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Prospective Modelling of Oil Supply in Tunisia

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ABSTRACT: The theoretical framework which we are developing refers essentially to Hubbert model in order to determine the peak oil in Tunisia and the exploitation speed of the remaining resources, while referring to the data supplied by the Directorate General for Energy. The study focuses on the comparison between the results of the several econometric adjustment techniques (linear, non-linear techniques, linear techniques with structural change and the non-parametric methods) applied to the Tunisian oil production during the period going from 1966 to 2011. The prospective study through the econometric models has allowed us to determine the two dates scaring all energy executives, namely the peak which took place in the middle of the eighties and the finiteness of resources planned in 2028. The obtained results have shown that it remains 495 million barrels to be produced in Tunisia, bearing in mind that the data supplied by the authorities announces the figure of 420 million barrels of the remaining proved reserve in the Tunisian underground. Thereby, we have noticed a strong link between the physical models for reservoir flows and the empirical specifications based on the decline curves.

Keywords: Oil production; non-linear techniques; non-parametric methods; peak oil.
JEL Classifications: C51; Q41; Q47

1. Introduction

In Tunisia natural gas and oil are the main energy resources. Tunisia has seen its energy balance going from a surplus balance at the beginning of the 80s to a deficit balance in 2000. This situation was mainly caused by a stagnation of national production as well as the fast increase in energy demand relative to the population and the economic growth; thereby, in 2011, the value added in the energy sector represented only 4,62 % of the GDP and 6,46 % in 2000 versus 12,9 % in the 80s.

The following central question arises to all the Tunisian political and industrial decision-makers nowadays: are we going to face an oil shortage, thereby facing more additional economic difficulties? Do the Tunisian underground have other surprises as those of the 60s and the 70?

Geologically it is difficult to realize discoveries model in Tunisia because the country covers several important oil systems where the majority of the reserves are situated outside the country. There are very few fields for a reliable assessment. According to the geologists, the estimation of the potential which will be discovered can be only realized correctly through a study of the existing oil systems and their generation, thereby requiring analyses, through the evaluation of the source rock "Rock-Evalde", of all the existing seismic surveys calibrated on all the existing wells (according to the experts). This work is generally reserved for the geologists and the physicists. Our work is based only on a statistical data analysis.

The objective of this work is to produce some prospective reflection avenues concerning oil future in Tunisia which seems uncertain since the Tunisian production has entered a phase of maturity since the 1980s. This prospective approach postulates the continuation of the observed trends without simultaneously denying that important events (particularly related to the increases in hydrocarbons prices on the international market and to the technological progress) can arise and change the evolution of the Tunisian oil industry.

2. Literature Review

The study focuses on the comparison of the results of the various techniques of the econometric adjustment applied to the Tunisian oil production during the period going from 1966 until 2011. However, the studies based on the exhaustion theory and the curves of symmetric decline Zaipu Tao and Mingyu Li (2007); Mikael Höök and all (2010); Steve Mohr and all (2010) did not converge towards a unanimity neither at the level of the estimation of the quantity of the available resources nor at the level of the deadlines and the supply security strategies.

From an empirical point of view, all the adjustment techniques of the curves rely on time series data for oil production (Q(t) or Q'(t)) or discovery (B(t) or B'(t)), where the data are generally available on an annual basis. A variety of statistical techniques are available to analyze these time series, either to understand the underlying mechanisms which generated the data or to make forecasts. The functional form supposed in oil supply modelling can be linear or non-linear. It is noteworthy that the linear term is related to the parameters rather than the explanatory variables (Myers, and al 2002); in this case, the equation can be estimated by non-linear regression techniques.

The curve adjustment relies on a supposed report between the time series of the explained variable and the series of one or several explanatory variables. This approach is a standard device of econometrics, but what distinguishes the adjustment of oil supply curves is the absence of a theoretical framework which is sufficient to justify the supposed report. Typically, the explanatory variable is simply the time or the exploration effort (ε).

Besides, the literature on the projection techniques of oil supply curve has generally drawn little attention to the distinction between the conventional and non-conventional supply. Most of the authors, referring to the works of Adelman have supposed that the "form" of the production process or the discovery can be estimated from historical data and that this form will not be sustainably affected by any future change at the prices level or the exploitation technologies.

Thereby, there is a tendency to neglect these variables, in spite of the possible potential errors, yet in the context of our work where oil prices generally exceed 80 $, price introduction is no longer necessary, and this conclusion will no longer be valid if we essentially deal with conventional crude oil supply. Nevertheless, price signals are essential when the supply consists of conventional and non-conventional raw products.

3. Methodology

The maximal cumulative production represents the physical limit that the production of a natural resource can achieve. This limit mainly depends on the level of cumulative production and on the way the accumulated production is achieved. Bell curves have different forms and can be symmetric or asymmetric and they are closely linked to the sigmoid functions. These curves generally appear as derived from "Sigmoid" functions. In other words they can be seen as the annual equivalent of a sigmoid behavior observed in time series data of the accumulated production.

Hubbert (1956) was among the pioneers in establishing an extrapolation base of the production curves of the finite future resources. The shape in bell curves represented the behavior of typical production, without giving any exact mathematical description (Hubbert 1956). Bell curves were an important cornerstone in this production prospective of finite resources. Hubbert has supposed that the production level starts and ends in zero. However, in this time interval, the production crosses several local extrema. The real form of a given production curve may vary, but it is finally limited by the amount of the finite resources in question. Later, Hubbert (1959) has used the symmetric logistic form as a tool to create perspectives after noticing the conformity of his hypothesis with the observed empirical data of oil production in the USA.

$$\frac{\partial Q}{\partial t} = bQ(1 - \frac{Q}{Q_\infty}) \quad (1)$$

Q_∞ is the finally recoverable resource, Q denote the cumulative production, $\frac{\partial Q}{\partial t}$ is the annual production and b is the constant which describes the physical growth rate. At the beginning of the extraction phase, the limit on production is not important because the extracted volumes represent only a small part of the ultimate recoverable resource (∞). As the cumulative production increases, it becomes an important part of the URR, the extraction becomes more difficult and the extraction rate decreases. As the limit on the ultimate recoverable quantity Q_∞exists, the production tends towards Zero.

The solution to the logistic equation expresses the accumulated production in function of time. It is represented by:

$$Q(t) = \frac{Q\infty}{(1+\frac{Q\infty - Q(0)}{Q(0)})exp(-kt)} = \frac{Q\infty}{(1+a*exp(-kt))} \quad (2)$$

The annual production, at any date, is the by-product of the accumulated production function, or simply a difference approximation $q(t) = Q(t) - Q(t-1)$.

4. Results

4.1 results of linear model: Projection of the Tunisian supply curves

The projection of oil production in Tunisia can be illustrated as follows:

- Projection of the cumulative production by supposing a logistic curve.
- Projection of the production as the first derivative of a logistic curve.

The logistic model for the cumulative production is expressed as follows:

$$Q_t = \frac{Q\infty}{1+\alpha e^{-\beta t}} + e_t \quad (3)$$

Where Q(t) represent the annual URR. This model has been adapted to the Tunisian data of the global cumulative production by using linear regression.

In order to determine the theoretical model, we are going to linearise the logistic equation:

$$Q_t = \frac{Q\infty}{1+\alpha e^{-\beta t}} \Rightarrow 1 + \alpha e^{-\beta t} = \frac{Q\infty}{Q_t} \Rightarrow \alpha e^{-\beta t} = \frac{Q\infty}{Q_t} - 1 \Rightarrow ln\left(\frac{Q\infty}{Q_t} - 1\right) = ln(b) + tln(a) \quad (4)$$

The estimated Q_∞ is equal to 1700 million barrels according to the Tunisian direction for energy. A simple characteristic of y_t is its long-term trend; if we think that a downward trend exists and is going to persist. It is possible to construct a simple model which is going to allow forecasting y_t. The simplest consists of a linear trend according to which the series is going to decrease with the same amount each period:

$$y_t = c + at \rightarrow \Delta y_t = y_t - y_{t-1} = a \quad (5)$$

Table 1 presents the estimation of the model's parameters.

Table 1. Parameters of the linear modal

Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	3.113894	0.168334	18.49829	0.0000
@TREND	-0.115752	0.006589	-17.56713	0.0000
R-squared	0.877703			
Adjusted R-squared	0.874859			

We notice the existence of heteroscedasticity[1] errors, because the probability associated with the statistics arch is equal to zero (Table 2).

Table 2. Heteroskedasticity test: GARCH

F-statistics	881.1273	Prob. F(1,42)	0.0000
Obs*R-squared	41.99811	Prob. Chi-Squared(1)	0.0000

Therefore we are seeking within the family ARCH (Table 3):

Table 3. Estimation of GARCH model parameters

Variable	Coefficient	Std. Error	z-Statistics	Prob.
C	2.263951	0.037214	60.83592	0.0000
@TREND	-0.087185	0.001017	-85.68557	0.0000
Variance Equation				
C	0.000438	0.000491	0.893214	0.3717
RESID(-1)^2	1.291269	0.813865	1.586588	0.1126
GARCH(-1)	-0.340281	0.100323	-3.391837	0.0007
R-squared	0.805194			
R-Adjusted Squared	0.800664			

The estimated model thereby spells as follows:

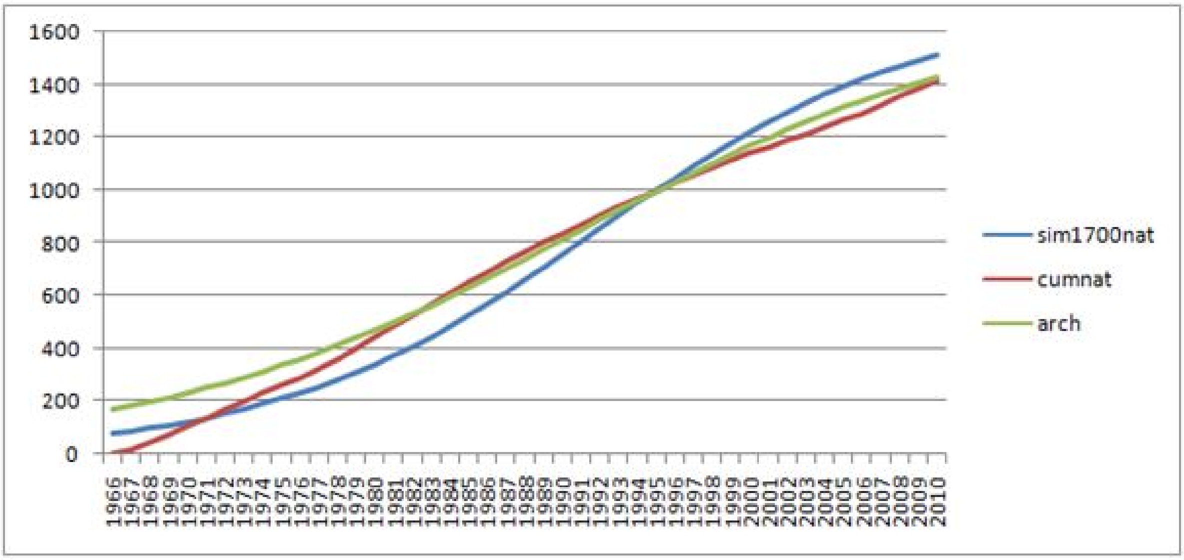
$$\mathrm{HUBNAT} = 2.263951 - 0.087185 * @\mathrm{TREND}$$

With $c = \ln(b) \Rightarrow b = e^{c}\ and\ \alpha = a$

With $t_{max} = \frac{1}{-\alpha} log \frac{1}{\beta}$

Figure 1 represents the logistic curves of the Tunisian cumulative oil production obtained from the equation 3: $Q_t = \frac{Q_\infty}{1+(e^{2.263951})*(e^{-0.087185t})}$

Figure 1. The logistic curves



[1] The presence of heteroscedasticity of errors in the series production reflects the uncertainty of the development of new fields, since the production cycle following the discovery cycle.

4.2 results of non-linear adjustment

The non-linear regression allows modelling complex phenomena not falling within the linear model framework. In this case, the non-linear adjustment allows us to solve the model by considering the Q_{field} as unknown.

$$q(t) = \frac{\partial Q(t)}{\partial t} = \frac{\partial \frac{Q_{field}}{1+ae^{-bt}}}{\partial t} = \frac{abQ_{field}e^{-bt}}{(1+ae^{-bt})} \rightarrow ln\big(q(t)\big) = ln\big(abQ_{field}\big) - bt - 2ln(1 + ae^{-bt}) \qquad (7)$$

The model's parameters are determined on the basis of optimality criterion (sum of least squares). The iterative Levenberg-Marquardt algorithm, which is a particularly robust and effective optimization method, is used to estimate the selected model parameters. Table 4 represents the parameters a, b and Q_{field} estimated by the non-linear least squares method. These parameters redraw the supply curve by taking into account the unknown Q_{field}.

Table 4. Parameters of the non-linear estimations

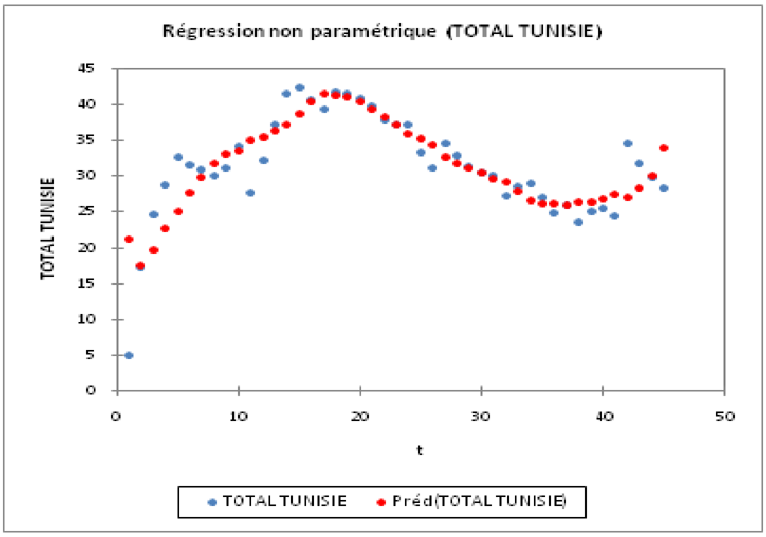
Parameter	Value	Standard-deviation
a	5,905	1,286
b	0,075	0,009
Q_{champ}	2009,491	159,120

4.3 Results of the non-parametric regression

With the objective of supplying a better forecasting quality of certain parameter such as the finally recoverable resources, we have resorted to parametric regression. In this case, unlike classical linear regression, the purpose here is not finding a unique model (a functional form) describing the growth of the resources, but obtaining an effective forecast method, without recurring to a physical understanding of the phenomenon.

Kernel Regression seems very useful to model time series such as oil supply growth. Kernel Regression which makes part of the non-parametric regression methods functions a bit like a black box, and it is realized following three phases: an adjustment phase during which we are going to try to find the best combinations of the method characteristics (model, core, bandwidth) on a sample test; a validation phase which allows validating the model on new observations; an application phase once the validation is satisfactory. Figure 2 represents the production profile obtained from a non-parametric regression.

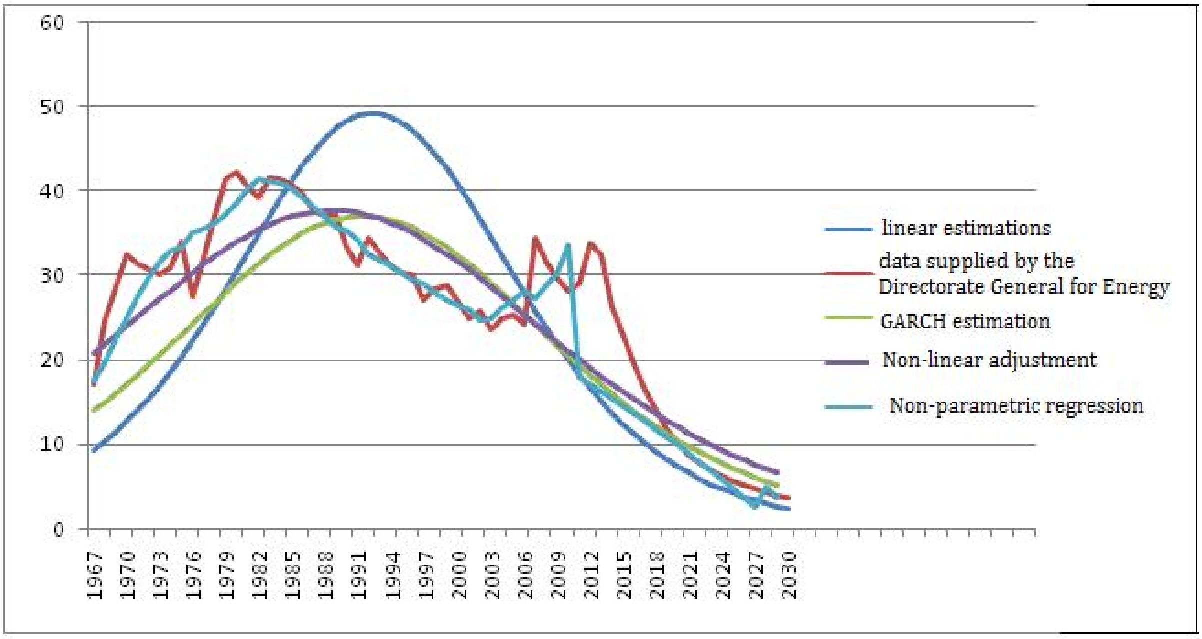
Figure 2. The non-parametric production profile of Tunisia



4.4. Comparison of the various adjustments

The derivative of oil supply logistic curve stemming from a linear model, GARCH, the non-linear models, the non-parametric models and the geologists' estimations are all represented in Figure 3.

Figure 2. The different production profils



5. Conformity of Multi-Cycle Hubbert Model

If we notice that more than Hubert cycle is produced, it's better to model oil supply in a multitude of cycles. The approach consists in reproducing an equation which gives the appearance of several cycles.

5.1 Method

Hubbert model based on the logistic function is represented by the equation 9:

$$Q(t) = \frac{Q_\infty}{1+e^{-a(t-t_0)}} \qquad (9)$$

Where $Q(t)$ is the cumulative production; Q_∞ is the finally recoverable reserves; t_0 is the year of maximal production, and a is a parameter.

The current level of production is the derivative of cumulative production with regard to time, and it is represented by the equation 10:

$$\frac{\partial Q}{\partial t} = \frac{a * Q_\infty * e^{-a(t-t_0)}}{[1+e^{-a(t-t_0)}]^2} \qquad (10)$$

First we solve $-a(t-t_0)$ as from the function of cumulative production and then we substitute it at the level of the current production:

$$e^{-a(t-t_0)} = \frac{Q_\infty}{Q(t)} - 1 \rightarrow -a(t-t_0) = Ln\left(\frac{Q_\infty}{Q(t)} - 1\right) \qquad (11)$$

$$\frac{\partial Q}{\partial t} = \frac{a * Q_\infty * e^{-a(t-t_0)}}{[1+e^{-a(t-t_0)}]^2} = \frac{a * URR * \left(\frac{Q_\infty}{Q(t)} - 1\right)}{[1+\left(\frac{Q_\infty}{Q(t)} - 1\right)]^2} \qquad (12)$$

5.2 Estimations

The approach relative to Hubbert multi-cycle curve modelling consists in creating a relation between the current and the accumulated production. The adopted alternative here is the engineers' point of view, which is establishing a report with the accumulated production instead of having a dependent time report t[2]:

[2] In economic sciences, the quantity of the produced goods this year is often determined by the quantity of the produced goods of the previous year and the same thing for the consumers' expenses. Thereby the rates of last

$$\frac{\partial Q}{\partial t} = \frac{a*\frac{Q_\infty{}^2}{Q(t)} - a*Q_\infty}{\left(\frac{Q_\infty}{Q(t)}\right)^2} = a*Q(t) - \frac{a}{Q_\infty}*Q(t)^2 \qquad (13)$$

By replacing $-\frac{a}{Q_\infty}$ by b_2, the equation 13 becomes:

$$\frac{\partial Q}{\partial t} = b_1 * Q(t) + b_2 * Q(t)^2 \qquad (14)$$

Starting from the equation 13, we obtain the finally recoverable reserves as being the report of b_1 by b_2 :

$$Q_\infty = -\frac{b_1}{b_2} \qquad (15)$$

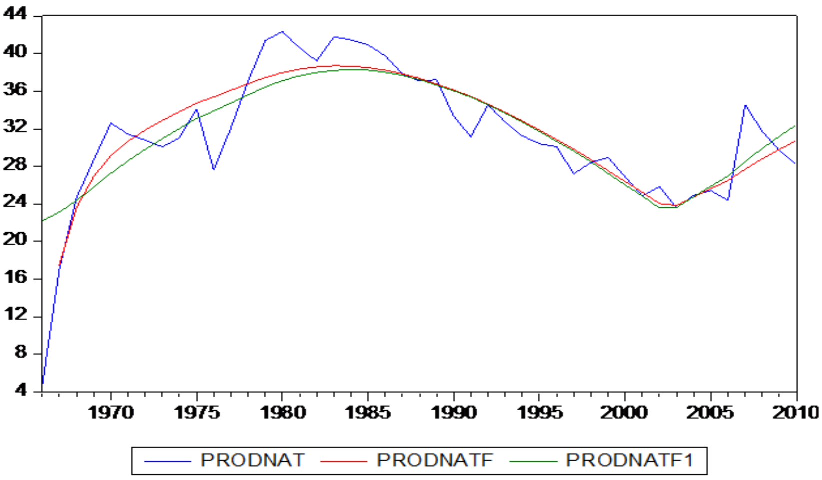
Here, we are not concerned with the resolution of Q_∞. Rather, we focus more in expressing the production rate according to cumulative production, figure 4. We have used this model, but with some rectifications; particularly, examining the effect of the institutional changes, we have added indicator variables:

$$QP = \beta_0 + b_1 Q(t) + b_2 Q(t)^2 + \beta_1 * DUM2003 + \beta_2 * DUM2003 * Q(t) \qquad (16)$$

Where β and DUM are respectively the parameters and the indicator variables or the dummy. The indicator variable is used in order for a second cycle to occur; thereby the slope-intercept must simultaneously change. Nevertheless, since there may be more than an institutional change, more than an indicator variable can be added. For example with two indicator variables, two different institutional events may occur.

The year of structural break shall appropriate with at least the minimum of the curve while following the theoretical concave nature of the Quadratic Hubbert Curve. Thereby the year of break represents the first inflection point. It is important to remember that it is not about finding the best break, but rather about finding a break from which a concave curve always starts. If a break which affects a multi-cycle Hubbert is convex, then it is not suitable for the theory and it is not a proper break. The cavity is necessary to find structural breaks.

Figure 3. Production profile with structural change



6. Conclusion

This work has allowed us to determine the two dates scaring all energy executives, namely the peak which took place in 1983 and the finiteness of the resources planned in 2028. The obtained results have shown that it remains 495 million barrels to be produced in Tunisia, bearing in mind that the data supplied by the authorities announce the figure of 420 million barrels of the remaining proved reserve

year's production or expenses are widely similar to those of this year and thus the variables are dependent on time.

in the Tunisian underground. Thereby, we have noticed a strong link between the physical models for reservoir flows and the empirical specifications based on the decline curves, since the conducted study is based on production as explanatory variables and not on the discoveries.

Concerning the results of the estimation of the finally recoverable crude oil quantity we notice that the figures stemming from the various techniques; the creaming curves and the peak theory are around 2100 million barrels. This work suggests that the estimated additional reserves of the new discoveries would not all the substitution of the extracted quantities (1581 million barrels) during the same period.

The decline in the national production of hydrocarbons and the exhaustion of the reserves were aggravated particularly by a reduction in the exploration effort, the exhaustion of the reserves of the main oil fields discovered in the 1960s (El Borma) and 1970 (Ashtart), the progressive departure of big oil companies and a tax system which is considered averagely incentive. It should nevertheless be pointed out that the hydrocarbon reserves remaining to discover are estimated by the US Geological Survey (USGS) index at 600 Mtep and that a considerable number of the listed oil prospects are not yet drilled[3].

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