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Cointegration Relationships to Estimate the Marginal Cost of Deficit in Planning a Hydrothermal System: The Case of Brazil

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ABSTRACT: The aim of this study is to analyze the behavior of gross domestic product (GDP) compared to electricity consumption in Brazil to estimate the curve of deficit marginal cost. The deficit cost is used as exogenous parameter in the chain of models for planning the operation and expansion of a hydrothermal system as part of the total cost of operation. The results show a cointegration relationship between GDP and electricity consumption; therefore, there is a long-term equilibrium relationship between GDP and electricity consumption. This relationship is used to estimate the curve of deficit marginal cost. The possible short-term imbalance can be mitigated using the vector error correction model (VEC).

Keywords: Energy deficit; Cointegration; Rationing; Energy planning
JEL Classifications: Q40; Q43; Q47; Q48

1. Introduction

Brazil has a large hydrothermal system, the main characteristic of which is the predominance of hydropower generated by several hydropower plants with large reservoirs with a multiannual regularization capacity, spread over several watersheds and interconnected by a large transmission network.

Given that the prevalence of hydraulic power (approximately 80% of total generation) requires properly regulated water storage reservoirs, it is important to estimate the values that allow the penalization of the emptying of reservoirs to prevent future power outages. This penalty is known as a deficit cost.

The marginal cost of electrical power deficit is defined as the economic loss suffered by a country due to a 1-MWh restriction or reduction in electricity supply. This cost is higher for greater deficits. Between 2001 and 2002, due to a lack of planning, an unprecedented crisis in Brazil affected electricity supply and distribution. Due to low rainfall, the power plants (hydro and thermal) were unable to meet the total demand. The alternative of installing new plants required large investments and long lead times for construction; therefore, the federal government had to introduce a rationing program. The immediate effect of this rationing was the retraction of the productive sector and the level of employment. The electricity consumption decreased sharply from 307.53 GWh in 2000 to 283.26 GWh in 2001 and 289.86 GWh in 2002, dramatically jeopardizing Brazil's economic growth.
The economic growth of a country is measured by an indicator called the gross domestic product (GDP). This indicator shows the value of all of the wealth generated in a country and is the production value within its geographical boundaries in a given period. Therefore, if a rationing of the electrical energy supply occurs, as reported previously, the loss or injury to the country can be measured by the GDP.

The planning of the short- medium- and long-term operation energy of an electrical system allows the available resources for electricity generation to be optimally utilized, providing a service with a high degree of reliability and reduced cost to society. In a hydrothermal system, the total operation cost to be minimized is composed of the operation cost of thermal power plants plus the cost of the electrical power deficit (deficit marginal cost × deficit).

The goal of the consideration of the deficit cost is to efficiently regulate the use of the water reserve, keeping reservoir levels balanced for future use. The deficit cost is expected to increase to the extent that decreases the reservoirs' storage or there is a future trend of low inflows. If the deficit cost is overestimated, water will be stored unnecessarily based on the fear of a possible future rationing, leading to a reduction in reservoir levels and an unnecessary increase in electricity prices (due to the use of more thermal power plants). On the other hand, if it is underestimated, excessive short-term use of reservoirs may occur, resulting in an increased risk of future deficits, which would imply that more thermal plants would be required to become operational to meet future demand, which would increase the price of electric power and lead to potential power shortages.

1.1. Different Forms of Constraint in Electric Energy Supply

Restrictions in electricity supply can occur in two ways. The first way, known as interruption, is a sudden restriction in power supply, which is unannounced and a surprise to the user. It is of short duration and is caused by equipment failure, operation failure or other unforeseen technical issues.

The second way, known as deficit, includes scheduled constraints in the power supply, of which users are given prior notice. This form of restriction is linked to the concept of rationing. The duration is longer (weeks or months), and its causes are of a structural nature, i.e., low reservoirs levels or insufficient capacity expansion in the generation system or transmission/distribution network relative to the growth in demand.

The methodologies used to model these two types of restrictions in the power supply are different. This article focuses only on "power deficit" and modeling the deficit cost curve in particular. There is a vast literature on the cost of power interruption, including the following noteworthy contributions (Hsu et al., 1994; Nooij et al., 2003, 2007; Kariuki and Allan, 1996; Woo and Train, 1988; Sullivan et al., 1996; Reichl et al., 2013; Adoghe et al., 2013).

In contrast, despite the importance of the subject, few works in the literature address the deficit cost curve. Souza and Soares (2007) present a study on the electricity consumption trends during the rationing in Brazil in the period 2001-2002, their findings show that during this rationing period, there was a structural break in the series of energy consumption. Nooij et al. (2009) analyze the social cost of regional electrical power deficit in times of scarcity and demonstrate that efficient rationing actions reduce the deficit social cost, in the aforementioned paper, the authors note the difference between efficient rationing cost and random rationing cost. Eletrobras (1986 and 1988) presents a methodology to estimate the social cost of power rationing using the input-output model and econometric models. A technical report of the OSINERGMIN Economic Studies Workshop (2012) estimates the rationing cost for the Peruvian electricity sector. Initially, the deficit cost is set according to the current legal framework. Next, using statistical techniques, the average deficit cost for the Peruvian electricity sector is estimated. In his master's dissertation, Loureiro (2009) conducted a historical review of the deficit cost and then estimates the deficit cost for the Brazilian power sector using simple regression techniques for the relationship between power consumption and GDP. Galetovic and Muñoz (2009) present an estimate for the deficit risk in Chile's interconnected power system between 2006 and 2010 based on the elasticity of demand and power price. Kelman et al. (2007) propose an alternative indicator to calculate the deficit risk by establishing the amount to be rationed from a conservative assumption for future inflows. Carpio (2006) develops a stochastic model to study the economic growth of Brazil, the convergence and stability of the economic growth model are investigated, revealing that the stock of water in the reservoirs is a basic variable in this model.

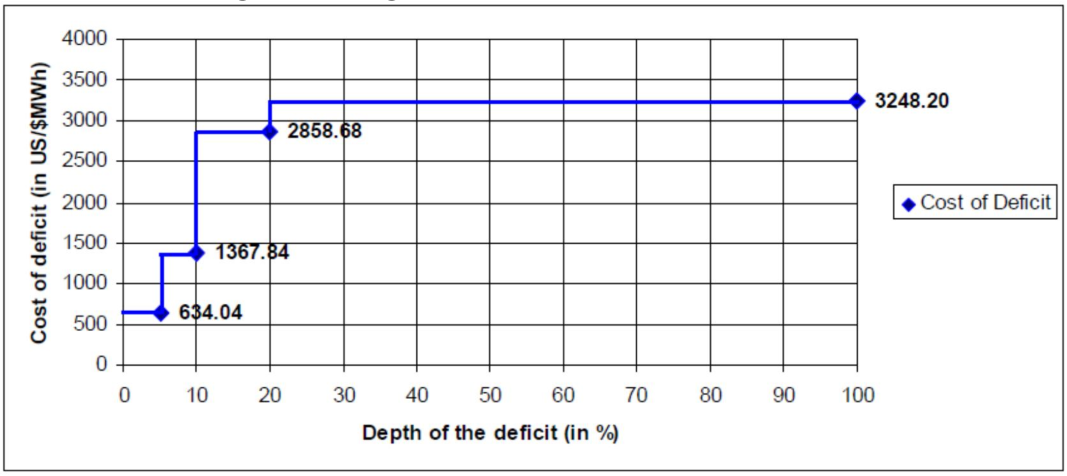
Despite the constant efforts of the National Agency of Electric Energy (ANEEL), there is currently no satisfactory methodology for estimating the curve of marginal cost for electric power deficits in Brazil.

1.2. Deficit Cost

The operation planning of an interconnected system aims at minimizing the expected value of the total operating costs plus the deficit cost; therefore, the deficit cost is an important parameter because it is crucial in the formation of operation marginal costs and consequently the market price. The cost of the deficit should reflect, in practice, how much the insufficiency of the electrical energy supply costs society. In any situation in which there is a deficit, the marginal cost of short-term operation is equal to the marginal cost of deficit.

The marginal cost deficit is expected to grow with the depth of the deficit: the deeper the deficit, the more expensive the next MWh of deficit should be. In Brazil, from the year 2003, the depth of the deficit is divided into four levels, (0% - 5%], (5% - 10%], (10% - 20%] and (20% - ∞), and a cost is defined for each level. The curve for the year 2013 is shown in figure 1.

Figure 1. Marginal Cost of the Electric Power Deficit for 2013[1].



Own elaboration with data from ANEEL (2013).

According to this curve, a 0.01% deficit has the same unit cost as a deficit of 5%, and a deficit of 10.1% has the same unit cost as a deficit of 20%. This has a strong impact on the price of electric power. This study presents a methodology to estimate the marginal cost curve of energy deficits at unitary deficit levels, aiming to smooth the curve in a balanced way and thus avoid future shortages and high energy costs in periods of water shortage in reservoirs.

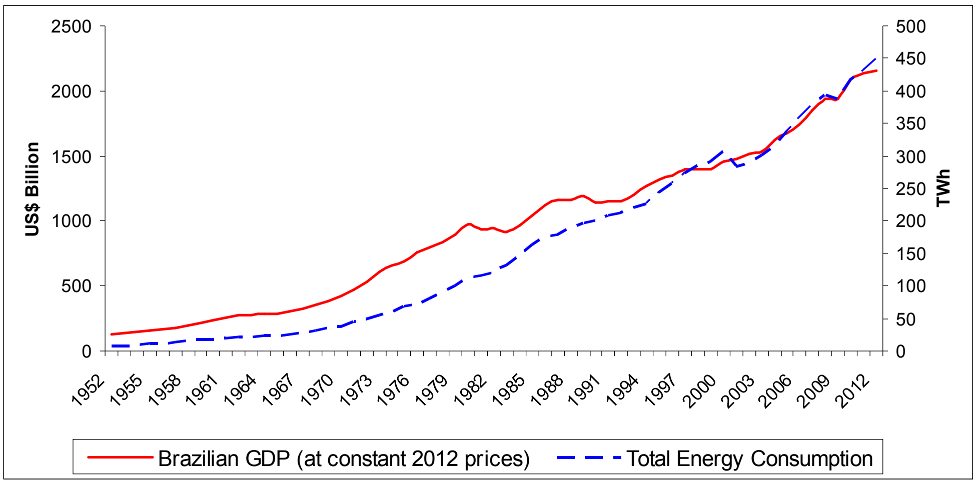
2. Methodology

The deficit cost curve presented in this study is based on the cointegration relationship between electricity consumption and economic growth in Brazil. As will be shown in section 3.2, there is a long-term robust equilibrium between electricity consumption and GDP. Wilmot (2013) studies the cointegration of crude oil spot prices, from different geographic regions.

2.1. Historical Data for the Period 1952 - 2012

Information concerning the power consumption and Brazilian GDP (at constant 2012 prices) for the period 1952 to 2012 was obtained from the IPEADATA (2013). GDP values have been converted from Brazilian reals into U.S. dollars using the exchange rate from December 31, 2012: US$ 1.00 = R$ 2.04.

Figure 2. Brazilian GDP and Electricity Consumption. Own elaboration.



[1] Using the exchange rate from December 31, 2012: US$ 1.00 = R$ 2.04.

Figure 2 shows a possible long-term relationship between GDP growth and the evolution of electricity consumption. A positive elasticity between these two variables is expected.
To estimate the elasticity between power consumption and GDP, the Cobb-Douglas function will be used:

$$GDP_t = CE_t^{\beta}.10^{\alpha+\varepsilon}$$

where *CE* is the variable electricity consumption in GWh, GDP is the variable gross domestic product at constant 2012 prices, α is a constant and ε is the random disturbance.
Using the properties of the logarithm, we obtain the following equation:

$$LogGDP_t = \alpha + \beta.LogCE_t + \varepsilon \quad (1)$$

The expression of the variables in logarithm form reduces the variability of the series, and equation (1) provides a long-term elasticity between variables (in Logs).

3. Cointegration Analysis

The basis of the importance of the concept of cointegration is that an autoregressive model between non-stationary but cointegrated series is not spurious and can therefore produce reliable estimates. There exists a stationary linear combination between non-stationary series that can be interpreted as indicating a long-run equilibrium between the variables.

To analyze the cointegration between the study variables, we must initially verify that the series are non-stationary and have the same degree of integration. To prove this, it is necessary to perform a unit root test.

3.1. Stationary Time Series

Among the various tests used to assess a stationary time series, stands out the test of Augmented Dickey-Fuller (ADF) that tests the hypothesis of a unit root, which is a condition of stationarity. In this article, we will use the ADF test, which can be expressed using the following equation:

$$\Delta Z_t = \alpha_0 + \alpha_1 T + \gamma.Z_{t-1} + \sum_{i=1}^{p} \beta_i.\Delta Z_{t-i} + \varepsilon_t \quad (2)$$

where Δ is the first difference operator, α_0 is the constant term, *T* is the trend, γ is the test coefficient for the presence or absence of a unit root, ΔZ_t is the own variable in the first difference, *p* is the order of the autoregressive model and ε_t is the structure of errors (which is assumed to have zero mean, constant variance and the absence of autocorrelation).

The test can be conducted without considering the intercept, considering the intercept and considering the intercept and trend. The hypothesis is performed on the coefficient γ; accepting the null hypothesis means that the series has a unit root and hence is stationary. The unit root tests (ADF) were performed for the two series under study, considering the intercept and trend, and the results are summarized in Table 1.

Table 1. Unit Root Tests

Variable	Model	Test Statistics (ADF)	Critical Values		
			1%	5%	10%
$Log(CE_t)$	Constant and trend	-0.847575	-4.1213	-3.48785	-3.17231
$Log(GDP_t)$	Constant and trend	-1.400504	-4.1213	-3.48784	-3.17231

Based on the results of Table 1, it is possible to accept the null hypothesis for the two series, i.e., there is a unitary root at the 1%, 5% and 10% levels of significance; therefore, the two series are non-stationary. Root unit tests (ADF) were performed for both series in the first difference, and the results are shown in Table 2.

Table 2. Unit Root Tests of Series in First Difference

Variable	Model	Test Statistics (ADF)	Critical Values		
			1%	5%	10%
$\Delta Log(CE_t)$	Constant and trend	-6.382236	-4.1213	-3.48785	-3.17231
$\Delta Log(GDP_t)$	Constant and trend	-5.190792	-4.1213	-3.48784	-3.17231

Based on the results of Table 2, the null hypothesis of a unit root is rejected for the two series in the first difference at the 1%, 5% and 10% levels of significance; therefore, the two series in the first difference are stationary.

We can conclude that the two series have the same degree of integration I(1), which means that it is possible that there is a cointegration relationship between the studied series.

3.2. Cointegration Tests

To verify the cointegration between the electrical energy consumption and GDP series, we use two tests developed by Johansen[2]: the trace test and the maximum eigenvalue test. Both tests provide the number of cointegrated equations. The results of the tests are presented in Tables 3 and 4.

Table 3. Trace Test

Null Hypothesis: No. Eq. Coint.	Test Statistics	Critical Value 5%
Neither	37.42883	20.26184
At least 1	3.927275	9.16455

Table 4. Maximum Eigenvalue Test

Null Hypothesis: No. Eq. Coint.	Test Statistics	Critical Value 5%
Neither	33.50156	15.89210
At least 1	3.927275	9.16455

Based on the first row of the results of the two tests, the hypothesis that there is no cointegration equation is rejected at a significance level of 5%. Based on the second rows, the hypothesis of there being least one cointegrating equation is accepted at a significance level of 5%. Thus, Johansen test results indicate that there is at least one cointegration relationship, i.e., that there is a long-term equilibrium between the two series. This was identified by both the trace statistic and the maximum eigenvalue.

Table 5 shows the vector cointegration obtained normalized to the variable *Log(GDP)*, where the number in parentheses is the corresponding standard deviation.

Table 5. Normalized Cointegration Vector

Log(GDP)	*Log(CE)*	*C*
1.000000	-0.8185	-1.0588
	(0.0311)	(0.0783)

Therefore, the cointegration equation that represents the long-term equilibrium relationship can be expressed as

$$Log(GDP_t) = 0.8185 Log(CE_t) + 1.0588 \quad (3)$$

Thus, the long-term elasticity between GDP and power consumption (in logs) is 0.8185.

4. Estimation of the Deficit Marginal Cost Curve

Representing the cointegration equation (3) without logarithms, we obtain:

$$GDP_t = (CE_t)^{0.8185}.10^{1.05881} \quad \text{or} \quad GDP_t = 11.45(CE_t)^{0.8185} \quad (4)$$

The marginal relationship is obtained by deriving equation (4):

$$GMg_t = \frac{d}{dCE_t}[GDP_t] = 9.3718(CE_t)^{-0.1815} \quad (5)$$

This marginal relationship represents the change in GDP obtained by a one-unit increase or decrease in electricity consumption. Because the purpose of this article is to model the deficit marginal

[2] According to the Akaike and Schwarz criteria, it was determined that the Johansen test must initially use one lag.

cost curve (DMC), we will consider reductions in consumption so that GMg_t can be understood as an economic cost to society due to a reduction in electricity consumption.

4.1. Algorithm for Estimating the Deficit Marginal Cost Curve in Deficit Unit Levels

Step 1. Start with the last year of the series (year without rationing). The present study considered 2012 as the reference year. Observe the power consumption for the reference year, CE_{2012}. Calculate the GDP marginal corresponding to the consumption of electricity in 2012, GMg_{2012}, according to equation (5).
Do: $i = 1$

Step 2. Perform an $i\%$ reduction in the electricity consumption CE_{2012} and calculate the corresponding GDP marginal, GMg^i, according to equation (5).

Step 3. Calculate the deficit marginal cost[3] DMC^i, corresponding to the $i\%$ reduction symmetrizing the GMg^i in GMg_{2012}, i.e., $DMC^i = GMg^i - GMg_{2012}$ in billions US$/GWh or, equivalently, $DMC^i = 10^3.(GMg^i - GMg_{2012})$ in US$/MWh.

Step 5. If $i = c_{max}$, where c_{max} is the maximum reduction percentage, finalize. Otherwise increase the reduction percentage by one unit, $i = i + 1$ and return to step 2.

4.2. Results

Using the steps of the algorithm shown in the previous section to estimate the deficit marginal cost curve and considering cmax = 30, the results obtained are shown in Table 6 and Figure 3. Although the results have shown that there is a long-term equilibrium between the GDP and electricity consumption series, some imbalance may occur in the short-term whose magnitude depends on the deviation from the long-term equilibrium.

Table 6. Deficit Marginal Cost (*DMC*)

Deficit	Cost US$/MWh
1%	213.55
2%	429.67
3%	648.41
4%	869.82
5%	1093.99
6%	1320.95
7%	1550.79
8%	1783.56
9%	2019.35
10%	2258.21
11%	2500.23
12%	2745.49
13%	2994.06
14%	3246.03
15%	3501.49

Deficit	Cost US$/MWh
16%	3760.52
17%	4023.22
18%	4289.69
19%	4560.02
20%	4834.33
21%	5112.72
22%	5395.30
23%	5682.20
24%	5973.53
250%	6269.43
261%	6570.03
27%	6875.46
28%	7185.88
29%	7501.43
30%	7822.28

One can make short-term forecasts for these two series and consequently make estimates of the deficit costs, but it will be necessary to eliminate such an imbalance. To this end, a VEC can be estimated, as it incorporates an error correction vector, connecting the short-term behavior and long-term behavior of the variables.

The VEC results are shown in Table 7. where CointEq (-1) corresponds to a vector error correction, values in parentheses are standard errors and values between brackets are the calculated results of the t-statistic.

[3] The deficit marginal cost DMC for the reference year 2012 is zero, as there was no deficit that year.

Figure 3. DMC Curve with Variation in the Deficit from 1% to 30%

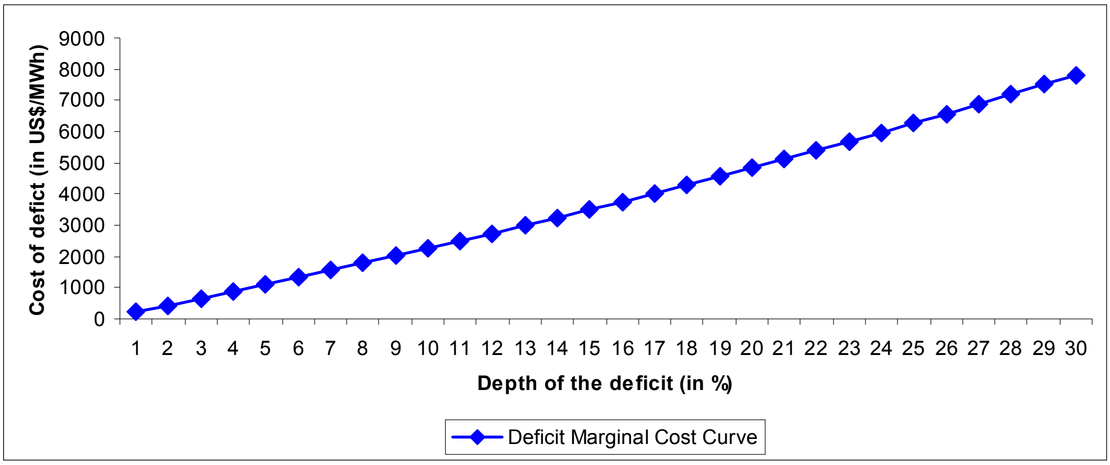


Table 7. Results of the Estimation for the Error Correction Vector, VEC

Variable	ΔLog(GDP)	ΔLog(CE)
CointEq(-1)	0.0613	0.1322
	(0.0210)	(0.0203)
	[2.91299]	[6.52031]
ΔLog(GDP)	0.2854	-0.1003
	(0.1516)	(0.1460)
	[1.88258]	[-0.68690]
ΔLog(CE)	0.0850	0.1989
	(0.1360)	(0.1310)
	[0.62510]	[1.51787]
R^2	0.3033	0.4518
Akaike AIC	-5.6019	-5.6765
Schwarz SC	-5.4962	-5.5708

5. Conclusions

The main objective of this study is to propose a new methodology to estimate the marginal cost curve of power deficits, which is very important for the electricity sector because Brazil is very dependent on hydroelectricity. Although the model focuses on the Brazilian system, changing the data and, with minor adjustments may be useful for hydrothermal systems of other countries of the world. In this regard, a study was conducted to understand the relationship between electricity consumption and economic growth during the period between 1952 and 2012. The results indicate that these two variables are cointegrated, i.e., that there is a long-term equilibrium relationship. This relationship shows that the variations in the power consumption are transmitted to the Brazilian GDP.

Based on this relationship, a method was proposed for modeling the deficit marginal cost curve. This method has four advantages over the current deficit marginal cost curve:

a) In the deficit marginal cost curve proposed in this article, the deficit depth is discretized into unit levels, with 30 levels considered. In the deficit curve currently used in Brazil, the deficit depth is discretized into just four levels. The differences are shown in Figures 1 and 3.
b) The relationship found here is robust and can be updated with future changes in the economic structure of the country, unlike the current deficit curve.
c) If necessary, one can obtain estimates for the deficit marginal cost using the VEC in Table 7. In the deficit marginal cost curve currently used, it is not possible to make predictions.

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