International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2020, 10(5), 243-255.

Asset Pricing With Higher Co-Moments and CVaR: Evidence from Pakistan Stock Exchange

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Received: 28 April 2020 **Accepted:** 18 June 2020 **DOI:** https://doi.org/10.32479/ijefi.10351

ABSTRACT

The development of asset pricing model is attributed to Markowitz (1952) which initiated towards Modern Portfolio Theory (MPT). The whole concept of MPT based on normality of returns assumption but in emerging economies volatility of returns is an important issue and sometimes markets only behave in either bullish or bearish patterns. Moreover, the volatility cannot be attributed and explained by variance rather it can be a result of extreme events (profits / losses) referred to as none elliptical distributions of returns. The objective of this study is to incorporate additional dimensions of risk in Markowitz Mean-Variance framework through inclusions of skewness kurtosis and coherent risk measure CVaR to obtain optimal portfolio with PGP approach. The study analyzes the portfolio returns of Mean-Variance (MV), Mean Variance Skewness (MVS), Mean Variance Skewness Kurtosis (MVSK) and Mean CVaR Skewness Kurtosis (MCVaRSK) models by using selected stocks of KSE-100 index over the time period of 2009-2018. The empirical findings suggest that portfolio returns impacted through inclusion of higher moments and CVaR and generated higher returns over the benchmark portfolio. The results of study are immensely useful for the fund managers and investors for stocks selection and construction of alternative portfolios.

Keywords: Portfolio Optimization, Mean CVaR Skewness Kurtosis, Multi Objective Optimization, Pakistan
JEL Classifications: C61, G10, G11

1. INTRODUCTION

The stock market in any country plays a significant role in economic development through saving mobilization. The developed liquid and efficient stock market mobilized savings easily which ultimately result in economic growth (Levine and Zervos, 1999). The Stock exchange provide platform to investors, to get optimize returns through diversification of investment. The investors and wealth managers always looked for those securities investment to which leads towards maximum returns therefore portfolio formation and the optimization has always been of major concern to them as the idiosyncratic risk may be reduced through creation of well diversified portfolio.

Portfolio is defined as pool of securities that is created by individual or institutional investors with a motive to earn returns/profit on investment. In order to optimal allocation of funds in stocks diversification is key, for the investors it is all about *"Don't put all your eggs in one basket"* the objective of diversification is to optimally select asset for the construction of portfolio aiming at possible reduction in risk to get maximum return. In the world of investment and finance this whole idea is attributed to the Markowitz (1952) "mean variance frame work" which leads towards Modern Portfolio Theory based on fundamental concept of tradeoff between risk and return. In existence of phenomena of normative theory investors are risk aversive and there only preference is to maximize expected returns, whereby the risk is measured through variance covariance of assets. Over the years this model is widely used in investment industry for optimal allocation of assets and construction of portfolios. One of the most important assumption of Markowitz mean variance frame work is the normality of returns which emphasizes that only mean and variance are sufficient indicators of portfolio optimization many studies theoretically and empirically rejected these assumptions and have reported the non-normality of returns (Fama, 1965; Kon, 1984; Harvey and Siddique, 2000).

The normality of return is an ideal phenomena which does not exist in case of emerging markets as they showed great volatility of returns and in presence of extreme events the returns might deviate from their average such markets are attributed to low liquidity, increased volatility and asymmetric information (Luciano and Marena, 2002; Ghysels and Valkanov, 2016; Sharif, 2018). For construction of portfolio according to investors decision the higher moments are relevant (Samuelson, 1958), the idea motivated many researcher to develop asset pricing model to address the issue as in the presence of extreme events i.e. Skewness and kurtosis the results obtained through MV model are inappropriate as probability of extreme events are underestimated (Jones, 2010). In 1976, Kraus and Litzenberger highlighted the use of higher order co moments and co skewness in order to explain the expected returns. Another view of higher order co moment presented in research of Friend and Westerfield (1980) that in addition to covariance, co-skewness required to be assed for asset pricing of individual assets moreover the presence of kurtosis indicates the higher probability of extreme events. Many academic researchers supported for inclusion of higher moments in construction of portfolios (Konno and Suzuki, 1992; Scott and Horvath, 1980; Lai et al., 2006; Saranya and Prasanna, 2014). In portfolio optimization the only risk left after diversification is systematic risk with the evolution of financial market financial regulatory body Basel committee on Banking Supervision (BCBS) make VaR as supplement to credit risk measure (Tian, Cai, & Fang, 2018). VaR is attributed as easy and practical approach to measure risk for portfolio managers (Miskolczi, 2016) but its non-convexity and subadditivity property ignores fat tail risk (Luciano and Marena, 2002). To overcome such issues CVaR or Expected Shortfall is efficient measure than VaR when the returns are non-normalized (Uryasev, 2000).

Pakistan is an emerging economy and its stock market showed a semi strong form of efficiency and the returns are highly volatile (Asghar et al., 2011; Snoussi and El-Aroui, 2012). Therefore use of MPT will provide suboptimal results (Chen, 2016). The current study is proposed due to the inability of MV theory to obtain optimize portfolios in cases of extreme events and fat tail risks moreover there is also need to incorporate coherent risk measure of CVaR for construction of optimal portfolio. Further, very few studies initiated to incorporate higher moments in mean variance frame work and as per observation so far no study has been witnessed which incorporate more accepted risk measure i.e. CVaR in MVSK framework in context of Pakistan.

The Polynomial goal programming (PGP) approach is used to solve the problem of investors decisions based on the multiple objectives to construct optimal portfolio. The PGP model developed by (Lai, 1991) is attributed as best approach to solve multiple conflicting criteria. Empirical findings indicate that the inclusion of higher moments and CVaR have strong influence on portfolio returns as compare to mean variance portfolio and benchmark portfolio. The results confirm the tradeoff between risk and return and contribute to the literature of portfolio optimization and these findings would also be useful for fund managers and investors to form optimal portfolios when the returns are non-normal.

The objective of this research is that most of the model so far only focused on the 2nd or third co moments model while very few studies in developing economy is available to account for the fourth moment. Moreover so far no study incorporated the higher order co moment in Mean CVaR portfolio optimization Model. The objectives are listed as under:-

- To compare the efficacy of portfolio optimization results obtained through different models i.e. from MV. MVS, MVSK, MCVaRSK for the stock returns of PSX.
- To develop a unifying portfolio optimization model that may overcome the deficiency of Markowitz MPT through analyzing higher co moments through inclusion of more efficient and widely accepted risk measure i.e. CVaR.
- To empirically investigate the results by analyzing stock returns and through portfolio construction from PSX by using polynomial goal programming technique.
- To better understand the PSX behavior that will help fund managers and practitioners to opt for alternative models for portfolio selection.

The remainder of the paper is organized as follows; section 2 incorporates brief literature review, Section 3; describes the research model data and methodology. Section 4; presents the empirical findings and results and section 5; consists of conclusions.

2. LITERATURE REVIEW

Investment management is mainly attributed to the security selection and asset allocation (Ross, 1976). In the world of finance the Markowitz (1952) first served the idea of risk and return which ensued to mean variance theory known as modern portfolio theory. This normative theory addressed the investor sentiment of risk and reward and remained widely used tool for investors for selection of stocks with an objective of portfolio optimization. Markowitz normative theory had certain limitations i.e. normality of returns and asset distribution is explained by its mean/expected value and variance, leading towards the issue that optimization result through MPT will be suboptimal when the returns are non-normal. (Copeland et al., 2013). This ideal phenomena of normal returns does not hold true in case of emerging markets due to presence of extreme events, low liquidity and asymmetric information (Luciano and Marena, 2002; Sharif, 2018). Several empirical investigation revealed the non-normality of returns (Baumol, 1963; Nazir et al., 2010).

The non-normality of return motivated many researchers to develop asset pricing model for the construction of optimal portfolio. To elaborate risk and return relationship time to time different models were proposed. Samuelson (1958) reported that for construction of portfolio according to investor’s decision the higher moments are relevant. Mirza and Reddy (2017) highlighted the use of higher order co moments i.e. co-skewness in order to explain the expected returns in empirical investigation conducted by Bollerslev et al. (1988) showed emerging markets exhibited great volatility of returns and have idiosyncratic and positive skewness. The efficacy of second moment i.e. variance covariance as a risk determinant was questioned and investigated by researchers in absence of normality of returns (Aggarwal et al., 1989; Tang and Choi, 1998; Lux and Marchesi, 2000). Another view of higher order co moment presented by in research of (Kraus and Litzenberger, 1976) whereby he incorporated the third moment in asset pricing model he presented that with addition to covariance, systematic skewness required to be incorporated for pricing of individual assets. Moreover the presence of kurtosis indicates the higher probability of extreme events. This portfolio selection problem was extended by (Davies et al., 2009) to hedge fund indexes. Jondeau and Rockinger (2006) study confirmed that in case of non-normality the first and second order moment model is ineffective and third moment or four moment can served as good approximation. Several academic researchers through empirical investigation of data have supported for inclusion of higher moments in construction of portfolios (Konno and Suzuki, 1992; Scott and Horvath, 1980; Lai et al., 2006; Saranya and Prasanna, 2014).

In portfolio optimization the only risk left after diversification is systematic risk with the evolution of financial market financial regulatory body Basel committee on Banking Supervision (BCBS) make VaR as supplement to credit risk measure. VaR is defined as the maximum loss over the given confidence level ($1-\alpha$). The concept of value at risk was introduced by Baumol (1963). Linsmeier and Pearson (2000) research reported that VaR as an acceptable risk measure for financial institutes, fund managers and regulators. Bali and Cakici (2004) attributed VaR as determinant of return in normal distribution further, Chabi-Yo et al., (2017) reported VaR reported same results as negative coskewness. Now a days VaR and Conditional Value-at-Risk (CVaR) are widely popular risk management tool (Guo et al., 2018). Further McKey and Keefer (1996) reported VaR inability to predict optimal position. Despite of the popularity of VaR it is reported by (Guo et al., 2018) that VaR is coherent risk measure when it is based on SD of normal distribution, further (Artzner et al., 1999) reported VaR inability to predict optimal position. Conditional Value at risk (CVaR) also referred to as tail VaR reports losses exceeding VaR, it is reported better performance measure than VaR (Uryasev, 2000). CVaR being possessing superior mathematical properties has an ability to measure and manage risk more efficiently (Artzner et al., 1999). CVaR considered as better technique for portfolio optimization and to evaluate risk (Uryasev, 2000). In order to address the inconsistencies in the mean variance frame work (Rockafeller and Uryasev, 2000) merge Mean-CVaR in Mean Variance framework. Being the popular risk measure and its reported efficiency to optimize portfolio another motive of the proposed study is to incorporate the CVaR in higher co moment model.

Portfolio optimization is a process of return optimization by selecting the best portfolio (asset allocation) (Agarwalla et al., 2017) within certain constraints it can be define as multi-functional, non-linear approach comprising varying objectives of investors. So for the multifunctional objectives polynomial goal programming is widely acceptable approach in literature in existence of varying objective of maximizing returns and minimizing variance and kurtosis. For non-linear optimization of portfolio the PGP approach proved to be the efficient technique (Roman et al., 2007). In order to manage bank balance sheet with conflicting objectives PGP was first introduced by (Kemalbay et al., 2005) for the portfolio construction with skewness this technique of portfolio optimization was used by (Tayi and Leonard, 1988) another dimensions mean-variance-skewness-kurtosis was added by Lai et al. (2006) and Škrinjarić (2013), subsequently PGP used for higher order comoments portfolio optimization by Lai (1991) and also used for optimization of hedge funds by Mhiri and Prigent (2010).

So based on the phenomena and after review of literature the research question arises:-

- Whether higher co-moments along with CVaR can be key indicator of gain or loss in developing economy like Pakistan?
- What is the degree and direction of these indicators have an impact on stock returns of PSX?
- CVaR measure in higher co moments frame work enhance the optimization of portfolio of stocks in PSX?
- How to optimize portfolio in presence of multipurpose objective?

3. DATA AND METHODOLOGY

PSX is the only representative market of stocks trading in Pakistan In 2016 the three stock exchanges (Lahore, Karachi and Islamabad) merged together and form single PSX (Economist, 2018). The performance of PSX is often measured with KSE-100 index which was declared as Asia's best in 2016. In 2017 the number of listed companies in PSX is 559 with market capitalization PKR 8.5 trillion. The importance of PSX is also accelerated with the initiative of OBOR in 2014 and in context of CPEC the 40% shares of PSX is transferred to the Chinese consortium for the purpose of strategic alliance after the competitive bidding process. On a positive note, so far, the market witnessed higher liquidity, less excessive volatility, and better returns for investors in the post-merger period compared to pre-merger period (CEIC, 2018). The resultant of the strategic alliance is that PSX is now listed to emerging market index of Morgan Stanley Capital Market Index (Sharif, 2018) which is served as catalyst to motivate new investor to invest in PSX and calls for research to identify the risks which may cause any sub- optimal investment decisions if may ignored.

For the purpose of analysis at first stage KSE-100 index companies stocks were selected. The data were obtained from website of PSX, SBP and standard Capital Securities (Pvt) Ltd, brokerage firm. The time span of stocks selected for valuation 10 years i.e. from 2009 to 2018. As most of the mutual funds in Pakistan report their performance on monthly basis and further, the monthly returns eliminate the element of noise which may distort the ability to draw real inferences about the investment strategy. Therefore monthly stock returns of 100 index companies were analyzed who constantly remain part of index. PSX is not efficient however a semi strong form of efficiency were observed by many studies (Ali and Mustafa, 2001; Hussain, 2017; Shamshair et al., 2018).

The returns of the stocks are mainly linked with the cash flows earned through its trade in market and also through the dividend receipts. In Pakistan the dividend payment are considered as one of the influential factor for investors as supported by the studies and results (Nazir et al., 2010; Hunjra et al., 2014). Another study conducted in context of Pakistan concluded that to form diversified portfolio 10 stocks should be selected (Ahuja, 2015). Therefore 10 stocks were selected for the construction of portfolio and for validity of higher co moment model with an additional and more acceptable risk measure of CVaR. The monthly returns of selected stocks are calculated by following the methodology of (Biglova et al., 2004).

$$Returns = \ln\left(\frac{P_{it}}{P_{it-1}}\right)$$

Returns are used in the portfolio optimization of Higher order co moment and CVaR detailed as under:

3.1. Model Formulation for Portfolio Optimization

Let w_k is weight of k assets in portfolio, W^t is transpose vector of weights assign to portfolios. Let X is distribution of stock returns the variables of the study are

Return/Mean = M = $(m_1, m_2, m_3,\ldots\ldots\ldots m_n)^t$,

V = Variance Covariance,
CVaR = Conditional Value at Risk
S = Skewness co-Skewness,
K = Kurtosis Co-Kurtosis

The measure of above mentioned variables are

$$M_p = \mathrm{E}(\mathrm{X}) = \mathrm{E}W^t(X-\bar{X})$$

$$\sum_{i=1}^{n} w_i m_i \quad (1)$$

$$V_p = \mathrm{V}(\mathrm{X}) = \mathrm{E}\,[W^t(X-\bar{X})(X-\bar{X})]$$

$$\mathrm{E}[W^t(X-\bar{X})^2]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \text{ where } i \neq j \quad (2)$$

$$\mathrm{S}_p = \mathrm{S}\,(\mathrm{X}) = \mathrm{E}\,[W^t(X-\bar{X})(X-\bar{X})(X-\bar{X})]$$

$$= \mathrm{E}[W^t(X-\bar{X})^3]$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} w_i w_j w_k s_{ijk} \quad (3)$$

$$K_p = \mathrm{E}\,[W^t(X-\bar{X})(X-\bar{X})(X-\bar{X})(X-\bar{X})]$$

$$= \mathrm{E}\,[W^t(X-\bar{X})^4]$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} w_i w_j w_k w_l s_{ijkl} \quad (4)$$

Further

$S_{\mathrm{iii}} = \mathrm{E}\,[X_i - \bar{X}_i]^3$ (skewness coefficient for individual stock) (5)

$S_{\mathrm{iij}} = \mathrm{E}\,[(X_i - \bar{X}_i)(X_j - \bar{X}_j)^2]$ (Skewness coefficient for two stock portfolio) (6)

$S_{\mathrm{ijk}} = \mathrm{E}\,[(X_i - \bar{X}_i)(X_j - \bar{X}_j)(X_k - \bar{X}_k)]$ (skewness coefficient for three stock portfolio (7)

Kurtosis co-kurtosis coefficients will be calculated as follows:-

$$K_{\mathrm{iiii}} = \mathrm{E}\,[X_i - \bar{X}_i]^4 \quad (8)$$

$$K_{\mathrm{iijj}} = \mathrm{E}\,[(X_i - \bar{X}_i)^2\,(X_j - \bar{X}_j)^2] \quad (9)$$

$$K_{\mathrm{ijjj}} = \mathrm{E}\,[(X_i - \bar{X}_i)\,(X_j - \bar{X}_j)^3] \quad (10)$$

$$K_{\mathrm{ijkl}} = \mathrm{E}\,[(X_i - \bar{X}_i)(X_j - \bar{X}_j)(X_k - \bar{X}_k)(X_l - \bar{X}_l)] \quad (11)$$

Value at Risk or VaR is defined as the value of maximum that will not exceed from the given level of confidence (Guo et al., 2018). Let X' random variable of loss then for given parameter $0< \alpha <1$ the VaRα for X' is defined as:-

$\mathrm{VaR}_\alpha = \min\{a: \mathrm{P}(X' \leq a) \geq \alpha\}$ i.e. the minimum loss not exceeded with α (12)

Conditional value at risk or CVaR which may also be referred to as conditional tail expectation or expected short fall may calculated as;-

$$CVaR_\alpha = \mathrm{E}[\,|X'|\,X' \geq VaR_\alpha] \quad \text{where } 0 < \alpha <$$

For discrete probability distribution for event Y_j' with probabilities P_j

$$CVaR_\alpha(X') = \frac{1}{1-\alpha} \sum_{j:f(X',Y') \geq VaR_\alpha(X')} P_j f(X',Y') \text{ for } (\mathrm{j} = 1,\ldots..,\mathrm{n}) \quad (13)$$

For continuous probability distribution. Let ϕ (.) be the standard normal cumulative distribution function and ϕ (.) is the standard normal density function then for any confidence level t ϵ (0.5, 1)

$$\phi_{(-zt)} = (1-t)$$

$$\int_{-\infty}^{-zt} \varphi(x)\,dx = 1-t$$

By using definition of VaR we can write it as

$$V\,[t, x_w] = z_t\,\sigma(x_w) - E(x)$$

CVaR is defined as the expected loss at confidence level by holding it over investment period where loss is ≥VaR, therefore CVaR at 100% confidence level t is

$$Y\,[t, x_w] = -E[x_w \backslash x_w \leq -V\,[t, x_w]]$$

If the min –CvaR exist then it will be mean variance efficient

Therefore CVaR is

$$Y\,[t, x_w] = j_t\,\sigma x_w - E(x_w)$$

Where $j_t = -\int_{-\infty}^{-zt} \frac{x\varphi(x)\,dx}{1-t}$ $\quad j_t > z_t \;\&Y[t,x_w] > V[t,x_w]$

Polynomial Goal Programming (PGP) will be used to get optimize portfolio with multiple objectives. Multi-objective optimization will be performed in two stages being considering following motives:-

$$\begin{cases} Max: M\;(X_p) \\ Min: V\;(X_p) \\ Min: CVaR_\alpha\left(X'_p\right) \\ Max: S\left(X_p\right) \\ Min: K\left(X_p\right) \\ Subject\,to\,W^t I = 1 where\, I = N*1 vector\, of\, 1 \\ w_i \geq 0 \end{cases} \quad (14)$$

Step II:

Let γ_1, γ_2, γ_3, γ_4 and γ_5 be the investor preferences over the given portfolio and let d_1, d_2, d_3, d_4, and d_5 non negative deviation from the parameter of optimized portfolio i.e. M'', V', $CVaR\alpha'$, S' and K' also called aspired value (Richta'rik, 2015).

By forming Polynomial Goal Programming Model multipurpose portfolio optimization converted to unifying model of portfolio optimization with an objective to minimize the deviations from the aspired values. So by implementing Minkowski distance (Lai et al., 2006)

$$Min: Z^* = \left|\frac{d_1}{M'}\right|^{\gamma_1} + \left|\frac{d_2}{V'}\right|^{\gamma_2} + \left|\frac{d_3}{CVaR_\alpha'}\right|^{\gamma_3} + \left|\frac{d_4}{S'}\right|^{\gamma_4} + \left|\frac{d_5}{K'}\right|^{\gamma_5} \quad (15)$$

Subject to:-

$$\begin{cases} W^t\,M + d_1 = M' \\ E\,[W^t\;(X - \bar{X}\;)^2\;] - d_2 = V' \\ CVaR_\alpha - d_3 = CVaR_\alpha{}' \\ E\left[W^t\left(X - \bar{X}\right)^3\right] + d_4 = S' \\ E\left[W^t\left(X - \bar{X}\right)^4\right] - d_5 = K' \\ For\,W^t I = 1 \\ w_i \geq 0 \\ d_i \geq 0 \end{cases}$$

By solving above equations the best value of w will be obtained for corresponding γ_i(i=1,….5) values. The R programming is used for

Table 1: Ranking of stocks based on coefficient of variation, skewness, kurtosis and CVaR

Stocks	Mean	Std. dev	Variance	Skewness	Kurtosis	CV	VaR	CVaR	Ranks C.V	Rank skewness	Rank kurtosis	Rank CVaR
ABOTT	0.0191971	0.082235	0.0067626	0.9303072	6.197667	4.283725	−0.092933	−0.1273661	3	6	6	3
APL	0.0097686	0.0657109	0.0043179	0.3373877	5.682908	6.726753	−0.08955806	−0.1392502	7	10	4	6
BATA	0.0131956	0.1126281	0.0126851	1.3915141	6.391803	8.535252	−0.1233456	−0.1636705	9	2	7	9
ENGRO	0.011487	0.0814464	0.0066335	0.3714696	3.635263	7.090286	−0.09180316	−0.1513686	8	8	1	7
INDU	0.0266732	0.097731	0.0095514	1.3058721	12.203935	3.664013	−0.1206709	−0.1743285	2	3	9	10
LUCK	0.0269386	0.086193	0.0074292	0.9051	6.716782	3.199607	−0.1072852	0.134751	1	7	8	5
MCB	0.0080548	0.0726276	0.0052748	1.1931486	5.456373	9.016649	−0.07828303	−0.1158474	10	5	3	1
PKGS	0.0142378	0.0864692	0.0074769	1.3043585	5.928126	6.073196	−0.09775049	−0.1186838	6	4	5	2
POL	0.0157865	0.0732112	0.0053599	0.3679387	4.045553	4.637583	−0.08900569	−0.1304724	4	9	2	4
THALL	0.020339	0.1056596	0.011164	2.623275	18.430007	5.194939	−0.0996195	−0.1516763	5	1	10	8

Source: Author's estimation and calculation.

the solving the equations and to estimate weights for construction of optimal portfolio under investors preferences.

4. EMPIRICAL RESULTS AND FINDINGS

The first step is to test the stock returns normality with Jarque-Bera test being identified as the best for testing of normality and the identification of non-normal returns of stocks. The hypothesis formulated for testing of normality as under:-
H_0 = The stocks returns are normally distributed.
H_1 = The stocks returns are not normally distributed

The rejection of null hypothesis is made on the basis of P-value related to JB test. The acceptance of alternative hypothesis is determined by the value of p at 5% level of significance. The test results showed that 82% of the stocks are non-normal which gives an importance of inclusion of higher order moments in Pakistan Stock exchange. For the purpose of portfolio construction 10 stocks were selected to determine the impact of the model on portfolio optimization to get optimal weight for each stock. Polynomial Goal Programming (PGP) is used to solve multipurpose motives to construct optimized portfolio. Descriptive statistics of selected stocks and the ranking were made on the basis of coefficient of variation, higher moments and CVaR indicates the significant impact of selection of stocks for the purpose of optimal portfolio.

Table 1 presents the descriptive statistics of each selected stock of KSE-100 index and the ranks according to the desirable criteria of selection. Coefficient of variation measure the ratio of risk to return, it is indicated from the result of the Table 1 that LUCK being highest due to lower risk to expected return but in terms of skewness and kurtosis it is ranked at 7 and 8 respectively, while MCB is ranked at the lowest in terms of risk per unit of return but varying ranks in respect of skewness kurtosis and CVaR. These variations in results motivated to investigate for inclusion of higher moment along with CVaR in Mean-Variance asset pricing model.

The Figure 1 represents the 10 individual stocks behavior evaluated on the individual criteria of return and risk.

Table 2: Optimal solution for individual criterion

Criteria	Mean	Variance	CVaR	CoSkewness	CoKurtosis
Optimal scores	0.02441	0.0026	0.0752	0.0009055	0.000026

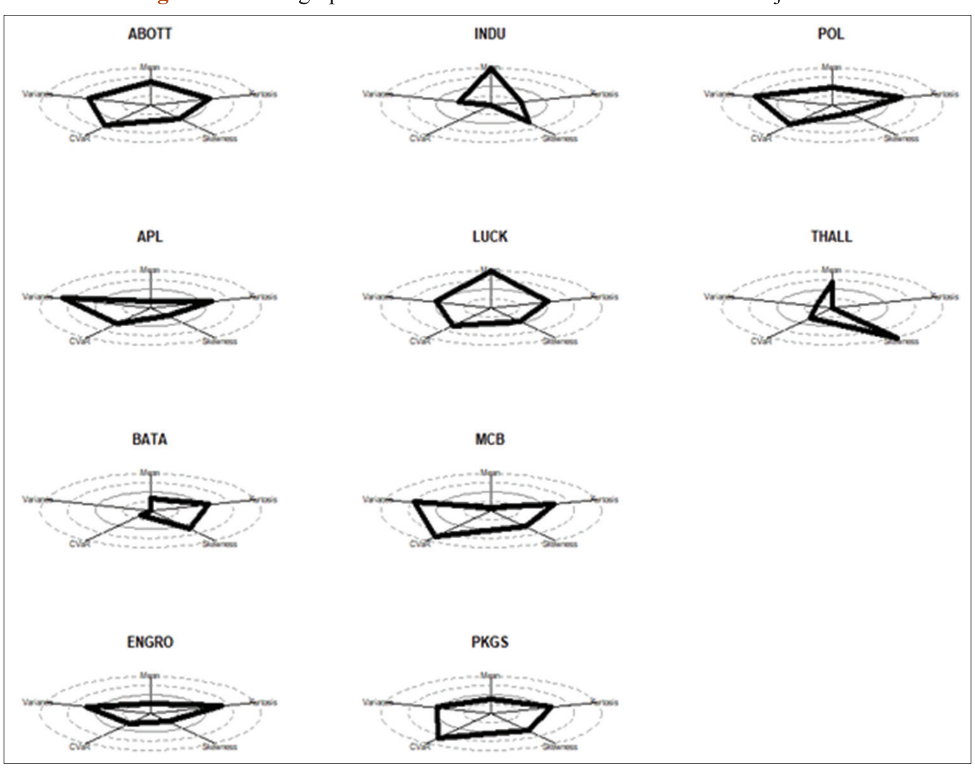
Source: Author's own estimation and calculation

Table 3: Optimal weight allocation under individual objective

Stocks	Mean	Variance	CVaR	Skewness	Kurtosis
ABOTT	0.05	0.01674	0.2756	0.05	0.254
APL	0.05	0.2105	0.05	0.076	0.166
BATA	0.05	0.05	0.05	0.052	0.05
ENGRO	0.05	0.1681	0.2255	0.05	0.166
INDU	0.2	0.05	0.05	0.064	0.05
LUCK	0.5	0.05	0.05	0.066	0.05
MCB	0.05	0.05	0.05	0.056	0.05
PKGS	0.05	0.05	0.05	0.05	0.05
POL	0.05	0.1441	0.0786	0.05	0.104
THALL	0.05	0.05	0.1103	0.494	0.05

Source: Author's own estimation and calculation

Figure 1: Radar graphs of 10 selected stocks evaluated on each objective



Source: Author's estimation and calculations

To solve the multipurpose objective of investment preference criteria Table 2 give the results of aspired values of each criteria.

The weights corresponding to the aspired levels of each stock is represented in Table 3.

The proposed model is solved by substituting the aspired levels in the in equation (15). The model is evaluated to solve for multi objective criteria according to the investor preference. The result of the model is compared with the equally weighted model which is used as the bench mark test. The R programming used by utilizing multiple packages to solve the optimization problem of portfolio (detail out puts are placed as Annexure-A and B). Through changing the investor preferences for the selected portfolio 18 portfolios were evaluated accordingly. The optimal values of objective as per the investor preferences criteria are presented in the Table 4 detailed as under:-

Table 4 results clearly indicates that the higher preference for the expected return increase the value of mean and vice versa. Further, the findings also indicate the presence of higher moments and the inclusion of CVaR in the mean variance optimized model have significant impact on the portfolio returns. The results of the model have evident impact on the returns with the additional dimensions of risk. If we compare the mean variance

Table 4: Optimal value of objectives and trade off in objective criteria

Investor preferences	1	2	3	4	5	6	7	8	9
γ1	1	3	3	3	1	1	0	0	0
γ2	1	1	1	1	3	0	1	0	0
γ3	0	2	1	2	1	0	0	1	0
γ4	0	1	0	2	0	0	0	0	1
γ5	0	0	0	1	0	0	0	0	0
Mean	0.024390	0.019590	0.015875	0.019700	0.017350	0.024410	00.02130	0.016460	0.018450
Variance	0.005200	0.004161	0.002910	0.004217	0.003295	0.005300	0.002590	0.002980	0.005270
CVaR	0.112600	0.088000	0.090400	0.087400	0.088040	0.112000	0.094500	0.075200	0.089280
Coskewness	0.000560	0.004200	0.000085	0.000473	0.000198	0.000512	0.000890	0.000104	0.000906
Cokurtosis	0.000302	0.000179	0.000040	0.000212	0.000081	0.000280	0.000447	0.004012	0.000440
Investor Preferences	10	11	12	13	14	15	16	17	18
γ1	0	1	1	1	1	1	1	1	1
γ2	0	0	1	1	1	0	1	0	0
γ3	0	1	0	0	1	1	0	1	1
γ4	0	0	1	0	1	1	1	0	1
γ5	1	0	0	1	1	0	1	1	1
Mean	0.015500	0.023900	0.017100	0.017470	0.019740	0.023260	0.016530	0.021810	0.020720
Variance	0.002650	0.004950	0.002849	0.002865	0.004200	0.005500	0.002800	0.004614	0.004130
CVaR	0.085200	0.094300	0.091900	0.081000	0.041260	0.043750	0.091100	0.041350	0.039950
Coskewness	0.000046	0.000481	0.000074	0.000109	0.000344	0.000704	0.000071	0.000527	0.000436
Cokurtosis	0.000026	0.000254	0.000035	0.000047	0.000155	0.000400	0.000034	0.000269	0.000208

Table 5: Assets allocation as per investors preference for construction of optimal portfolio

Portfolio	1	2	3	4	5	6	7	8	9
Γi	(1,1,0,0,0)	(3,1,2,1,0)	(3,1,1,0,0)	(3,1,2,2,1)	(1,3,1,0,0)	(1,0,0,0,0)	(0,1,0,0,0)	(0,0,1,0,0)	(0,0,0,1,0)
ABOTT	0.05	0.08	0.13	0.11	0.11	0.05	0.17	0.28	0.05
APL	0.05	0	0.14	0.01	0.08	0.05	0.21	0.05	0.076
BATA	0.05	0.1	0.02	0.1	0.08	0.05	0.05	0.05	0.052
ENGRO	0.05	0.01	0.11	0.02	0.08	0.2	0.17	0.23	0.05
INDU	0.27	0.13	0.06	0.17	0.11	0.5	0.05	0.05	0.064
LUCK	0.43	0.1	0.11	0.15	0.12	0.05	0.05	0.05	0.066
MCB	0.05	0.07	0.14	0.06	0.11	0.05	0.05	0.05	0.056
PKGS	0.05	0.1	0.11	0.1	0.11	0.05	0.05	0.05	0.05
POL	0.05	0.24	0.04	0.05	0.09	0.05	0.14	0.08	0.05
THALL	0.05	0.23	0.05	0.23	0.12	0.05	0.05	0.11	0.494
	10	11	12	13	14	15	16	17	18
	(0,0,0,0,1)	(1,0,1,0,0,0)	(1,1,0,1,0)	(1,1,0,0,1)	(1,1,1,1,1)	(1,0,1,1,0)	(1,1,0,1,1)	(1,0,1,0,1)	(1,0,1,1,1)
ABOTT	0.254	0.073	0.21	0.174	0.064	0.05	0.22	0.062	0.078
APL	0.166	0.05	0.06	0.068	0.08	0.05	0.056	0.05	0.052
BATA	0.05	0.05	0.06	0.056	0.07	0.05	0.05	0.068	0.064
ENGRO	0.166	0.05	0.06	0.124	0.054	0.054	0.108	0.05	0.05
INDU	0.05	0.2331	0.08	0.116	0.24	0.496	0.068	0.374	0.314
LUCK	0.05	0.4123	0.08	0.116	0.2	0.102	0.072	0.166	0.178
MCB	0.05	0.05	0.05	0.05	0.062	0.05	0.074	0.06	0.082
PKGS	0.05	0.05	0.06	0.05	0.05	0.08	0.106	0.068	0.054
POL	0.104	0.0745	0.28	0.182	0.078	0.062	0.176	0.054	0.052
THALL	0.0571	0.0571	0.05	0.054	0.096	0.1	0.06	0.094	0.092

Source: Author's estimation and calculations

Figure 2: Comparison of cumulative returns for portfolio of Mean-variance and Mean-CVaR, with benchmark

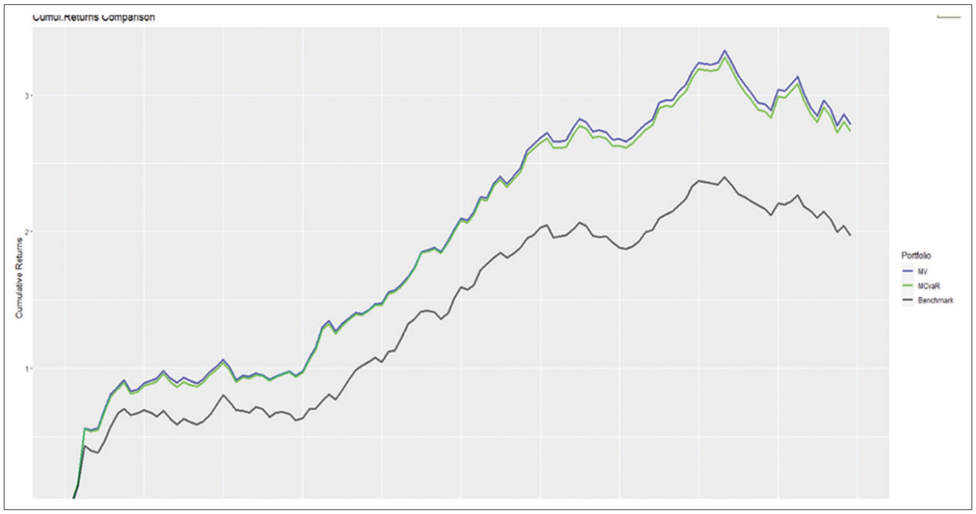
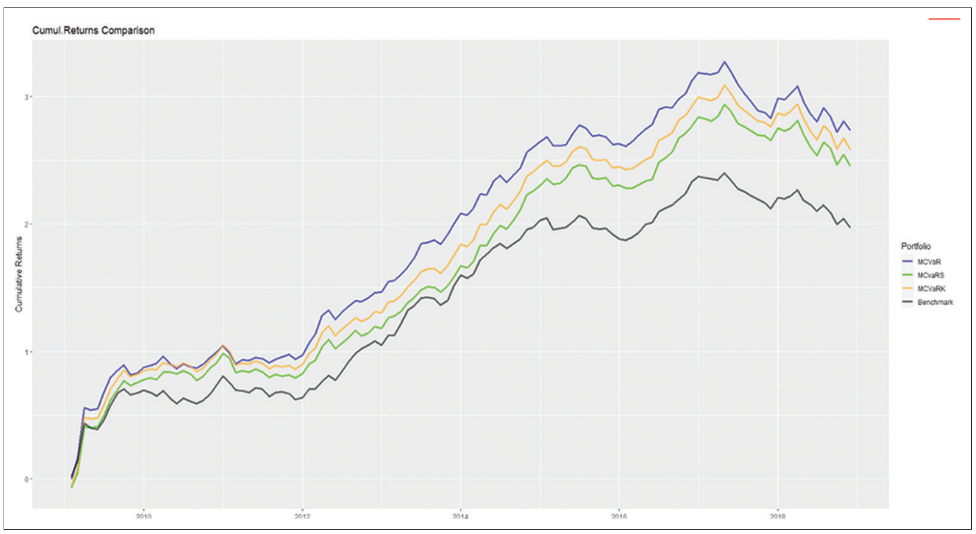


Figure 3: Cumulative returns for the portfolio models MCVaR, MCVaRS, MCVaRK vs benchmark



portfolio (portfolio1) with mean-CVaR portfolio it is evident that the highest expected return is achieved in both of these portfolio but with the Mean-CVaR optimization the additional risk dimensions value is lower with the same level of return as compare to the only Mean-Variance criteria. As for choosing only Mean-Variance criteria investor opted to choose for extreme events. The optimal combination of stocks which satisfies the investor criteria is presented in Table 5.

Table 5 indicates the optimal allocation of stocks as per investor's preference criteria. For the performance evaluation of portfolios under higher moment preference and CVaR the portfolio cumulative returns has been calculated and compared with the benchmark .

The Figures 2-5 represents cumulative return comparison of models Mean-Variance, Mean-CVaR, MCVaRS, MCVaRK, MVS and MVK, MVCVaRSK, MVSK with the bench mark portfolio over the research time frame of 2009-2018. In Figure 2 both the portfolios i.e., Mean-Variance and Mean-CVaR outperform as compared to the bench mark portfolio. The Figure 3 presents the model comparison of portfolio formed under Mean-CVaR, Mean-CVaR-Skewness, Mean-CVaR-Kurtosis vs Benchmark the cumulative return performance of alternative model formed with higher co moments along with more coherent risk measure of CVaR yield higher returns as compared to the bench mark. Further, from Figure 4 the risk return tradeoff is clearly evident as if investor wants to minimize additional dimensions of risk then he has to forego additional returns associated with higher risk. The comparison of graph 3 and 5 demonstrate that the higher order comoments models i.e. Mean-CVaR-Skewness, Mean-CVaR-Kurtosis yield higher cumulative returns over the benchmark but for MVCVaRSK the results are close to and for MVSK models the mix trend is observed in performance comparison to benchmark portfolio. The results strongly recommend the higher dimensions of risk should be incorporated in case of non-normal return behavior of stock market and to determine the true optimization process which ultimately will be beneficial for investor exposure to high volatile periods and to protect against extra ordinary losses in emerging markets where the returns are non-normal.

Figure 4: Cumulative return comparison of portfolio models MV, MVS, MVK with benchmark portfolio

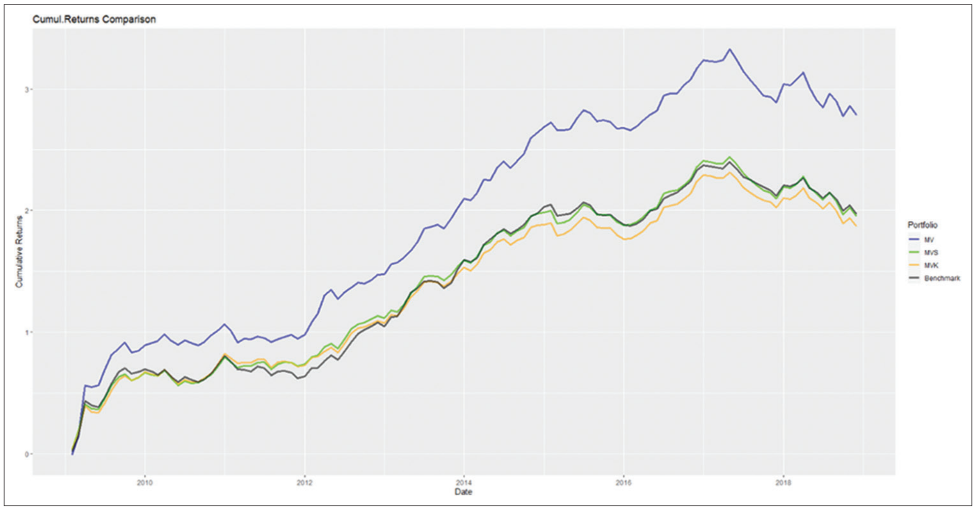
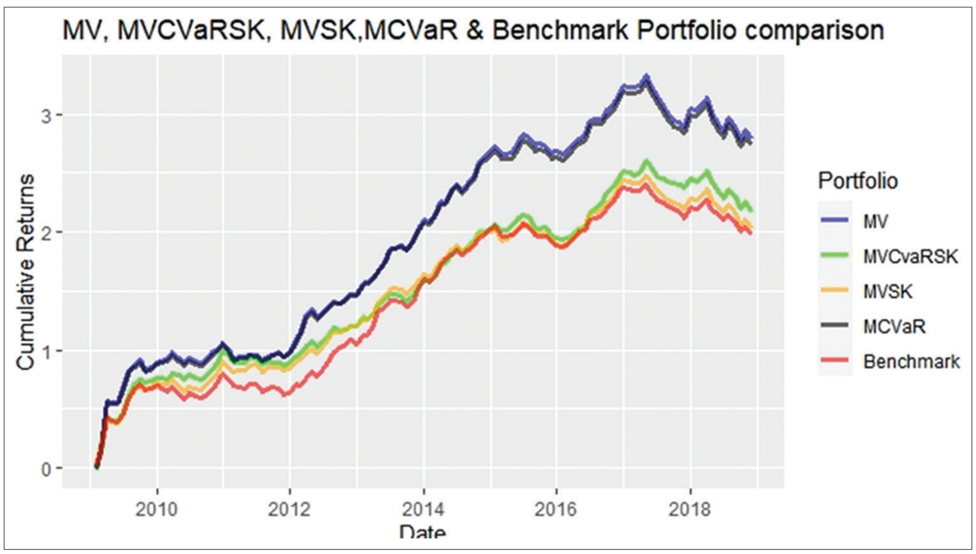


Figure 5: Cumulative return comparison of portfolio models MVCVaRSK, MVSK, MCVaR with benchmark portfolio



5. CONCLUSION

In this study we expand the Markowitz theory of portfolio optimization through inclusion of higher order co-moments and more sophisticated risk measure of CVaR. The Markowitz theory is criticized due to its assumptions of normality of stocks which is not held true in practical world. Emerging markets like Pakistan which are not efficient showed the behavior of non-normality of returns. The presence of such phenomena search for inclusion of higher order comoments and more sophisticated risk measure to avoid high volatility of returns.

The empirical findings of this study indicate the inclusion of higher dimensions of risk will have an impact on returns of portfolio. In order to get an optimized return in emerging market like PSX investor needs to incorporate higher dimensions of risk in order avoid extreme volatility. To have stable consistent returns there is a strong need to consider these multidimensional risk instead of forming portfolio based on Mean-Variance criteria. So, the study concluded that ignoring higher dimensions of risk will lead towards mispricing which ultimately have an impact on sustainable returns.

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ANNEXURE-A

Descriptive statistics and normality test results

Stocks	Mean	St. dev	Variance	Skewness	Kurtosis	CV	P-value
ABL	0.0132490	0.0684599	0.0046868	1.0404605	5.088675	5.167172	5.72494E−10
ABOTT	0.0191971	0.082235	0.0067626	0.9303072	6.197667	4.283725	2.22045E−15
ACPL	0.0152765	0.0978478	0.0095742	0.5385335	4.118813	6.405097	0.002720356
AICL	0.0011868	0.0994497	0.0098902	0.244262	8.459326	83.794646	0
APL	0.0097686	0.0657109	0.0043179	0.3373877	5.682908	6.726753	5.91682E−09
ATLH	0.0116856	0.0937027	0.0087802	0.0125565	5.463582	8.018656	2.91516E−07
ATRL	0.0157917	0.114399	0.0130871	1.5904872	8.986616	7.24424	0
BAF	0.0124939	0.0696305	0.0048484	0.0564588	2.626702	5.573141	0.686402
BAHL	0.0095237	0.0581736	0.0033842	−0.1938788	3.520002	6.1083	0.3556675
BATA	0.0131956	0.1126281	0.0126851	1.3915141	6.391803	8.535252	0
BNWM	0.0081143	0.1351684	0.0182705	0.3552532	5.260212	16.657954	9.32837E−07
BOP	0.0083749	0.1188198	0.0141182	1.3211184	5.304217	14.187585	9.02611E−14
BWCL	0.0212822	0.1510597	0.022819	1.4929462	7.188005	7.097929	0
BYCO	0.0094793	0.1168321	0.0136497	1.3863993	5.990078	12.325016	0
CHCC	0.0200870	0.11266	0.0126923	0.8198316	4.436603	5.608606	9.03646E−06
CPPL	0.0213287	0.134509	0.0180927	1.1882017	5.690443	6.306495	1.89848E−14
DAWH	0.0075672	0.1208359	0.0146013	−1.1896743	11.902864	15.968329	0
DJKC	0.0186727	0.1011919	0.0102398	0.6488998	3.942549	5.419242	0.001886303
EFUG	0.0054123	0.0924185	0.0085412	1.1174289	8.157586	17.075729	0
ENGRO	0.0114870	0.0814464	0.0066335	0.3714696	3.635263	7.090286	0.09686022
FABL	0.0096511	0.0882839	0.007794	0.7654514	3.899505	9.147555	0.000466533
FCCL	0.0165151	0.0934915	0.0087407	0.168559	2.68277	5.660959	0.592052
FFBL	0.0101181	0.06663	0.0044396	0.2677728	3.259853	6.585237	0.4229188
GADT	0.0238194	0.1257337	0.015809	−0.0257125	3.84189	5.278628	0.1714316
GHGL	0.0030057	0.0863794	0.0074614	−0.1128659	6.051556	28.738194	8.32537E−11
GLAXO	0.0060694	0.0822851	0.0067708	1.6080962	9.959318	13.557465	0
HBL	0.008887	0.0794016	0.0063046	0.9794826	5.927871	8.934561	5.51781E−14
HCAR	0.0309188	0.1315333	0.017301	0.9053408	4.401018	4.254149	2.78847E−06
HMB	0.005458	0.0675408	0.0045618	0.3110622	3.128357	12.374605	0.3767184
HUBC	0.0161201	0.0583456	0.0034042	1.172332	6.795398	3.619428	0
HUMNL	−0.0034459	0.1333675	0.0177869	−1.6213181	13.923737	−38.70292	0
IBFL	0.0096277	0.0979634	0.0095968	1.085166	6.094566	10.175141	5.55112E−16
ICI	0.0266152	0.105779	0.0111892	2.4295189	14.465737	3.974381	0
INDU	0.0266732	0.097731	0.0095514	1.3058721	12.203935	3.664013	0
INIL	0.014834	0.1004213	0.0100844	0.4868312	2.82543	6.76966	0.09378377
JDWS	0.020386	0.0867598	0.0075273	1.1259308	6.353972	4.255854	0
JGICL	−0.000665	0.0756632	0.0057249	0.1888686	7.247467	−113.77258	0
JLICL	0.0267217	0.0870202	0.0075725	1.389514	10.543068	3.256539	0
JSCL	0.0052808	0.1703259	0.0290109	2.1750501	10.995278	32.253616	0
KAPCO	0.0039217	0.0497851	0.0024786	0.075372	4.170727	12.694758	0.03165467
KEL	0.0138202	0.1198242	0.0143578	1.8860013	7.853089	8.670214	0
KOHC	0.0252254	0.1333188	0.0177739	1.3210785	6.583232	5.285108	0

(Contd...)

Continued

Stocks	Mean	St. dev	Variance	Skewness	Kurtosis	CV	P-value
KTML	0.0269588	0.1416213	0.0200566	1.5097217	7.680061	5.253251	0
LUCK	0.0269386	0.086193	0.0074292	0.9051	6.716782	3.199607	0
MARI	0.0313516	0.1372763	0.0188448	1.0396643	7.429576	4.378611	0
MCB	0.0080548	0.0726276	0.0052748	1.1931486	5.456373	9.016649	3.36176E−13
MEBL	0.016474	0.0599336	0.003592	0.4227311	3.070468	3.638075	0.1755405
MLCF	0.0280955	0.140134	0.0196375	1.6813687	8.856107	4.987778	0
MTL	0.0176176	0.0778218	0.0060562	−0.2965133	3.481092	4.417265	0.2408071
MUREB	0.0259865	0.1302678	0.0169697	2.0461208	10.509455	5.012897	0
NATF	0.0176728	0.1244824	0.0154959	0.528806	7.328598	7.04374	0
NBP	0.0015016	0.0772281	0.0059642	−0.0510279	3.906418	51.431256	0.1271944
NCL	0.0203414	0.1069232	0.0114326	0.4757667	3.859941	5.256426	0.01792444
NESTLE	0.0205733	0.0850316	0.0072304	0.6769748	4.120329	4.133107	0.00053023
NRL	0.0121948	0.0942501	0.0088831	0.7454379	5.874556	7.728719	5.89173E−12
OGDC	0.0111942	0.0676015	0.00457	0.5698864	3.987516	6.038964	0.003859068
OLPL	0.0127581	0.0954599	0.0091126	0.1931386	4.936221	7.482282	6.41112E−05
PAEL	0.0130202	0.136915	0.0187457	0.7155574	3.530691	10.515571	0.003524119
PAKT	0.0366319	0.1216899	0.0148084	1.659154	6.150523	3.321969	0
PMPK	0.0280412	0.1347536	0.0181585	1.0759546	5.665124	4.805559	3.10418E−13
PICT	0.0211536	0.1131466	0.0128021	2.1148829	9.146733	5.348803	0
PIOC	0.0183629	0.1464737	0.0214545	1.0644758	7.385358	7.976627	0
PKGS	0.0142378	0.0864692	0.0074769	1.3043585	5.928126	6.073196	0
POL	0.0157865	0.0732112	0.0053599	0.3679388	4.045553	4.637583	0
POML	0.0225542	0.1131676	0.0128069	1.3848949	7.138461	5.017589	0
PPL	0.0033394	0.0596033	0.0035526	0.0407968	3.166014	17.848475	0.9190471
PSMC	0.0156965	0.0986564	0.0097331	0.6032087	3.635517	6.28526	0.01090135
PSO	0.0095799	0.0840597	0.007066	0.4643415	4.303332	8.774601	0.0018442
PTC	−0.0004927	0.0748886	0.0056083	1.2566113	5.699875	−151.98558	3.33067E−15
SCBPL	0.0099828	0.0658683	0.0043386	0.8046955	4.137681	6.598157	7.72248E−05
SHEL	0.0049221	0.078997	0.0062405	0.2195452	3.5606	16.049509	0.2879114
SHFA	0.0241239	0.0977624	0.0095575	1.3596463	6.723966	4.052519	0
SML	0.0296827	0.1462997	0.0214036	1.3491014	5.40606	4.928779	1.33227E−14
SNBL	0.0037692	0.0905135	0.0081927	1.2498947	5.878234	24.014139	3.33067E−16
SNGP	0.0136899	0.1032841	0.0106676	0.2792946	2.69595	7.544573	0.3740753
SSGC	0.0089943	0.0951877	0.0090607	0.1735969	3.104158	10.58306	0.7274347
THALL	0.020339	0.1056596	0.011164	2.623275	18.430007	5.194939	0
TRG	0.0326925	0.1670759	0.0279144	1.395618	6.261428	5.110528	0

Source: Author’s Estimation and Calculations

ANNEXURE-B

Figure 1: Stock performance over each individual criteria of preference for construction of portfolio

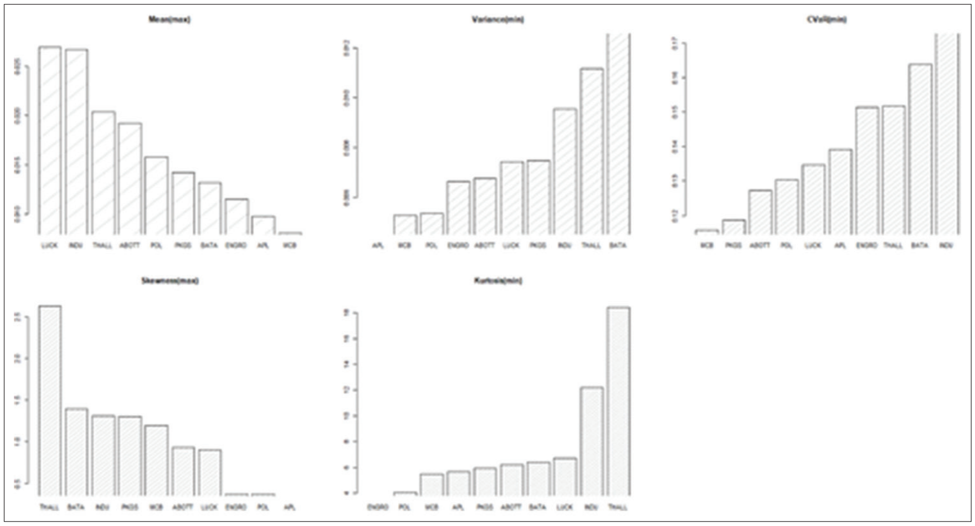

Figure 2: Mean variance optimized portfolio returns with benchmark portfolio

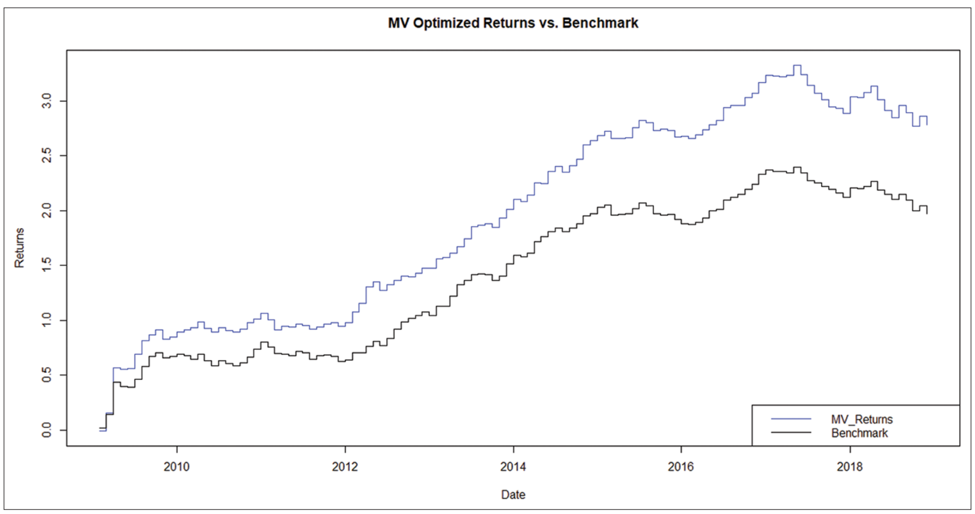

Figure 3: Mean-CVaR portfolio comparion with benchmark portfolio