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Trading Forward in the Brazilian Electricity Market

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ABSTRACT: We study the interaction between forward and spot electricity markets in a scenario where buyers and sellers are price takers in the forward market and trade through marketers, who play a Cournot game. Our model's main features come from the Brazilian electricity market, where a free contract market coexists with a regulated contract market, and the spot price is the output of a stochastic dynamic algorithm. We are able to show that the price of energy bought (sold) forward decreases (increases) with the number of marketers, and that, as a result, full hedging is achieved in the limit. We also investigate the effects on prices of changes in the number of market participants and in aggregate consumption and supply, an exercise that yields important policy recommendations for the Brazilian regulator.

Keywords: Forward and spot markets; marketers; Brazilian electricity market; Cournot; hedging.
JEL Classifications: C61; C72; G10; L13; L94

1. Introduction

In this paper we investigate the interaction between forward and spot markets in a model based on the Brazilian electricity market. The analysis we develop here serves two purposes. First, it presents a model that captures the main features of the Brazilian market. Second, it brings new elements to the analysis of contract markets, in particular the role played by marketers.

The Brazilian electricity sector underwent two major overhauls in the last two decades. The first started in 1995, when Law # 8.987, known as the "Concessions Law", was passed by the Brazilian Congress. By establishing the legal framework to regulate the concession of public services, it ushered in a new era in the electricity sector. Several distributions and a few generation companies were privatized, a regulator and a system operator were created, and a wholesale market was structured. This new framework was designed to promote competition in generation and commercialization, and provide open access to the transmission and distribution grids, while keeping distribution and transmission under (incentive) regulation.

The second overhaul took place in the wake of an energy crisis in 2001-02 that forced the federal government to take drastic measures to curtail consumption, and the advent of a new government, in 2003, that came to power with a program calling for reform of the electricity sector. Its agenda came to fruition with the introduction of the so-called "New Electricity Sector Model", in 2004.

This "new" model changed several aspects of the original design of the Brazilian electricity market, but it kept open a contract market where free (i.e. not captive) consumers and generators could trade electricity forward. This opened the door for marketers, agents who purchase and resell energy and/or help close deals between buyers and sellers, to enter the market. In Brazil, these marketers can be either independent or affiliated with generators and/or distributors. The presence of marketers is not an exclusive feature of the Brazilian electricity market. Several other markets around the world, like the PJM and the Texas markets in the United States, also have marketers.

There is no shortage of papers in the literature that study the interface between spot and contract markets. Some of them are theoretical papers interested in the general features of this interaction. Others are applications to product markets like that of electricity.

The seminal result in the theoretical literature about the interaction between spot and forward markets is Allaz and Vila (1993). Their main model (which applies to several situations, not only energy markets) is a two-period game where (duopoly) producers first buy or sell forward (binding and observable) contracts and then, in the second period, play a Cournot game in quantities in a spot market. A key assumption is perfect foresight, which entails no arbitrage and consequently a forward price equal to the price that will obtain in the spot market. They show that forward markets can emerge even in the absence of uncertainty and also that Cournot spot markets with forward markets are efficient in the limit, as the number of trading periods goes to infinity.

Several later papers show that the conclusion that forward markets are socially desirable even in the absence of uncertainty may not hold under different assumptions than those used by Allaz and Vila (1993). Mahenc and Salanié (2004), for instance, are interested in a situation where price-setting duopolists produce differentiated products. They show that in this case producers end up buying forward their own production. As a result, equilibrium prices are higher than they would be in the absence of forward trading. Green and Le Coq (2010) try to answer a different question, namely how the length of contracts affects the possibility of collusion in a repeated price-setting game. They conclude that firms can always sustain some collusive price above marginal cost if they sell the right number of contracts, whatever their discount factor. As the length of contracts increases, however, collusion becomes more difficult to sustain.

There is also a large chunk of the literature that focuses on the electricity sector. The seminal paper in this area is Green and Newbery (1992), the first to apply the concept of supply function equilibrium developed by Klemperer and Meyer (1989) to electricity markets. It is important at this point to mention that in the model we develop here suppliers (generators) don't submit supply schedules in the spot market. The reason is that the Brazilian spot market doesn't allow supply-side (or demand-side) bids. Suppliers simply forward technical data to the system operator, which is then used as input to calculate the spot price, as will be explained later. In Green and Newbery (1992), in contrast, generators submit a supply schedule of prices for generation and receive the market-clearing price, which varies with demand. They show that the Nash equilibrium in supply schedules yields a high markup on marginal cost and substantial deadweight losses, and use their findings to explain the early outcomes observed in the British electricity spot market.

Powell (1993) models the contract market in Britain, where financial contracts known as "contracts for differences" (CfDs) are traded. Demand for electricity comes mostly from distribution companies with mean-variance utility. Generators are price setters in the contract market and quantity setters in the spot market. He shows that when generators collude (either in the contract market or both markets) hedging is partial, and the expected spot price exceeds marginal cost and is lower than the futures price. Other early contributions to the study of the UK electricity market are Von der Fehr and Harbord (1993) and Wolfram (1998).

Green (1999) is another important reference in this literature. He models the electricity market in the UK as a two-stage game of a spot market and a hedging contract (CfDs) market, just like Powell (1993). Generators strategies in the spot market are different, however. They simultaneously submit supply functions[1] and the Pool (market operator) considers bids in ascending order. He shows that prices and the amount of hedging depend on the conjectures of the generators in the contract market (Bertrand or Cournot), and that generators may cover most of their output in the contract market and still raise prices above their marginal costs in the spot market.

More recent contributions to the literature are Bushnell (2007) (US market), Ciarreta and Espinosa (2010) (Spanish market), and Adilov (2010).

The existing literature on the Brazilian electricity market is mostly in Portuguese and doesn't go much beyond providing accounts of the historical evolution of the electricity sector and describing the current system. Exceptions are Dutra and Menezes (2005), who study the properties and outcomes of the auctions carried out in the regulated part of the Brazilian contract market, and Wolak (2008), who presents a proposal for short-term price determination in the wholesale market.

[1] Green (1999) works with linear supply functions most of the time.

In addition to modeling the Brazilian electricity market, ou paper adds to the literature by focusing on the implications of the presence of marketers. We do this in a model where suppliers and consumers of electricity behave competitively in the forward market, a feature borrowed from the Brazilian market, where there were 1727 consumers, 534 generators, and 153 marketers as of May 2013[2]. It comes as no surprise that papers that study other electricity markets model the forward market differently. Green (1999) and Powell (1993), for instance, ignore the presence of marketers and model the supply side of the market as a duopoly. As for the demand side of the contract market, Green (1999) assumes buyers determine the market-clearing price, and Powell (1993) assumes they set quantities[3].

Our model allows us to investigate the effects of changes in the number of marketers on the amount of hedging and the prices of energy bought and energy sold forward. It also gives us the opportunity to examine the impact of changes in other aspects of the forward market. For instance, we can study the implications of a larger number of consumers on marketers' profits, or how prices and quantities are affected by risk aversion or the degree of uncertainty.

The paper is organized as follows. Section 2 describes the main features of the Brazilian electricity market. Section 3 develops the basic framework of analysis and presents the main findings. It is divided into two sub-sections, one that investigates the case of a monopolist marketer, and another that considers the case of several marketers playing a Cournot game. Section 4 concludes and the Appendix presents the main proofs.

2. The Brazilian Electricity Market

One of the main features of the "New Electricity Sector Model", introduced in 2004, is the existence of two separate energy trading environments. In the first one, named the Regulated Contracting Environment (RCE)[4], energy is sold by electric utilities, independent power producers, self-generators and power marketers, and the only buyers are distribution companies, who are required to contract their entire forecast demand for captive consumers. Contracts are auctioned off over time with delivery dates of one, three, and five years after the date of the auction, and separate auctions for "new" and "existing" electricity[5]. Contracts for new electricity are longer (duration of more than 15 years) than those for existing electricity (eight years). Distribution companies are required to contract 100% of their expected power needs, but there are annual "adjustment" auctions where they can buy additional energy when their forecasts are off the mark. In the regulated environment, marketers are only allowed to participate in these adjustment auctions.

The second trading environment is called the Free Contracting Environment (FCE), and brings together electric utilities, independent power producers, self-generators, marketers, importers, exporters, and free consumers (those that do not need to buy power from distribution companies, typically industrial and commercial firms). Buyers and sellers are free to enter bilateral contracts and negotiate prices, quantities and delivery dates and conditions. Marketers can be either independent or affiliated with generators and/or distributors. They may either purchase and resell energy or only help close deals between buyers and sellers.

The FCE, also known as the "free market" in Brazilian electricity sector parlance, has been growing steadily in the past few years. It consisted of around 1,650 free and special[6] consumers in 2012, which accounted for approximately 27% of total consumption in the Brazilian electricity system (ABRACEEL, 2012).

Differences between the energy contracted and the energy effectively produced or consumed by market participants are liquidated in the spot market at the so-called "Liquidation of Differences

[2] These numbers refer to agents registered with the Brazilian Electricity Commercialization Clearinghouse (or CCEE by its Portuguese acronym), and were obtained from the CCEE's website http://www.ccee.org.br on May 13, 2013.

[3] Allaz and Vila (1993)'s model is not based on any electricity market, but it is worth mentioning that their supply side is a duopoly and that their demand side is comprised of speculators.

[4] "Ambiente de Contratação Regulada" and "Ambiente de Contratação Livre", respectively, in Portuguese.

[5] "New" electricity refers to power to be generated by plants yet to be built, and "existing" electricity refers to power generated by existing plants.

[6] Special consumers are those entitled to buy energy from incentivized sources (wind, small hydroelectric plants, biomass and solar).

Price." In contrast with other spot markets around the world, no short-term energy trading takes place in the Brazilian market. It is purely a mechanism for multilateral clearing of energy surpluses or deficits. Generators, in particular, do not decide how much energy to produce. That is determined by the system operator based on a dynamic programming model explained below.

The spot price is computed weekly (by subsystem[7]) and is based on the marginal operational cost of the system, with lower and upper bounds set by the regulator. Since the Brazilian system is preponderantly hydroelectric, the spot price is computed by a stochastic dynamic programming algorithm that seeks to find the optimal balance between using water today and storing it for future use. To use as much water as possible today to produce electricity is the best short term solution, but that would increase the likelihood of electricity shortfalls in the future. On the other hand, to conserve water today by keeping reservoirs full is the most reliable solution, but it requires higher thermal generation and, thus, higher electricity costs and prices.

3. Model and Findings

In this section, we are interested in investigating the impact of power marketers in an electricity market with the characteristics of the Brazilian market. In order to do that, we need to model two separate but interlinked markets, the contract and the spot market. In addition, we need to take into account the fact that the contract market is actually divided into two sub-markets, a regulated (the RCE) and a free market (the FCE).

Let's start with the contract market. By definition, free consumers do not participate in the regulated market, so they buy all of their power in the free market. On the supply side, however, we need to model how generators decide to allocate their sales between the regulated and free markets. In the Brazilian market, a generator who wants to sell power to distribution companies must first submit bids in auctions carried out within the RCE. If his bids are successful, he will enter long term contracts with the distribution companies. Therefore it makes sense to assume that generators take their commitments in the regulated market as given when they make decisions about how much to sell in the free market[8].

The spot market[9] is modeled as a mechanism that yields a random spot price, which is in sync with how the spot price is calculated in the Brazilian hydroelectric-dominated system. In addition to the forecast demand, inputs to the algorithm used by the Brazilian system operator to compute the spot price are stochastic variables such as the level of water reservoirs, precipitation, evaporation, and other uses of water (irrigation, water supply etc.)[10].

In what follows, we first study the case where there is a single marketer present in the contract market. We then investigate the implications of adding more marketers to that market.

3.1 Contract market with one marketer

There are two periods in our model. In period 0, a forward contract market opens with n electricity suppliers (indexed by *k*), m consumers (indexed by *i*), and one marketer, who buys forward contracts from suppliers and sells them to consumers at a premium. The spot market opens in period 1, when differences between observed and contracted quantities of electricity are settled at the spot price. Forward contracts are also settled in period 1.

The spot market is a random mechanism that yields a spot price p. We model the spot price as a random variable with mean μ and variance σ^2. Both suppliers and consumers are risk averse and have negative exponential utility functions given by $u(\pi) = -e^{-\alpha\pi}$, where profit π follows a normal distribution.

The profit function of consumer *i* is given by

$$\pi_i^c = r_i f_i(R_i) - p(R_i - y_i^c) - q^c y_i^c - c_i f_i(R_i), \tag{1}$$

[7] Subsystems of the Brazilian electricity system defined according to transmission constraints.

[8] We intend to investigate the possible opportunities for strategic behavior available to suppliers as a result of their presence in both markets in future work.

[9] Even though technically it is not a market, we will continue to use this term.

[10] For a detailed exposition of the stochastic dual dynamic programming based algorithms used by the Brazilian system operator, see Maceira et al. (2008).

where q^c is the price of a unit of contracted electricity as quoted by the marketer to the consumer, r_i is the given retail price of its product, R_i is the actual amount of electricity used by the consumer to produce $f_i(R_i)$ units of its product, f_i is her production function, c_i is her constant marginal (and average) production cost, and y_i^c is the quantity she buys forward.

Revenue in (1) is equal to the output the consumer produces from a volume R_i of electricity (recall that a free consumer in the forward market is a producer in her product market) times the retail price of her product. We normalize the marginal production cost to zero, so the cost side of (1) equals the sum of the cost of buying energy in the spot market and the cost of buying it in the contract market. Notice that the quantity she buys in the spot market is the difference between how much electricity she actually consumes and how much she buys forward.

Since the consumer's utility function has a negative exponential form, her maximization problem can be expressed in terms of the certainty equivalent measure:

$$\max E\left(\pi_i^c\right) - \frac{a_i^c}{2}\mathrm{var}\left(\pi_i^c\right), \qquad (2)$$

where a_i^s is her coefficient of risk aversion, and the decision variable is y_i^c.

Before we can compute the expected value and the variance of consumer i's profit, we need to understand how she forms expectations about her sales in the product market. It would be impractical to model each consumer's product market, so we assume she can perfectly forecast how much she will be producing and selling in period 1. This implies R_i is given[11], and so

$$\begin{aligned} E\left(\pi_i^c\right) &= r_i f_i\left(R_i\right) - \bar{p}(R_i - y_i^c) - q^c y_i^c \\ \mathrm{var}\left(\pi_i^c\right) &= \mathrm{var}\left(r_i f_i\left(R_i\right) - p(R_i - y_i^c) - q^c y_i^c\right) = \left(R_i - y_i^c\right)^2 \sigma^2 \end{aligned} \qquad (3)$$

where $\bar{p}$ is the expected value of the spot price.

The solution to problem (2) can be easily calculated[12]:

$$y_i^c = A_i - B_i q^c, \qquad (4)$$

where $A_i = R_i + \bar{p}\big/\left(a_i^c\sigma^2\right)$ and $B_i = 1\big/\left(a_i^c\sigma^2\right)$. Notice that $A_i > 0$ and $B_i > 0$.

The supplier is a price taker in both the spot and contract markets. Accordingly, his profit function is given by:

$$\pi_k^g = p\left(F_k - y_k^g\right) + q^g y_k^g - v_k F_k, \qquad (5)$$

where y_k^g is the quantity of output sold forward, q^g is the unit price of contracted electricity quoted by the marketer to the supplier, F_k is the supplier's actual electricity output[13] net of his sales in the regulated market, and v_k is his constant marginal (and average) cost.

Since the supplier does not know exactly how much power he will supply to the system, we model F_k as a random variable. We assume that p and F_k are independently distributed. The rationale for this is that no individual supplier's dispatched power has a discernible effect on the spot price calculated by the system operator. We also normalize marginal costs to zero.

Given his negative exponential utility function, the supplier's problem can be expressed in terms of the certainty equivalent measure

$$\max E\left(\pi_k^g\right) - \frac{a_k^g}{2}\mathrm{var}\left(\pi_k^g\right) \qquad (6)$$

where a_k^g is the supplier's coefficient of risk aversion.

Notice that since

[11] An equivalent assumption would be that R_i has mean R_i^e and variance 0.

[12] The first order condition yields $\bar{p} - q^c + a_i^c\left(R_i - y_i^c\right)\sigma^2 = 0$.

[13] That is, how much it is required to generate by the system operator.

$$E\left(\pi_k^g\right)=E\left(pF_k-py_k^g+q^g y_k^g\right)=E\left(pF_k\right)-y_k^g\overline{p}+q^g y_k^g \tag{7}$$

and

$$\begin{aligned}\operatorname{var}\left(\pi_k^g\right)&=\operatorname{var}\left(pF_k-py_k^g+q^g y_k^g\right)\\&=\operatorname{var}\left(pF_k\right)+\left(y_k^g\right)^2\operatorname{var}\left(p\right)-2y_k^g\operatorname{cov}\left(pF_k,p\right)\\&=\operatorname{var}\left(pF\right)+\sigma^2\left(y_k^g\right)^2-2y_k^g\operatorname{cov}\left(pF_k,p\right),\end{aligned} \tag{8}$$

the first order condition is given by

$$-\overline{p}+q^g-\frac{a_k^g}{2}\left[2\sigma^2 y_k^g-2\operatorname{cov}\left(pF_k,p\right)\right]=0\,. \tag{9}$$

Therefore:

$$\begin{aligned}&\sigma^2 a_k^g y_k^g=a_k^g\operatorname{cov}\left(pF_k,p\right)+q^g-\overline{p}\\&\Rightarrow y_k^g=\frac{\operatorname{cov}\left(pF_k,p\right)}{\sigma^2}-\frac{\overline{p}}{a_k^g\sigma^2}+\frac{q^g}{a_k^g\sigma^2}\end{aligned} \tag{10}$$

Now use the fact that p and F_k are independent to obtain

$$\begin{aligned}\operatorname{cov}\left(pF_k,p\right)&=E\left(pF_kp\right)-E\left(pF_k\right)E\left(p\right)=E\left(p^2F_k\right)-E\left(pF_k\right)E\left(p\right)\\&=E\left(p^2\right)E\left(F_k\right)-E\left(p\right)E\left(F_k\right)E\left(p\right)=E\left(F_k\right)\left[E\left(p^2\right)-\overline{p}^2\right]\\&=\sigma^2E\left(F_k\right)\end{aligned} \tag{11}$$

The solution to the problem is then

$$y_k^g=C_k+D_kq^g\,, \tag{12}$$

where $C_k=\overline{F}_k-\overline{p}\big/\left(a_k^g\sigma^2\right)$, $D_k=1\big/\left(a_k^g\sigma^2\right)$, and $\overline{F}_k\equiv E\left(F_k\right)$. It can be easily seen that $D_k>0$.

The marketer is a monopolist in the contract market. It quotes a selling price to consumers and a buying price to suppliers. He is risk neutral and thus wants to maximize his profits, given by the spread $d=q^c-q^g$ times the quantity traded y. In our model, all trades go through marketers, so $y=\sum_{i=1}^m y_i^s=\sum_{k=1}^n y_k^g$. Quantity demanded is equal to quantity supplied in the contract market, and so $A-Bq^c=C+Dq^g$, where $A=\sum_{i=1}^m A_i=R+\left(\overline{p}/\sigma^2\right)\sum_{i=1}^m\left(a_i^c\right)^{-1}$, $B=\sum_{i=1}^m B_i=\left(\sigma^2\right)^{-1}\sum_{i=1}^m\left(a_i^c\right)^{-1}$, $C=\sum_{k=1}^n C_k=\overline{F}-\left(\overline{p}/\sigma^2\right)\sum_{k=1}^n\left(a_k^g\right)^{-1}$, $D=\sum_{k=1}^n D_k=\left(\sigma^2\right)^{-1}\sum_{k=1}^n\left(a_k^g\right)^{-1}$, $R=\sum_{i=1}^m R_i$, and $\overline{F}=\sum_{k=1}^n\overline{F}_k$ [14]. In addition, we make the assumption that $R=\overline{F}$, which means that actual aggregate consumption of electricity by buyers who participate in the forward market is equal to the sum of the expected net individual supplies of sellers. In other words, we are assuming that individual forecasting mistakes made by suppliers cancel out, and so expected aggregate (net) supply is equal to actual aggregate (net) supply, i.e. $\sum_{k=1}^n F_k=\sum_{k=1}^n\overline{F}_k$ [15][16].

The marketer solves the following maximization problem:

$$\begin{aligned}&\max\left(q^c-q^g\right)y\\&s.t.\quad q^c-q^g\geq 0,\end{aligned} \tag{13}$$

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[14] Notice that R and $\overline{F}$ are functions of m and n, respectively.

[15] This assumption is made for technical reasons. If we were not to make it, our results would remain qualitatively unchanged, but prices and quantities in the forward market would include an extra term that depended on the difference between R and $\overline{F}$.

[16] Notice that, by definition, $\sum_{k=1}^n F_k=\sum_{i=1}^m R_i$. We are ignoring any possible differences between actual and contracted consumption in the regulated market (RCE).

The proposition below follows from the solution to (13). The proof can be found in the Appendix.

Proposition 1: The equilibrium quantities and prices in a forward market where (a) suppliers and consumers are price takers, (b) the marketer has monopoly power, (c) suppliers have the same coefficient of risk aversion, and (d) consumers have the same coefficient of risk aversion, are the following:

$$q^c = \overline{p} + R\left(\frac{a^c\sigma^2}{2m}\right), \qquad q^g = \overline{p} - \overline{F}\left(\frac{a^g\sigma^2}{2n}\right),$$
$$d = \left(\frac{R\sigma^2}{2}\right)\left(\frac{a^c}{m} + \frac{a^g}{n}\right), \quad y_i^c = R_i - \frac{R}{2m} \text{ and } y_k^g = \overline{F}_k - \frac{\overline{F}}{2n}, \tag{14}$$

where $a_i^s = a^s \; \forall i = 1,\dots,m$ *and* $a_k^g = a^g \;\; \forall k = 1,\dots,n$.

Upon inspection, we can immediately see that the forward price of energy sold (by suppliers) is lower than the expected spot price, whereas the price of energy bought (by consumers) is higher than the expected spot price. Accordingly, suppliers sell forward less than their expected (net) supply and consumers buy forward less than their consumption of electricity.

Risk-averse agents want to hedge against risk. In our model, they do that in the forward market, and any factor that increases the risk (of being exposed to the spot market) or makes the agent more risk-averse increases his demand for hedging, affecting forward prices accordingly. Therefore the following results should come as no surprise:

(i) The forward price paid by (to) consumers (suppliers) is higher (lower) the more risk averse they are. This makes sense because more risk-averse agents assign more value to less exposure to the spot market.

(ii) The forward price paid by (to) consumers (suppliers) is higher (lower) the larger the variance of the spot price, since this means more risk.

Other interesting exercises of comparative statics can be carried out by focusing on the effects of changes in aggregate consumption and supply, number of consumers, and number of suppliers. For instance, notice that the forward price paid by consumers increases with total consumption, for a fixed number of consumers. This is so because when average consumption is higher, each consumer individually has more energy to trade, and this increases her risk of exposure to the spot price[17]. Conversely, the forward price received by suppliers is negatively related to expected aggregate power available in the forward market, for a fixed number of suppliers. The reason is that when average expected supply is higher, the marketer is able to exploit the suppliers' higher risk of exposure to the spot price[18].

Similar reasoning can be used to explain how forward prices in markets with the same aggregate consumption but different numbers of consumers compare. Average exposure to spot price risk is smaller in the market with more consumers, and accordingly the forward price faced by those consumers is also smaller. Similarly, if two markets have the same aggregate expected power supply but different numbers of suppliers, the forward price received by suppliers is higher in the market where they are in greater number.

A particularly relevant result for the Brazilian electricity market is a combination of the cases discussed in the two previous paragraphs. Consider a situation where some captive consumers migrate from the regulated market to the (free) contract market, leading to a decrease in demand in the former and an increase in the latter. In terms of our model, this means that *m*, the number of consumers, increases while total demand for electricity and the number of generators *n* do not change. Since *R* goes up and, by assumption, $\overline{F} = R$, we can see from (14) that the price of energy sold forward decreases after the migration takes place. This is so because now suppliers have on average more energy to sell in the contract market, and so their exposure to the spot market increases.

[17] Notice that the individual demand function (4) is linear, and thus elastic for relatively small quantities and inelastic for relatively large quantities.

[18] Similarly, the individual supply function in (12) is linear, and thus elastic for relatively small quantities and inelastic for relatively large quantities.

On the other hand, the behavior of the price of energy bought forward depends on what happens to the ratio R/m. If it is larger after the migration, then q^c increases. If it is lower, q^c decreases. The latter is a surprising result, since the marketer has monopoly power in the contract market. The explanation is that since average consumption decreases, the average consumer is exposed to less spot price risk. As a consequence, the elasticity of demand for contracts increases, for risk sharing becomes less important to the average consumer.

An example will help shed some light on this matter. We consider two different ways of increasing the aggregate demand for power. In the first, a new consumer enters the market, raising the number of consumers from *m* to $m+1$, and aggregate demand from R to R'. Let's say that the average consumption after entry is the same as that before entry, i.e. $R/m = R'/(m+1)$. Then, according to (14), there is no change in the price consumers buy their energy forward. The second case has the number of consumers staying at *m* but aggregate demand again rising to R'. This corresponds to a situation where the extra demand $R'-R$ is split (evenly or not) between all *m* consumers. The result is now a higher forward price faced by consumers. When we compare the two cases, we notice that the same increase in aggregate demand has different effects on price. The explanation is that when aggregate demand increases but the number of consumers doesn't, each consumer buys all her additional energy in the forward market (see (4)), a behavior that can be attributed to our assumption of constant risk aversion. When, on the other hand, an additional consumer enters the market, she splits her consumption between the forward and spot markets (again, see (4))[19].

Let's now turn to the marketer. First notice that spread *d* is strictly positive, and, as expected, increases with the degree of risk aversion of suppliers, with that of consumers, and with the variance of the spot price. Moreover, since half of the system's expected (net) energy is traded in the contract market[20], the marketer's profit is equal to

$$\pi_d = d\,y = \left(\frac{R\sigma^2}{2}\right)\left(\frac{a^c}{m}+\frac{a^g}{n}\right)\left(\frac{R}{2}\right) = R^2\left(\frac{\sigma^2}{4}\right)\left(\frac{a^c}{m}+\frac{a^g}{n}\right) \tag{15}$$

Notice that profit decreases when aggregate consumption remains constant but the number of consumers increases, consequence of a reduced spread. A smaller spread also explains why profit decreases when the number of suppliers is larger but aggregate power supply doesn't change.

When there is migration of consumers to the free market, what happens to the marketer's profit depends on the behavior of average consumption. If, for instance, average consumption stays the same, profit increases. To see this, let $R' > R$ be the total demand for electricity after the migration, and let $m' > m$ be the number of consumers now in the market. Since, by construction, $R'/m' = R/m$, we have

$$\pi'_d = d'y' = R'^2\left(\frac{\sigma^2}{4}\right)\left(\frac{a^c}{m'}+\frac{a^g}{n}\right) = \left(\frac{\sigma^2}{4}\right)\left(\frac{R'^2a^c}{m'}+\frac{R'^2a^g}{n}\right) > \left(\frac{\sigma^2}{4}\right)\left(\frac{R^2a^c}{m}+\frac{R^2a^g}{n}\right) = \pi_d \tag{16}$$

Similar reasoning can be used to show that when average consumption after the migration is larger, the marketer's profit again increases. When average consumption is smaller, however, the effect on profit is dubious.

The next section discusses a model with more than one marketer.

3.2 Contract market with more than one marketer

According to Proposition 1, the monopolist marketer obtains a strictly positive spread and, consequently, makes positive profit through its operations in the forward market. This should entice other firms to enter the market as marketers. The situation where there are many marketers is the focus of this section.

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[19] We intend to investigate the consequences of relaxing the assumption of constant risk aversion in the future.

[20] The first way to see this is: $y \equiv \sum_{k=1}^{n} y_k^g = \sum_{k=1}^{n}\left(\bar{F}_k - \bar{F}/2n\right) = \bar{F} - \bar{F}/2 = \bar{F}/2$. Another way is: $y \equiv \sum_{i=1}^{m} y_i^c = \sum_{i=1}^{m}\left(R_i - R/2m\right) = R - R/2 = R/2 = \bar{F}/2$.

There are now *H* identical marketers and they play a Cournot game. Marketer *h's* profit function is $\pi_h = \left(q^c - q^g\right) y_h^b$, where y_h^b is the quantity of energy traded by marketer *h*. Marketer *h* has to solve the following problem, where $y = \sum_{h=1}^{H} y_h^b$:

$$\begin{aligned} &\max_{y_h^m} \left(q^c - q^g\right) y_h^b \\ &s.t. \quad y = A - Bq^c = C + Dq^g \end{aligned} \tag{17}$$

This problem is equivalent to

$$\max \left(\frac{A-y}{B} - \frac{y-C}{D} \right) y_h^b \tag{18}$$

where the constraint has already been plugged into the objective function.

Proposition 2: The equilibrium quantities and prices in a forward market where (a) generators and suppliers are price takers, (b) there are several identical marketers who play a Cournot game, (c) all generators have the same coefficient of risk aversion and (d) all suppliers have the same coefficient of risk aversion, are given by:

$$\begin{aligned} &q^c = \bar{p} + \left(\frac{a^c \sigma^2}{m}\right)\left(\frac{R}{H+1}\right), \quad q^g = \bar{p} - \left(\frac{a^g \sigma^2}{n}\right)\left(\frac{\bar{F}}{H+1}\right), \\ &d = \bar{F}\left(\frac{\sigma^2}{H+1}\right)\left(\frac{a^c}{m} + \frac{a^g}{n}\right), \quad y_i^c = R_i - \frac{R}{m(H+1)}, \\ &y_k^g = \bar{F}_k - \frac{\bar{F}}{n(H+1)}, \quad y_h^b = \frac{\bar{F}}{H+1} \text{ and } y = \left(\frac{H}{H+1}\right)\bar{F}. \end{aligned} \tag{19}$$

As far as how they depend on degrees of risk aversion and variances is concerned, prices and quantities bought and sold forward have similar properties to those they featured in the monopolist marketer case, so we will not comment on them. As a matter of fact, the results of Proposition 2 boil down to those of Proposition 1 when $H = 1$. The total amount of electricity traded through forward contracts is again less than the expected (net) energy available, but no longer exactly equal to half of it (except, of course, when $H = 1$).

We turn our attention to the effects of an increased number of marketers on the equilibrium values of the variables. First, it is easy to see that the price of energy bought forward decreases with the number of marketers. That is exactly what a Cournot model should yield: The more marketers there are, the stronger the competition between them, and this drives down the price they charge consumers. In the limit, they can charge no more than the expected spot price. Analogously, the price of energy sold forward increases with the number of marketers, again as a consequence of the enhanced competition between marketers. In the limit, the expected spot price is achieved. It comes as no surprise that the spread charged by marketers goes to zero as the number of marketers increases without bound.

We can also see immediately upon inspection of the formulas for y_i^c and y_k^g that, as the number of marketers increases; the energy sold forward by a supplier approaches its expected (net) production, while the energy bought forward by a consumer approaches its actual consumption. This is a trivial consequence of the fact that, since the price paid by consumers decreases and the price received by suppliers increases with the number of marketers, consumers and suppliers are faced with stronger incentives to hedge their positions in the contract market.

Inspection of (19) also reveals that portfolios held by individual marketers shrink in size and total amount of energy traded in the contract market moves toward the available expected (net) energy in the system. This was expected, since both consumers and suppliers are trading forward almost all the energy they need or have, respectively. This indicates that the role played by the spot market tends to diminish due to increasing competition between marketers.

An interesting question is how the equilibrium with several marketers compares to the equilibrium of a competitive market with no marketers. This comparison will allow us to make comments about efficiency. Consider a competitive market with *n* suppliers, *m* consumers, and no marketers. By analogy to (4), we have $y_i^c = R_i + \left(\bar{p} - q\right) / \left(a_i^c \sigma^2\right)$, where *q* is the forward price.

Similarly, $y_k^g = \overline{F}_k - (\overline{p} - q)/(a_k^g \sigma^2)$. Aggregate quantity supplied is equal to aggregate quantity demanded in equilibrium, and so, under the assumption that $a_i^c = a_i \ \forall i$ and $a_k^g = a^g \ \forall k$:

$$\begin{aligned} &\sum_{i=1}^{m} y_i^c = \sum_{i=1}^{m} R_i + m\left(\frac{\overline{p}-q}{a^c\sigma^2}\right) = \sum_{k=1}^{n} \overline{F}_k - n\left(\frac{\overline{p}-q}{a^g\sigma^2}\right) = \sum_{k=1}^{n} y_k^g \\ &\Rightarrow \left(\frac{\overline{p}-q}{\sigma^2}\right)\left(\frac{m}{a^c} + \frac{n}{a^g}\right) = 0 \\ &\Rightarrow q = \overline{p} \end{aligned} \tag{20}$$

where we used the fact that $\sum_{k=1}^{n} \overline{F}_k = \sum_{i=1}^{m} R_i$. It comes as no surprise that the equilibrium forward price in this competitive market is equal to the expected spot price. Therefore, the equilibrium forward quantities are given by $y_i^c = R_i$ and $y_k^g = \overline{F}_k$, which means that there is full hedging by both suppliers and consumers.

We can now compare the equilibrium in a market with *H* marketers given by (19) with the efficient (competitive) solution. Notice that the distance between them, as measured by the distance between the corresponding forward prices and the expected spot price, goes to zero as the number of marketers increases without bound. In other words, the equilibrium in the presence of marketers approaches the efficient solution as the competition between marketers themselves increases.

Alternatively, we can look at the magnitude of the existing inefficiency in the presence of marketers, measured as the difference between the electricity traded forward in a competitive market and that traded forward in a market with *H* marketers:

$$\overline{F} - \left(\frac{H}{H+1}\right)\overline{F} = \left(\frac{1}{H+1}\right)\overline{F} \tag{21}$$

Notice that it goes to zero as the number of marketers increases. Perhaps more interesting is the inefficiency doesn't depend on any of the other parameters of the model, being a function only of the number of marketers and the aggregate expected supply of electricity (equal to aggregate consumption).

4. Conclusions

In this paper, we modeled the interaction between marketers, suppliers, and consumers in an electricity forward market when there are no bids allowed in the spot market. This is essentially how the Brazilian electricity market is set up, with the spot price being the output of a stochastic dynamic programming algorithm whose objective is to find the optimal balance between using water today and storing it for future use.

We first obtained results that are standard in the literature. Forward prices paid by consumers are increasing in their degree of risk aversion and the variance of the spot price, while prices at which suppliers sell their electricity forward decrease with those same indicators. We also showed that a monopolist marketer will be able to charge prices that yield a positive spread, and that the spread increases with the risk aversion of suppliers and consumers, as well as with the variance of the spot price.

Our results also allowed us to carry out comparative statics analyses where the number of consumers or the number of suppliers changes. For instance, the forward price faced by consumers diminishes when the number of consumers increases but aggregate consumption doesn't change. Similarly, if two markets have the same aggregate expected power supply but different numbers of suppliers, the forward price received by suppliers is higher in the market where they are in greater number.

Another important contribution of our study, one that is particularly relevant for the Brazilian electricity market, comes from the analysis of what happens when (captive) consumers migrate from the regulated market to the contract (free) market. We pointed out that one of the consequences of that migration is that the price of energy sold forward decreases, which, other things being equal, hurts suppliers. It is important for the electricity regulator to be aware of this, for policy changes that expand the demand side of the Brazilian electricity forward market will possibly be opposed by generators.

As for the effects of the migration on the price of energy bought forward, we came to the conclusion that it depends on the behavior of average consumption. If average consumption is larger after the migration, then the price consumers pay increases. If average consumption is lower, then it decreases. The second possibility is a non-standard result, since the marketer has monopoly power in the contract market. To understand it, notice that when average consumption decreases the average consumer is exposed to less spot price risk. This means that risk sharing becomes less important to her, and so marketers face increased competition from the spot market, which dilutes their market power. This is a matter of practical importance for the Brazilian electricity regulator. The free contract market (FCE) in Brazil has typically been expanded by allowing consumers with lower demand (currently the limit is 3MW) to join the market. This means that new consumers entering the market will have lower consumption levels, which, according to our model, will lower the price of energy bought forward and increase hedging by consumers.

We were also able to show that the price of energy bought forward decreases with the number of marketers. The more marketers there are, the stronger the competition between them, and this drives down the price they charge consumers. In the limit, they can charge no more than the expected spot price. Analogously, the price of energy sold forward increases with the number of marketers, and equals the spot price in the limit. As a consequence, the total amount of energy traded in the contract market approaches the system's expected available energy (net of regulated trades). This means that spot markets may become less important when there is increased competition between marketers in the contract market. It also implies that inefficiency, measured as the difference between total hedging (the competitive solution) and hedging in the presence of marketers, decreases with the number of marketers. The policy recommendation here is clearly to foster competition between marketers.

There are many ways in which our model can be improved. We look forward to the opportunity of investigating issues such as the strategic interaction between free and regulated (where the electricity demanded by captive consumers is traded) contract markets, price competition between marketers, the effects of allowing suppliers and consumers to submit bids to the spot markets, and others.

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Appendix

Proof of Proposition 1:

Since $A - Bq^c = C + Dq^g$, we can rewrite the problem (13) as

$$\max\left(\frac{A-C}{B} - \frac{D}{B}q^g - q^g\right)\left(C + Dq^g\right) \qquad s.t. \quad q^c - q^g \geq 0 \tag{22}$$

We will first solve the unconstrained problem and then show that the constraint is satisfied at the optimum.

But first let's show that the objective function is concave. Let

$$T(q^g) \equiv \left(\frac{A-C}{B} - \frac{D}{B}q^g - q^g\right)\left(C + Dq^g\right) = \frac{(A-C)C}{B} + $$
$$+\frac{(A-C)D}{B}q^g - \frac{DC}{B}q^g - \frac{D^2}{B}(q^g)^2 - Cq^g - D(q^g)^2,$$

Since $D > 0, B > 0$, we have

$$\frac{\partial T}{\partial q^g} = \frac{(A-C)D}{B} - \frac{DC}{B} - \frac{2D^2}{B}q^g - C - 2Dq^g$$
$$\frac{\partial^2 T}{\partial (q^g)^2} = \frac{-2D^2}{B} - 2D < 0.$$

Thus the first order condition is both necessary and sufficient for a maximum. The first order condition for this problem is given by

$$\frac{(A-C)D}{B} - \frac{DC}{B} - \frac{2D^2}{B}q^g - C - 2Dq^g = 0$$
$$\Rightarrow \frac{2D^2}{B}q^g + 2Dq^g = \frac{AD - 2DC - BC}{B}$$

This equation can be solved to obtain

$$q^g = \frac{AD - 2DC - BC}{B\left(\frac{2D^2}{B} + 2D\right)} = \frac{AD - C(2D+B)}{2D(D+B)}, \tag{23}$$

and so

$$\begin{aligned} q^c &= \frac{A-C}{B} - \frac{D}{B}\left(\frac{AD - C(2D+B)}{2D(D+B)}\right) = \frac{A-C}{B} - \frac{AD - C(2D+B)}{2B(D+B)} \\ &= \frac{2(A-C)(D+B) - AD + C(2D+B)}{2B(D+B)} = \frac{AD + 2AB - BC}{2B(D+B)} \\ &= \frac{AD + B(2A-C)}{2B(D+B)} \end{aligned} \tag{24}$$

The condition $q^g \leq q^c$ is satisfied if

$$\frac{AD-C(2D+B)}{2D(D+B)} \le \frac{AD+B(2A-C)}{2B(D+B)}$$

Since $B>0$ and $D>0$, this is equivalent to

$$\left[AD-C(2D+B)\right]B \le \left[AD+B(2A-C)\right]D$$
$$\Leftrightarrow ABD-2BCD-CB^2 < AD^2+2ABD-BCD$$
$$\Leftrightarrow ABD+BCD+AD^2+CB^2>0,$$

Now let's use the simplifying assumptions that suppliers have the same coefficient of risk aversion, i.e. $a_k^g = a^g \ \forall k=1,\ldots,n$, and that consumers also have the same coefficient of risk aversion, i.e. $a_i^c = a^c \quad \forall i=1,\ldots,m$. Then $A=R+(m\bar{p})/(a^c\sigma^2)$, $B=m/(a^c\sigma^2)$, $C=F-(n\bar{p})/(a^g\sigma^2)$, and $D=n/(a^g\sigma^2)$. Therefore:

$$ABD+CBD+AD^2+CB^2=\left(R+\frac{m\bar{p}}{a^c\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)+\left(\bar{F}-\frac{n\bar{p}}{a^g\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)$$

$$+\left(R+\frac{m\bar{p}}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)^2+\left(\bar{F}-\frac{n\bar{p}}{a^g\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)^2=\left(R+\bar{F}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)+\left[\frac{m\bar{p}}{a^c\sigma^2}-\frac{n\bar{p}}{a^g\sigma^2}\right]\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)$$

$$+\left(R+\frac{m\bar{p}}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)^2+\left(\bar{F}-\frac{n\bar{p}}{a^g\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)^2=\left(R+\bar{F}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)+\left(\frac{m\bar{p}}{a^c\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)$$

$$-\left(\frac{n\bar{p}}{a^g\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)+R\left(\frac{n}{a^g\sigma^2}\right)^2+\left(\frac{m\bar{p}}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)^2+\bar{F}\left(\frac{m}{a^c\sigma^2}\right)^2-\left(\frac{n\bar{p}}{a^g\sigma^2}\right)\left(\frac{m}{a^c\sigma^2}\right)^2$$

$$=\left(R+\bar{F}\right)\left(\frac{m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)+R\left(\frac{n}{a^g\sigma^2}\right)^2+\bar{F}\left(\frac{m}{a^c\sigma^2}\right)^2>0$$

which proves that the restriction is satisfied at the optimum.

Let's go back to condition (23). It can be rewritten as

$$q^g=\frac{\left(R+\frac{m\bar{p}}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right)-\left(\bar{F}-\frac{n\bar{p}}{a^g\sigma^2}\right)\left(\frac{2n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}{2\left(\frac{n}{a^g\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}$$

$$=\frac{\frac{Rn}{a^g\sigma^2}+\frac{mn\bar{p}}{a^ca^g\left(\sigma^2\right)^2}-\frac{2\bar{F}n}{a^g\sigma^2}-\frac{\bar{F}m}{a^c\sigma^2}+\frac{2n^2\bar{p}}{\left(a^g\right)^2\left(\sigma^2\right)^2}+\frac{mn\bar{p}}{a^ca^g\left(\sigma^2\right)^2}}{2\left(\frac{n}{a^g\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}$$

$$=\frac{\frac{\left(R-2\bar{F}\right)n}{a^g\sigma^2}-\frac{\bar{F}m}{a^c\sigma^2}+\frac{2mn\bar{p}}{a^ca^g\left(\sigma^2\right)^2}+\frac{2n^2\bar{p}}{\left(a^g\right)^2\left(\sigma^2\right)^2}}{2\left(\frac{n}{a^g\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}$$

$$=\frac{\frac{\left(R-\bar{F}\right)n}{a^g\sigma^2}-\bar{F}\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}{2\left(\frac{n}{a^g\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}+\frac{\frac{2n\bar{p}}{a^g\sigma^2}\left(\frac{m}{a^c\sigma^2}+\frac{n}{a^g\sigma^2}\right)}{2\left(\frac{n}{a^g\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}$$

$$=\bar{p}+\frac{\left(R-\bar{F}\right)}{2\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}-\bar{F}\left(\frac{a^g\sigma^2}{2n}\right) \tag{25}$$

Similarly, (24) can be expressed as

$$q^c = \frac{\left(R + \frac{m\overline{p}}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2}\right) + \left(\frac{m}{a^c\sigma^2}\right)\left(2R + \frac{2m\overline{p}}{a^c\sigma^2} - \overline{F} + \frac{n\overline{p}}{a^g\sigma^2}\right)}{\left(\frac{2m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right)}$$

$$= \frac{\frac{Rn}{a^g\sigma^2} + \frac{mn\overline{p}}{a^c a^g\left(\sigma^2\right)^2} + \frac{\left(2R - \overline{F}\right)m}{a^c\sigma^2} + \frac{2m^2\overline{p}}{\left(a^c\right)^2\left(\sigma^2\right)^2} + \frac{mn\overline{p}}{a^c a^g\left(\sigma^2\right)^2}}{\left(\frac{2m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right)}$$

$$= \frac{R\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right) + \left(R - \overline{F}\right)\frac{m}{a^c\sigma^2}}{\left(\frac{2m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right)} + \frac{\frac{2m\overline{p}}{a^c\sigma^2}\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right)}{\left(\frac{2m}{a^c\sigma^2}\right)\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right)}$$

$$= \overline{p} + R\left(\frac{a^c\sigma^2}{2m}\right) + \frac{R - \overline{F}}{2\left(\frac{n}{a^g\sigma^2} + \frac{m}{a^c\sigma^2}\right)} \tag{26}$$

Since $R = \overline{F}$, we can write $q^c = \overline{p} + R\left(a^c\sigma^2\right)/(2m)$ and $q^g = \overline{p} - \overline{F}\left(a^g\sigma^2\right)/(2n)$. Now plug (26) into (4) to get:

$$y_i^c = R_i + \frac{\overline{p}}{a^c\sigma^2} - \left(\frac{1}{a^c\sigma^2}\right)\left[\overline{p} + R\left(\frac{a^c\sigma^2}{2m}\right)\right]$$

$$= R_i + \frac{\overline{p}}{a^c\sigma^2} - \frac{\overline{p}}{a^c\sigma^2} - \frac{R}{2m}$$

$$= R_i - \frac{R}{2m}$$

Similarly, plug (25) into (12) to obtain

$$y_k^g = \overline{F}_k - \frac{\overline{p}}{a^g\sigma^2} + \left(\frac{1}{a^g\sigma^2}\right)\left[\overline{p} - \overline{F}\left(\frac{a^g\sigma^2}{2n}\right)\right]$$

$$= \overline{F}_k - \frac{\overline{p}}{a^g\sigma^2} + \frac{\overline{p}}{a^g\sigma^2} - \frac{\overline{F}}{2n}$$

$$= \overline{F}_k - \frac{\overline{F}}{2n}$$

Finally, the spread can be calculated as

$$d = q^c - q^g = \overline{p} + R\left(\frac{a^c\sigma^2}{2m}\right) - \overline{p} + \overline{F}\left(\frac{a^g\sigma^2}{2n}\right)$$

$$= \left(\frac{\sigma^2}{2}\right)\left(\frac{Ra^c}{m} + \frac{\overline{F}a^g}{n}\right),$$

which, given that $R = \overline{F}$, boils down to $d = \left(\frac{R\sigma^2}{2}\right)\left(\frac{a^c}{m} + \frac{a^g}{n}\right)$.

Proof of Proposition 2:

Problem (18) can be rewritten as

$$\max_{y_h^b}\left(\frac{AD + BC - (D + B)y}{BD}\right)y_h^b \tag{27}$$

The first order condition for this problem is:

$$
\begin{aligned}
&-\frac{(D+B)}{BD}y_h^b+\frac{AD+BC-(D+B)y}{BD}=0\\
&\Rightarrow -(D+B)y_h^b+AD+BC-(D+B)y_h^b-(D+B)\sum_{j\neq h}y_j^b=0\\
&\Rightarrow y_h^b\left(2(D+B)\right)=AD+BC-(D+B)\sum_{j\neq h}y_j^b \qquad (28)\\
&\Rightarrow y_h^b=\frac{AD+BC}{2(D+B)}-\frac{\sum_{j\neq h}y_j^b}{2}
\end{aligned}
$$

Since marketers are symmetric, we have $y_h^b=y/H$ and $\sum_{j\neq h}y_j^b=(H-1)y/H$. Therefore

$$
\begin{aligned}
&y_h^b=\frac{AD+BC}{2(D+B)}-\frac{(H-1)y}{2H}\\
&\Rightarrow y=\frac{H(AD+BC)}{2(D+B)}-\frac{H(H-1)y}{2H}\\
&\Rightarrow y+\frac{(H-1)}{2}y=\frac{H(AD+BC)}{2(D+B)} \qquad (29)\\
&\Rightarrow y=\left[\frac{H(AD+BC)}{2(D+B)}\right]\left[\frac{2}{H+1}\right]=\frac{H(AD+BC)}{(H+1)(D+B)}
\end{aligned}
$$

and $y_h^b=(AD+BC)/\left[(H+1)(D+B)\right]$. Now we plug the formulas for *A*, *B*, *C* and *D* into (29) to get

$$
\begin{aligned}
y_h^b&=\frac{\left(R+\frac{m\overline{p}}{a^c\sigma^2}\right)\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\left(\overline{F}-\frac{n\overline{p}}{a^g\sigma^2}\right)}{(H+1)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}\\
&=\frac{\frac{Rn}{a^g\sigma^2}+\frac{mn\overline{p}}{a^ca^g\left(\sigma^2\right)^2}+\frac{\overline{F}m}{a^c\sigma^2}-\frac{mn\overline{p}}{a^ca^g\left(\sigma^2\right)^2}}{(H+1)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)} \qquad (30)\\
&=\frac{\overline{F}\left(\frac{m}{a^c\sigma^2}+\frac{n}{a^g\sigma^2}\right)}{(H+1)\left(\frac{n}{a^g\sigma^2}+\frac{m}{a^c\sigma^2}\right)}=\frac{\overline{F}}{H+1}
\end{aligned}
$$

and $y=\left(H/(H+1)\right)\overline{F}$, where we used the fact that $R=\overline{F}$.

The next step is to calculate prices:

$$
\begin{aligned}
q^c&=\frac{A-y}{B}=\frac{R+\frac{m\overline{p}}{a^c\sigma^2}-\left(\frac{H}{H+1}\right)\overline{F}}{\frac{m}{a^c\sigma^2}} \qquad (31)\\
&=\frac{Ra^c\sigma^2}{m}+\overline{p}-\left(\frac{H}{H+1}\right)\left(\frac{a^c\sigma^2}{m}\right)R=\overline{p}+R\left(\frac{a^c\sigma^2}{m}\right)\left(\frac{1}{H+1}\right)
\end{aligned}
$$

and

$$q^g = \frac{y-C}{D} = \frac{\left(\frac{H}{H+1}\right)\overline{F} - \overline{F} + \frac{n\overline{p}}{a^g\sigma^2}}{\frac{n}{a^g\sigma^2}} = \frac{\frac{n\overline{p}}{a^g\sigma^2} - \left(\frac{1}{H+1}\right)\overline{F}}{\frac{n}{a^g\sigma^2}} = \overline{p} - \overline{F}\left(\frac{a^g\sigma^2}{n}\right)\left(\frac{1}{H+1}\right) \tag{32}$$

The spread can now be easily calculated:

$$d = q^c - q^g = \overline{F}\left(\frac{s^2}{H+1}\right)\left(\frac{a^c}{m} + \frac{a^g}{n}\right),$$

where we again used $R = \overline{F}$.

Quantities contracted by suppliers and consumers can be obtained from (4), (12), (31) and (32):

$$y_k^g = \overline{F}_k - \frac{\overline{p}}{a^g\sigma^2} + \frac{1}{a^g\sigma^2}\left(\overline{p} - \overline{F}\left(\frac{a^g\sigma^2}{n}\right)\left(\frac{1}{H+1}\right)\right) = \overline{F}_k - \frac{\overline{p}}{a^g\sigma^2} + \frac{\overline{p}}{a^g\sigma^2} - \frac{\overline{F}}{n(H+1)} = \overline{F}_k - \frac{\overline{F}}{n(H+1)} \tag{33}$$

and

$$y_i^c = R_i + \frac{\overline{p}}{a^c\sigma^2} - \frac{1}{a^c\sigma^2}\left(\overline{p} + R\left(\frac{a^c\sigma^2}{m}\right)\left(\frac{1}{H+1}\right)\right) = R_i + \frac{\overline{p}}{a^c\sigma^2} - \frac{\overline{p}}{a^c\sigma^2} - \frac{R}{m(H+1)} = R_i - \frac{R}{m(H+1)} \tag{34}$$

Finally, we can check our calculations as follows:

$$\sum_{k=1}^{n} y_k^g = \sum_{k=1}^{n}\left(\overline{F}_k - \frac{\overline{F}}{n(H+1)}\right) = \overline{F} - \frac{\overline{F}}{H+1} = \left(\frac{H}{H+1}\right)\overline{F}$$

$$\sum_{i=1}^{m} y_i^c = \sum_{i=1}^{m}\left(R_i - \frac{R}{m(H+1)}\right) = R - \frac{R}{H+1} = \left(\frac{H}{H+1}\right)R$$

$$\sum_{h=1}^{H} y_h^b = \sum_{h=1}^{H}\frac{\overline{F}}{H+1} = \left(\frac{H}{H+1}\right)\overline{F}$$