, The GJR-GARCH in mean effect is totally absent for these horizons.

Overall, given the findings mentioned above, we deduce that the AR (1) - GJR - GARCH (1,1) with a normal distribution turns out to be the best specification to synthesise the EUR/USD forward premium. This asymmetric and bivariate analysis is certainly more intuitive and advantageous than the univariate analysis based on symmetric GARCH -M linear modeling neglecting the asymmetry that can take the forward premium on the foreign exchange market.

Table 5: Estimation of the AR (1)-GJR-GARCH (1,1) model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j} + \gamma \varepsilon_{t-1}^2 I_{t-1}$$

The series studied	1 month Forward premium	3 month Forward premium	6 month Forward premium	9 month Forward premium	12 month Forward premium
Constant (M)	-0.0,00,002 (-0.93,394)	8.6599 ^{e-06} * (2.19375)	4.4961 ^{e-06} (1.10165)	-1.9145 ^{e-06} (-0.69527)	-2.8556 ^{e-06} (-0.66293)
AR (1)	-0.112662* (-2.97856)	-0.1869* (-5.18064)	0.07828* (3.04246)	0.1853* (18.36496)	0.1053* (8.73551)
Constant (V)	0* (90.40361)	1.5274 ^{e-08} * (29.41631)	1.16335 ^{e-08} * (30.22960)	1.205 ^{e-09} * (8.15303)	2.9961 ^{e-09} * (9.57050)
Arch	1.024435* (16.55232)	0.2928* (18.71985)	0.26041* (27.60535)	0.3628* (16.93131)	0.2431* (21.34680)
GARCH	0.000034 (0.00306)	0.4560* (25.22553)	0.62164* (58.87611)	0.7419* (144.12833)	0.7897* (170.40240)
D	-0.963534* (-15.41036)	-0.1171* (-7.07610)	0.04794* (2.93989)	0.1255* (4.29548)	0.0534* (3.31005)

Estimate made by Rats 7.0 software. The digital resolution was achieved via the algorithm BHHH (Berndt et al., 1974). D is the skewness. The values in parentheses are the t-student. The exponent * indicates that the coefficient is statistically significant. GARCH: Generalized autoregressive conditional heteroscedastic

Table 6: Estimation of the AR (1)-GJR-GARCH-M (1,1) model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j} + \gamma \varepsilon_{t-1}^2 I_{t-1}$$

$$(\varepsilon_t / t) \sim N(0, h_t)$$

The series studied	1 month forward premium	3 month forward premium	6 month forward premium	9 month forward premium	12 month forward premium
Constant (M)	0.0,00,010* (2.31363)	0.0,00,024* (4.43948)	0.0,00,011* (2.39,207)	-0.0,00,001 (-0.37,459)	-0.0,00,001 (-0.21,519)
AR (1)	-0.1,25,426* (-2.69688)	-0.1,51,876* (-4.59,646)	0.0,69,609* (2.70,852)	0.1,89,291* (18.09,144)	0.1,13,052* (9.23,074)
β	-357.02,667* (-2.71,267)	-596.0496* (-4.77,271)	-95.94876 (-1.75,817)	-27.80,099 (-0.85,653)	-24.9,61,633 (-0.68,316)
Constant (V)	0.0000* (44.57,795)	0.0000* (21.62,984)	0.0000* (28.9444)	0.0000* (8.62,132)	0.0000* (9.78,175)
Arch	0.3,70,414* (22.03,647)	0.1,94,730* (22.18,277)	0.2,60,250* (27.86,947)	0.3,63,079* (16.47,822)	0.2,60,344* (19.55,872)
GARCH	0.1,29,634* (6.63,926)	0.3,66,911* (12.55,396)	0.6,17,053* (53.53,388)	0.7,41,668* (140.83,585)	0.7,80,006* (160.27,286)
D	-0.3,44,214* (-13.6692)	-0.0,33,149* (-2.00,951)	0.0341* (2.02,313)	0.1,10,365* (3.59,068)	0.0,53,118* (2.8,28,83)

Estimate made by Rats 7.0 software. The digital resolution was achieved by the algorithm BHHH (Berndt et al., 1974). D is the skewness. Values in parentheses are the t-student statistics. The superscript * indicates that the coefficient is statistically significant, GARCH: Generalized autoregressive conditional heteroscedastic

5. CONCLUSION

Our empirical analysis is based on daily data related to the spot and the 1 month, 3 month, 6 month, 9 month and 1 year forward exchange rates from January 8, 1999 to January 8, 2016. Our choice was based on the application of ARCH/GARCH modeling, given its descriptive and predictive advantages. Indeed, ARCH modeling allows to correct the heteroscedasticity and integrate the information content of the conditional variance that varies in time. Thus, at first, we estimate a symmetric linear model with the consideration of the average effect and the conditional variance effect in a univariate framework namely GARCH-in Mean. The results showed that the GARCH-M model is not a good specification for the EUR/USD at 3, 6 and 12 months forward premiums.

Secondly, we proceed the estimation of GJR-GARCH(1,1), AR(1)-GJR-GARCH(1,1) and AR(1)-GJR-GARCH-M(1,1) models that fits within a linear, bivariate and asymmetrical framework. The results relating to the model estimates indicate that shocks affecting (hitting) the conditional variance are quite persistent over time and that this high persistence of shocks of the conditional variance can reveal the presence of regime change in the process of explaining the variance as confirmed by Lamoureux and Lastrapes (1990). In addition, they show the existence of an asymmetry in the dynamics of the conditional variance characterizing the forward exchange premiums at 3 months and 6 months. However, the GJR-GARCH in mean effect is completely absent for the forward premium for a period of 6 months, 9 months and 1 year. That said, the comparative analysis conducted as part of this class of models pleads for the AR(1)-GJR-GARCH model to retrace the better the studied series. This appears to be legitimate given the inability of ARCH/GARCH standard models to consider certain phenomena such as cyclic oscillatory behavior, sudden shocks and asymmetry of the series volatility.

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