IJEEP

INTERNATIONAL JOURNAL OF ENERGY ECONOMICS AND POLICY

EJ EconJournals ISSN: 2146-4553

International Journal of Energy Economics and Policy

ISSN: 2146-4553

available at http: www.econjournals.com

International Journal of Energy Economics and Policy, 2015, 5(4), 1109-1124.

EJ EconJournals

Dynamics of Switching from Polluting Resources to Green Technologies

Supratim Das Gupta*[a]

University of South Carolina, Columbia, SC, USA and University of Guanajuato, Guanajuato, Mexico.
*Email: dasgupta@ugto.org

[a]I am deeply indebted to my Ph.D. Dissertation Committee comprising of John McDermott, Christian Jensen and Jason Murray for their support throughout this project. All remaining errors are mine

ABSTRACT

We have total energy produced by a firm using a non-renewable resource and a perfect substitute backstop. The average cost of the backstop is significantly higher relative to the non-renewable resource initially; average backstop costs are modeled to fall with investments in knowledge. Investments in knowledge are thought to bring about more efficient techniques to use alternative energies (better technical know-how for wind, solar) reducing their average costs. The knowledge stock is modeled as an impure public good such that an individual firm only partially benefits from its own knowledge accumulation. We find a firm in equilibrium invests less in the backstop relative to the social planner and that the planner solution also leads to faster exhaustion of the depletable resource. Introducing flow pollution, we find the time of switch to the backstop in the planner solution depends on the relative magnitudes of the average pollution cost and the average cost of the backstop. An increase in the pollution cost implies slower extraction of the exhaustible resource and a later switch (compared to the case without pollution); however for a very high pollution cost, the extraction rate rises and switch to the backstop is made sooner leaving some of the exhaustible resource in the ground. We solve both the models explicitly and use sophisticated numerical techniques in Mathematica.

Keywords: Exhaustible Resources, Backstop, Knowledge Stock, Investment, Pollution, Numerical Methods
JEL Classifications: C61, O32, O33, Q32, Q42

1. INTRODUCTION

Efforts to use alternative technologies (backstop technologies) at a greater scale have been on the rise in most developed countries recently. An example can be the increased reliance on natural gas, hydro and wind for electricity generation in the U.S. (reduction in use of coal) in the past 2 years[1]. But significant cost differences between conventional dirty sources of energy (coal and gasoline) and these backstop technologies still exist. Data from the U.S. Energy Information Administration (EIA)'s Annual Energy Outlook 2014 estimates U.S. average levelized costs for electricity generation for plants entering service in 2019 (in 2012 \$/MWh) to be 95.6 for Conventional Coal, 102.6 for Biomass and 130.0 for Solar. This paper addresses the question of the optimal path of investment in backstops (in the event of substantial cost differences initially) when there exists a possibility to switch from a non-renewable resource to a backstop technology. We analyze this question from the point of view of a firm in equilibrium and in the case of a social planner.

Early work by Das Gupta and Heal (1974) and Stiglitz (1974a, 1974b) show when exhaustible resources are essential in production, positive consumption levels can be maintained with physical capital accumulation from early periods. However, consumption declines in the long run with an asymptotic exhaustion of the non-renewable resource. Tsur and Zemel (2003, 2005) analyze a switch from an exhaustible resource to a backstop;

1 In this paper, by "alternative" we mean cleaner technologies. Examples for alternative renewable technologies are solar, wind, hydro and biomass. Examples for nonrenewable alternative technologies are nuclear and natural gas.

our work is closest to Tsur and Zemel (2005) in that the switch from the exhaustible resource to the backstop depends on the amount of (average) cost reduction in the backstop. Reduction in the average backstop cost comes through R and D investments in the stock of knowledge[2]. More recent papers on the literature of regime switching include (Boucekkine et al. 2003, 2011, 2013a, 2013b), Acemoglu et al. (2012) where the authors focus on a switch from a polluting resource to a backstop. The first part of this paper excludes pollution and focuses only on the path of investment and the time of switch to the backstop in a model where total energy is produced from either an exhaustible resource or a backstop. However, to our knowledge, this study is unique in the modeling of R and D investments in the knowledge stock reducing average backstop cost: We introduce aggregate R and D investments as being an impure public good[3]. This implies that the benefit of investing in the knowledge stock (to achieve a lower backstop cost) is shared unequally between a representative firm and other firms in the economy. Since the social planner fully internalizes the benefits of investing in knowledge, we find intuitively that the planner carries out greater R and D investments relative to a firm in the equilibrium solution. Consequently, the planner achieves greater reduction in the average cost of the backstop at the time of switch.

We include pollution as a flow in the second part of the paper. The aggregate flow of pollution in any period which affects all firms in an economy is modeled as external to an individual firm. Schou (2000), in the context of a Cobb-Douglas production function with human capital and an exhaustible resource and zero extraction costs, analyze the steady-state growth rates for output, extraction and human capital when the non-renewable resource creates ow pollution problems[4]. Flow pollution has been shown to have adverse health effects by Currie and Schmieder (2008) and Graff Zivin and Neidell (2011). The latter study finds ozone pollution[5] affects the marginal product of labor even when total labor supply is unaffected (study in the context of agricultural workers in a farm in California). Currie and Schmieder (2008) find significant negative effects of pollutants on birth outcomes: Using data from U.S. Environmental Protection Agency's (EPA) Toxics Release Inventory (TRI), the authors particularly find pronounced effects on gestation and birth weight. Our results change significantly for the planner solution after pollution is introduced to the model. Based on the relative magnitudes of the unit pollution cost and the average backtop cost, we find the planner might switch to the backstop sooner or later compared to the case without pollution.

2 Intuitively, the switch occurs when the marginal cost of the exhaustible resource equals the falling marginal cost of the backstop.

3 Similar to Gray and Grimaud (2010), R and D spillovers are partial in that some part of the scope of diffusion is retained by a firm. But firms draw on a shared pool of knowledge when carrying out investment activities. In this paper, the knowledge stock is private to a firm (with some given initial stock) and investment only increases the representative firm's stock of knowledge.

4 We model pollution as a flow to keep the model tractable and obtain numerical solutions. Recently, pollution as stock with a ceiling constraint has been modeled by Amigues et al. (2012), Amigues and Moreaux (2013) and Boucekkine et al. (2013b).

5 Ozone forms from the complex interactions between nitrogen oxides (NO_x) and volatile organic chemicals (VOCs), both of which are directly emitted in the presence of heat and sunlight.

In cases where the non-renewable resource has no value in terms of adding to lifetime net profits because of very high pollution costs, switching sooner to the backstop is optimal. On the other hand, when pollution costs are relatively low, extracting the non-renewable resource at a slow rate, investing in the backstop in lower amounts and switching completely to the backstop later is optimal. This would be true for economies with high energy demands (e.g. India and China), that in spite of high levels of air pollution creating health hazards, these economies continue to be heavily reliant on traditional dirty energy sources. It is because these economies lack the infrastructure to use backstop technologies effectively and also have a substantial fraction of the population engaged in mining activities.

The contribution of this work lies in including R and D investments in knowledge as an impure public good with the simultaneous inclusion of flow pollution. Aggregate flow pollution in any period causes damage to all firms but each firm treats the aggregate flow as constant. Results for the social planner would be similar in case a constant marginal extraction cost is included (instead of pollution cost): Relative to the model without any cost for the non-renewable resource, the planner would extract greater amounts and switch sooner to the backstop when costs become too high. In the event of very high costs, the planner would leave some of the exhaustible resource unexploited. However, one of our main contributions is the fact that we solve both models explicitly: In cases where analytical solutions cannot be obtained, we use complex numerical techniques in Mathematica.

The following sections of this paper are organized as follows. Section 2 describes the main model of the paper (not including pollution). Sections 3 and 4 analyze the competitive equilibrium solution and the social planner solutions. In Section 5, we add flow pollution with Section 6 providing a summary of main results and ideas for future research.

2. MODEL

We consider an economy consisting of a continuum of measure one of identical firms. Each firm is owned by an infinitely-lived household. We abstract from population growth and for simplicity, we normalize the total population in the economy to be unity. A representative firm in the economy produces a "composite commodity" using a non-renewable natural resource and a perfect substitute backstop technology[6]. We consider a representative backstop technology in our model. Avoiding aggregation issues when there are many alternative technologies in reality to non-renewable resources such as coal and oil, we assume that the backstop is relatively expensive to the non-renewable resource initially (as would be clear later, we fix the parameters to depict this). Energy, or the sum of the use of the exhaustible resource and the backstop, is the only input for a representative firm. It must be noted that there is no alternative use of the exhaustible resource and it is only used to produce energy. The composite commodity in turn is used for both consumption and investment.

6 Das Gupta (2015) models total energy production as the sum of a polluting natural resource use and the use of a clean technology.

The production costs for a firm are divided into cost of the non-renewable resource and the backstop. We assume the exhaustible resource to have zero cost of extraction. We make this simplifying assumption in order for the optimal path of energy use (for a representative firm) to first involve use of the non-renewable resource before switching to the backstop. In other words, it is to make the backstop initially uncompetitive with the exhaustible resource. The only cost associated with the non-renewable resource is the opportunity cost of using much of it in the present so as to leave little for future generations. We introduce flow pollution from use of the exhaustible resource in a later section. On the other hand, the average cost of the backstop is positive which can be reduced through investments in the stock of knowledge at each time period. Alternative or backstop technologies may have a higher embodied technical progress (Boucekkine et al., 2004) or can be more sophisticated. Knowledge can be thought of as the technical know-how to operate alternative clean technologies like wind and solar. Investments are expenditures in R and D by a representative firm: These may bring about more efficient techniques to use alternative clean technologies lowering their average costs. Subsequently, we denote use of the non-renewable resource, the backstop, the average cost of the backstop technology and investments in knowledge in units of the composite commodity.

This paper assumes innovation in technical know-how to operate backstop technologies are developed in the lab. They do not come about through using the clean technologies themselves (or not through a learning-by-doing process). Learning-by-doing enhancing the stock of knowledge about the renewable energy source, which further helps to reduce its marginal cost has been modeled in Hartley et al. (2014) (see also Chakravorty et al. 2011). We model increments in the knowledge stock as purely through investments or R and D expenditures to keep the model tractable numerically. We note again that one of the main contributions of this work lies, when solutions cannot be found analytically, in the extensive use of computational techniques to arrive at numerical solutions.

The production function for the composite commodity is given by

$$y = e^{1-\alpha}, 0 < \alpha < 1 \tag{1}$$

where y denotes the composite commodity and e denotes total energy use.[7] We assume the price of the composite commodity y to be constant at unity. Finally, all firms in the economy are assumed to be price takers and entry and exit is not permitted in the model.

Total energy production e is in turn given by

$$e = r + b \tag{2}$$

where $r \geq 0$ and $b \geq 0$ represent exhaustible resource and backstop use by a representative firm. The exhaustible resource and the backstop are modeled as being perfect substitutes in the production of total energy. Furthermore, each household is assumed to have an identical initial endowment of the stock of the exhaustible resource. A firm (owned by an infinitely-lived household) draws down this initial stock of the exhaustible resource according to

$$\int_0^\infty r\,dt \leq s_0 \tag{3}$$

which implies,

$$\dot{s} = -r \tag{4}$$

where $s_0 > 0$ denotes the initial stock of the exhaustible resource.

The average cost of the backstop in the production of total energy is given by

$$M(n) = q + \frac{a}{N^{1-\beta} n^{\beta}}, q > 0, a > 0, 0 \leq \beta \leq 1 \tag{5}$$

where n denotes the stock of knowledge for a representative firm. The above specification is similar to that adopted by Tsur and Zemel (2005) in that increases in the stock of knowledge carried out by a representative firm helps reduce the future average cost of the backstop. Here N represents the aggregate stock of knowledge in the economy and is given by

$$N = \int_0^1 n_j\,dj \tag{6}$$

Such that n_j stands for the knowledge stock of the j[th] firm in the economy. $q > 0$ and $a > 0$ are the cost parameters of the model. A representative firm carries out investment to increase its knowledge stock represented by the equation

$$\dot{n} = \sqrt{i} \tag{7}$$

where $i \geq 0$ denotes investment for a representative firm. Firms in an economy are identical in that they are endowed with initial knowledge stock $n_0 > 0$. Equation (7) shows a concave function implying that its cheaper for a firm to invest little amounts every period rather than a lot in any one period.

Equations (5-7) indicate the divergence between the optimal (planner) and equilibrium solutions. Individual firms, thinking alike that their own investment decisions only have a negligible impact on the aggregate stock of knowledge in the economy N, take it as given. Equation (5) shows that only the part ($\frac{a}{N^{1-\beta} n^{\beta}}$) of the average backstop cost falls with knowledge accumulation while the part q is fixed. In the equilibrium solution an additional unit of investment in knowledge by a firm reduces the average backstop cost by the term n^{β} whereas $N^{1-\beta}$ captures by how much backstop costs would decrease if the aggregate knowledge stock increased by one unit. In other words, a representative firm in equilibrium only partially benefits from its own investment decisions. On the contrary, the social planner fully internalizes the positive externality of investments in the stock of knowledge and invests a greater amount compared to an individual firm in the equilibrium solution. In an essence similar to Gray and Grimaud (2010), we model R and D spillovers being partial as some part of the scope of diffusion of an innovation is retained by only the representative firm.

7 We understand that $y = e^{\alpha}$ might have been an easier specification. However, we implicitly assume $0 < 1 - \alpha < 1$ and $0 < \alpha < 1$ to be the respective shares of energy and labor. In the numerical part of the paper, we assume $\alpha = 0.667$.

However, we differ in that R and D activities or investment are purely private to a firm which increases its own knowledge stock; individual firms do not draw on a shared pool of technological knowledge when carrying out R and D. In equation (5) the degree of public versus private benefit of knowledge accumulation is captured by the parameter β. When $\beta = 0$, all benefits of investment are fully external to a representative firm and it cannot reduce the average backstop cost in any way. At the other extreme of $\beta = 1$, the average cost of the backstop only depends on the individual stock of knowledge of a representative firm. In this case, the full extent of externality is internalized by an individual firm and the equilibrium and planner solutions coincide.

3. COMPETITIVE EQUILIBRIUM SOLUTION

3.1 The Problem

The representative firm maximizes the Present Discounted Value (PDV) of net profits over infinite periods. Net profits are given by the market value of the composite commodity net of input costs and investments in knowledge. We have

$$\pi = y - M(n)b - i \tag{8}$$

Substituting from above in (8), the problem for a representative firm is given by

$$\max_{\{b,r,i\}} \int_{t=0}^{\infty} ((b+r)^{1-\alpha} - \left(q + \frac{a}{N^{1-\beta} n^{\beta}}\right) b - i) e^{-\rho t} dt \tag{9}$$

subject to (4), (7) and (6) where $\rho > 0$ is the discount rate. The initial stocks of the non-renewable resource s_0 and that of knowledge n_0 are given.

The current-valued Hamiltonian of the above problem is

$$H = (b+r)^{1-\alpha} - \left(q + \frac{a}{N^{1-\beta} n^{\beta}}\right) b - i + \lambda_1 \sqrt{i} - \lambda_2 r + \theta_1 b + \theta_2 r + \theta_3 i$$

s and n are the states of the system and b, r and i are the controls. We denote the shadow prices by λ_i's and the Lagrange multipliers associated with the controls by respective θ_i's. $\lambda_i, \theta_i \geq 0$. The first-order conditions and the transversality conditions are given by

$$\frac{\partial H}{\partial b} = (1-\alpha)(b+r)^{-\alpha} - \left(q + \frac{a}{N^{1-\beta} n^{\beta}}\right) + \theta_1 = 0, \theta_1 b = 0 \tag{10}$$

$$\frac{\partial H}{\partial r} = (1-\alpha)(b+r)^{-\alpha} - \lambda_2 + \theta_2 = 0, \theta_2 r = 0 \tag{11}$$

$$\frac{\partial H}{\partial i} = -1 + \frac{\lambda_1}{2\sqrt{i}} + \theta_3 = 0, \theta_3 i = 0 \tag{12}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial n} = \rho\lambda_1 - \frac{a\beta}{N^{1-\beta} n^{1+\beta}} b \tag{13}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial s} = \rho\lambda_2 \tag{14}$$

$$\lim_{t\to\infty} e^{-\rho t} \lambda_1(t) n(t) = 0 \tag{15}$$

and, $$\lim_{t\to\infty} e^{-\rho t} \lambda_2(t) s(t) = 0 \tag{16}$$

As all firms are identical, the average knowledge stock for the economy N equals the individual stock of knowledge n for a representative firm. Imposing the aggregate consistency condition $N = n$ in the above first-order conditions, we can write (10) and (13) as

$$(1-\alpha)(b+r)^{-\alpha} - \left(q + \frac{a}{n}\right) + \theta_1 = 0, \theta_1 b = 0 \tag{17}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial n} = \rho\lambda_1 - \frac{a\beta}{n^2} b \tag{18}$$

the other conditions given by (11), (12), (14), (15) and (16) remain unchanged.

The above necessary conditions imply either interior or corner solutions for the control variables b, r and i. However, when $i = 0$, the marginal benefit of investing in knowledge is infinity compared to a marginal cost of 1. So investment would always be positive in the model $i > 0$. From (7) and (12), setting $\theta_3 = 0$ we get

$$\dot{n} = \sqrt{i} = \frac{\lambda_1}{2} \tag{19}$$

Equation (19) would be a key equation for analyzing the model as it relates the time path of investment to that of the shadow price for the knowledge stock λ_1. Equation (14) shows the Hotelling rule is satisfied as the shadow price of the stock of the non-renewable resource λ_2 always rises at the constant rate of discount.

3.2 Energy, Investment and Profit Profiles

Analyzing the above necessary conditions for optimality, we find the representative firm would first only use the non-renewable resource followed by an instant of simultaneous use of both the non-renewable resource and the backstop and then periods of only using the backstop technology. This can be explained with the help of the following cases.

Case I: The interior case of $b > 0$ and $r > 0$. From (10) and (11), we get $\theta_1 = \theta_2 = 0$. This implies

$$(1-\alpha)(b+r)^{-\alpha} = \left(q + \frac{a}{n}\right) \tag{20}$$

and, $$(1-\alpha)(b+r)^{-\alpha} = \lambda_2 \tag{21}$$

The non-renewable resource and the backstop being perfect substitutes (equal marginal benefits), (20) and (21) would be satisfied only at one instant in time when the marginal cost of the backstop equals the shadow price of the exhaustible resource stock. Total energy use $\hat{e}$ in this case is given by

$$\hat{e} = \left(\frac{1-\alpha}{q + \frac{a}{n}}\right)^{\frac{1}{\alpha}} = \left(\frac{1-\alpha}{\lambda_2}\right)^{\frac{1}{\alpha}} \tag{22}$$

Case II: The corner case of $r > 0$ and $b = 0$. This implies $\theta_1 > 0$ and $\theta_2 = 0$ from (10) and (11). Here

$$(1-\alpha)r^{-\alpha} = \left(q+\frac{a}{n}\right)-\theta_1 \quad (23)$$

and, $(1-\alpha)r^{-\alpha} = \lambda_2$ (24)

Total energy use by a firm would equal its use of the non-renewable resource. From (24)

$$\hat{e} = \hat{r} = \left(\frac{1-\alpha}{\lambda_2}\right)^{\frac{1}{\alpha}} \quad (25)$$

With $0 < \alpha < 1$, (25) implies falling non-renewable resource extraction over time at a constant rate with the rise in λ_2. Equation (23) determines the value of $\theta_1 = \left(q+\frac{a}{n}\right)-\lambda_2$ which must be falling over time. Furthermore, substituting $b = 0$ in (18) we get

$$\dot{\lambda}_1 = \rho\lambda_1 \quad (26)$$

We get the interesting result that both the co-states λ_1 and λ_2 grow at the constant rate ρ when $r > 0$ and $b = 0$. For periods when the backstop is not used, the shadow price of the knowledge stock would increase at a rate higher than that given by (18) as additions to the stock of knowledge become even more valuable with a possibility to switch to a cheaper backstop in future.

Case III: The other corner case is of $b > 0$ and $r = 0$. This implies $\theta_1 = 0$ and $\theta_2 > 0$ from (10) and (11). In this case total energy use follows the time path of backstop use for a representative firm. We get

$$\hat{e} = \hat{b} = \left(\frac{1-\alpha}{q+\frac{a}{n}}\right)^{\frac{1}{\alpha}} \quad (27)$$

Given that $0 < \alpha < 1$, backstop use rises over time with growing knowledge accumulation. However, in reality there are geophysical limits or a certain carrying capacity of the earth which puts a check on this rising energy use. From equations (10) and (11) we find the equilibrium value of $\theta_2 = \lambda_2 - \left(q+\frac{a}{n}\right)$ which is rising over time. The time paths of the co-states λ_1 and λ_2 are given by (18) and (14).

From (22), (25) and (27), we can conclude that a representative firm's profile for total energy use would include

1. Only exhaustible resource use whenever $\lambda_2 < \left(q+\frac{a}{n}\right)$[8]
2. An indeterminate division between the exhaustible resource and the backstop when $\lambda_2 = \left(q+\frac{a}{n}\right)$
3. Only backstop use whenever $\lambda_2 > \left(q+\frac{a}{n}\right)$.

While extraction costs are assumed to be zero for the non-renewable resource, we assume its initial shadow cost $\lambda_2(0)$ to be sufficiently low such that its rational for a firm to start producing energy using only the exhaustible resource ($\lambda_2(0) < \left(q+\frac{a}{n_0}\right)$).

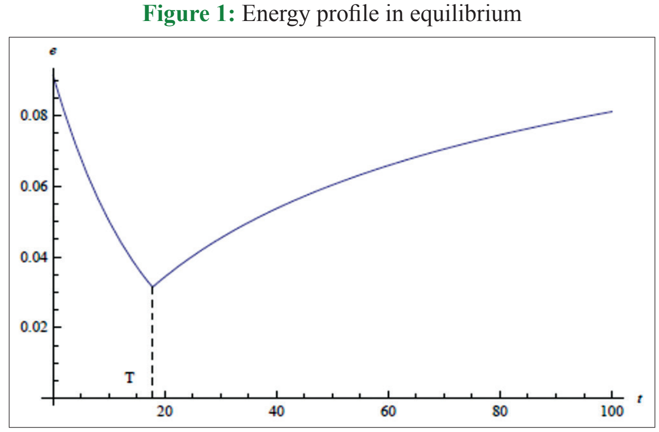
Since leaving any stock of the non-renewable resource in the ground is not optimal, the firm would exhaust it completely before switching over to the backstop. A representative firm would extract the non-renewable resource (and invest in the knowledge stock) in a manner such that this time of switch also corresponds to the date when the marginal costs of *b* and *r* become equal. Denoting the date of exhaustion of the initial resource stock s_0 as *T*, we can summarize the production of total energy *e* for a firm in equilibrium in Figure 1. For all practical purposes, we capture the above three cases in two phases, Phase 1 $(r > 0, b = 0)$ and Phase 2 $(r = 0, b > 0)$. We assume that only the exhaustible resource is used until T and the backstop is used from the very next instant. That is to say, non-renewable resource use at *T* would equal backstop use ($\hat{r} \equiv \hat{b}$ at *T*). In Figure 1, the energy profile for a firm is V-shaped and continuous but with a kink at the time of switch from the exhaustible resource to the backstop. To check if the transversality condition (16) is satisfied for the energy profile for a firm in equilibrium, we can modify it for a finite horizon case as

$$e^{-\rho T}\lambda_2(T)s(T) = 0 \quad (28)$$

where *T* denotes the date of exhaustion for the stock of the non-renewable resource. The condition is satisfied as s(T) = 0.

Turning to the path of investment in the stock of knowledge for a representative firm, we find from equations (19) and (26) that investment over time grows at the constant rate of discount until the time of switch *T* to the backstop technology. Moreover, combining equations (19) and (26), we can see that the path of investment as a function of the knowledge stock *n* would exhibit a similar

Figure 1: Energy profile in equilibrium



8 Given no alternative use of the exhaustible resource except producing energy, and perfect substitutability between the exhaustible resource and the backstop, a firm would find it rational to use the cheaper input at first before switching to the relatively expensive backstop.

profile to the path of investment over time.[9] Figure 2 shows the path of investment for the equilibrium solution as a function of the knowledge stock (n_0 in the figure represents the initial stock of knowledge). After growing at a constant rate until the time of switch to the backstop, Figure 2 shows that investment starts to fall as the stock of knowledge gets larger. In Figure 2, $\hat{n}$ refers to the stock of knowledge at T or the time of switch to the backstop. The fall in investment after T can be seen by combining equations (18) and (19): Intuitively, after a firm switches to the backstop technology b and given that the model does not allow the individual firm to switch back to the exhaustible resource r (as its already exhausted), additions to the stock of knowledge are not as valuable as before the switch to the backstop. This is verified by comparing equations (18) and (26) which shows that the shadow price of the knowledge stock λ_1 grows at a slower rate when $b > 0$ as opposed to when $r > 0$. So the average cost of the backstop falls at a faster rate when a representative firm only uses the non-renewable resource.

Now we consider the situation of profits for a firm in the equilibrium solution. Net profits for a representative firm are given by total production net of the cost of the backstop and investments in knowledge. Each household is assumed to consume its entire profits. The net profit over time for a firm in equilibrium is shown in Figure 3. Firm profits fall in the phase of only non-renewable resource use due to a rising λ_2 (as the stock of the exhaustible resource keeps falling) combined with increasing investments in the knowledge stock[10]. The jump in profits profile occur as backstop costs kick in at the time of switch to the backstop technology or T. The magnitude of this jump equals the average cost of the backstop *M(n)* at the time of switch. With knowledge being accumulated at a decreasing rate by a firm after T and combined with growing backstop use, profits rise slowly in this phase.

4. SOCIAL PLANNER SOLUTION

The social planner fully internalizes the external benefits of investments in the knowledge stock which helps reduce the average cost of the backstop. As mentioned previously and compared to the equilibrium solution, it is as if $\beta = 1$ for the planner solution. We can impose the aggregate consistency condition $N = n$ before maximization; equation (5) now changes to

$$M(n) = q + \frac{a}{n} \tag{29}$$

The maximization problem for the planner solution can then be written as

$$\max_{\{b,r,i\}} \int_{t=0}^{\infty} ((b+r)^{1-\alpha} - \left(q + \frac{a}{n}\right)b - i)e^{-\rho t}dt \tag{30}$$

subject to (7), (4) and (6). We write the subsequent Hamiltonian as

$$H = (b+r)^{1-\alpha} - \left(q + \frac{a}{n}\right)b - i + \lambda_1\sqrt{i} - \lambda_2 r + \theta_1 b + \theta_2 r + \theta_3 i$$

As before, s and n are the states of the system and b, r and i are the controls. We denote the shadow prices by λ_i's and the Lagrange multipliers associated with the controls by respective θ_i's $\lambda_i, \theta_i \geq 0$.

From the first-order conditions, we get (17), (11), (12), (14-16) as before. Equation (18) now changes to

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{a}{n^2}b \tag{31}$$

From the necessary conditions for optimality, the time paths for b, r and i ould be identical to those for the equilibrium solution. Investment in the knowledge stock is always positive as in the equilibrium solution and its time path would be given by (19). These imply that the optimal energy profile is also V-shaped in the planner solution. That is the planner uses only the non-renewable resource at first before switching completely to the backstop technology (this time of switch corresponds to the date of exhaustion for the exhaustible resource)[11]. However, level differences at each point in time arise between the two energy

9 We show the investment profile over time in the Appendix.

10 Although we have a continuum of identical firms in the economy and there is no trading of the non-renewable resource stock between firms, even if the possibility of trading is allowed, firms' would buy and sell the exhaustible resource only at the shadow price of λ_2.

11 The modified transversality condition in equation (28) would be satisfied as the non-renewable resource is fully exhausted at the time of switch.

Figure 2: Investment profile in equilibrium

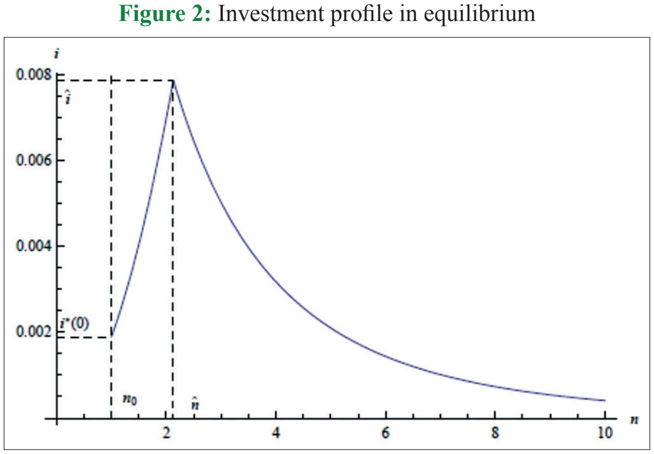
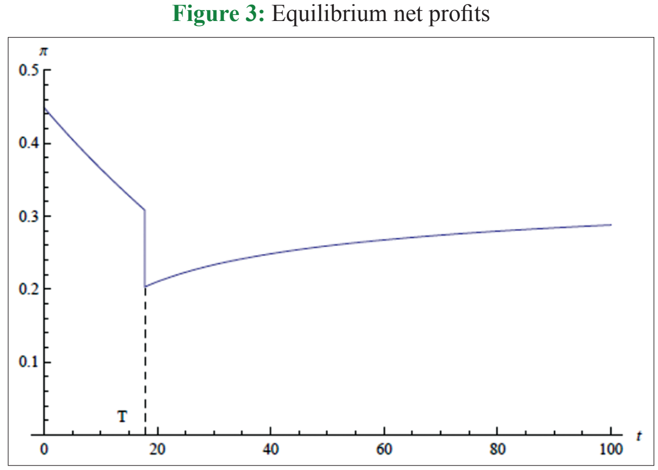


Figure 3: Equilibrium net profits



profiles because of different initial values of the shadow price of the non-renewable resource stock and a different T for a firm in equilibrium and the social planner. Figure 4 combines the energy profiles for the planner and equilibrium solutions (in Figure 4 and in the following figures, S.P. stands for social planner and C.E. for competitive equilibrium). The points of kink, which show the times at which the non-renewable resource stock is exhausted and the switch is made to the backstop technology, is different across the two solutions. The planner starts with a higher non-renewable resource extraction from the initial period and sooner exhausts its given stock. Since the planner extracts more of the exhaustible resource stock every period relative to a firm in equilibrium, investment would always be higher in the optimal solution compared to the equilibrium one. Investment being higher until the time of switch to the backstop T implies a higher knowledge stock at T for the planner solution. As Figure 4 shows, this higher knowledge stock also allows the social planner to have a greater use of the backstop after the time of switch.

We compare the investment profiles for the equilibrium and planner solutions in Figure 5. As discussed previously, investment profiles as a function of the knowledge stock n would have a similar shape as the investment time paths (n_0 once again represents the initial stock of knowledge). The graphs for both the equilibrium and planner solutions show that investment rises at a constant rate until the time of switch to the backstop technology and then falls. Investments in knowledge are more valuable during periods of only non-renewable resource use for both the representative firm in equilibrium and the social planner. Figure 5 is one of the central ones of the paper. It shows precisely the underinvestment problem in knowledge in the equilibrium solution compared to the optimal one. Hence the planner affects a larger decline in the average cost of the backstop.

Finally, we compare the net profits between the equilibrium and planner solutions. Figure 6 combines both the graphs. The planner extracts more out of the non-renewable resource stock and thus invests greater amounts relative to a firm in equilibrium. Over time, as the rate of exhaustible resource extraction falls but investments in knowledge keep increasing (until the switch to the backstop), profit for the optimum solution would fall below that for the equilibrium solution briefly. At the time of switch T, net profits are higher for the planner solution[12] compared to the equilibrium one: This is because both $\hat{n}$ and $\hat{r} \equiv \hat{b}$ are higher for the planner solution. Net profits continue to be higher for the social planner even after T as the total value of production dominates a higher backstop use and greater investment.

4.1 Comparison of the Equilibrium and Planner Solutions

We compute the time of switch to the backstop T, the key values for the control variables and the shadow prices using numerical methods in Mathematica. We explain some of the techniques used in the Appendix.

Figure 4: Energy profiles for the planner and equilibrium solutions

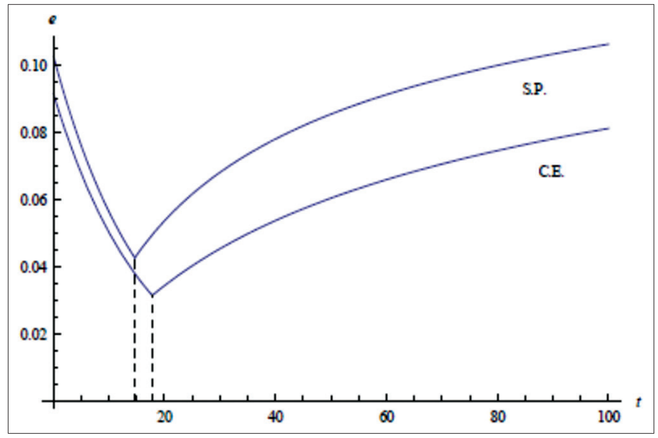


Figure 5: Investment profiles for the planner and equilibrium solutions

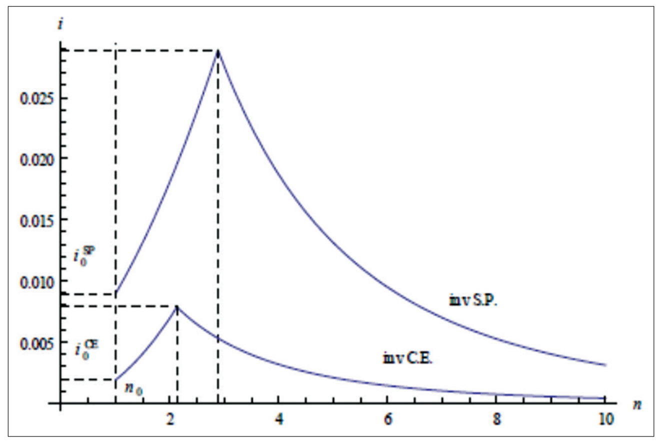
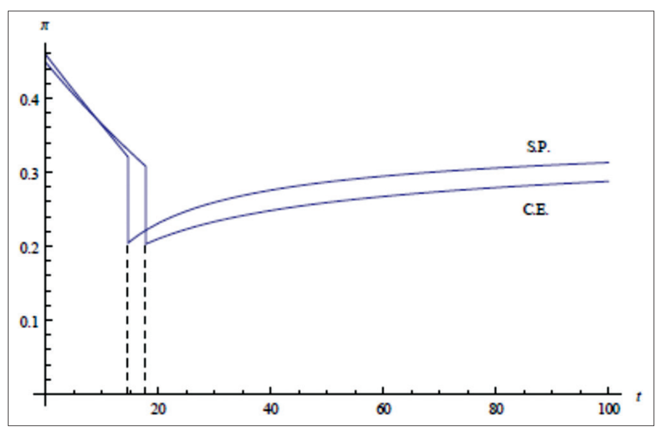


Figure 6: Net profits for the planner and equilibrium solutions



We use standard values in the literature of α = 0.667 and ρ = 0.04 and fix the other parameter values to be β = 0.33, q = 1, a = 5, $s_0 = 1$ and $n_0 = 1$. We do not have any intuition on the value of the externality parameter β and assume that only $\frac{1}{3}$ of the benefits of investments in knowledge accrue to an individual firm in equilibrium. We summarize the results in Table 1[13]. That the social

12 The jump in profits for both solutions at the switch point equals the average cost of the backstop given by (5).

13 In parity with previous notation, b_T^* and r_T^* represent the values of $\hat{b}$ and $\hat{r}$ at T respectively.

Table 1: Comparison of equilibrium and planner solutions

Values	Equilibrium	Planner
$\lambda_1^*(0)$	0.087	0.189
$\hat{\lambda}_1 = \lambda_1^*(T)$	0.177	0.34
$\hat{n} = n_T$	2.129	2.884
T	17.803	14.655
$b_T^* \equiv r_T^*$	0.031	0.043
$r^*(0)$	0.091	0.103
$i^*(0)$	0.002	0.009
$\lambda_2^*(0)$	1.642	1.521
PDV of net profits	7.865	8.035

PDV: Present Discounted value

planner values investment in the stock of knowledge much more is reflected in the values of $\lambda_1^*(0)$: The corresponding value for the planner is more than twice than that for the representative firm. As a result, initial investment $i^*(0)$ is significantly lower in the equilibrium solution compared to the planner solution. This leads to a lower accumulated knowledge stock for a firm in equilibrium relative to the social planner at the time of switch to the backstop: The accumulated knowledge stock is 2.88 for the optimal solution and 2.13 for the equilibrium solution. We see that the representative firm extracts the non-renewable resource at an inefficiently slow rate: An individual firm takes 17.8 years whereas it takes 14.66 years for the planner to run out of the exhaustible resource and switch completely to the backstop. Expectedly, this is reflected in the higher initial shadow price of the exhaustible resource $\lambda_2^*(0)$ for a firm in equilibrium. Since the planner extracts more out of the initial stock of the non-renewable resource, the value of the backstop at the time of switch is higher for the planner at 0.04 compared to 0.03 for the representative firm in equilibrium.

5. INTRODUCING FLOW POLLUTION TO THE MODEL

We introduce flow pollution to the above model and explore how the results are affected for the planner and equilibrium solutions. Flow pollution has been proven to have adverse health effects as in studies by Currie and Schmieder (2008) and Graff Zivin and Neidell (2011). Flow pollution as a cost in profits for a representative firm was included by Schou (2000). The author studies the difference in long-run growth rates between the equilibrium and planner solutions and finds that a greater influence of the negative externality of pollution cost would improve the long-run growth rate for both the solutions. Schou (2000) also finds that since the non-renewable resource is essential in production, the consumption path for the market economy may approach zero in the long-term. In our model, pollution at any instant of time is caused by the aggregate use of the exhaustible resource in production by all firms. Furthermore, the representative firm ignores the effect of its own action on total pollution; however the effect of total pollution (in a given period) on firm profits is negative. For simplicity and in order to get possible solutions, we assume the average cost of pollution to be constant.

5.1 The Equilibrium and Planner Solutions

The structure of the model is identical to the previous one when there was no pollution. The only difference lies in the net profit function for a representative firm. Since a firm in equilibrium now faces the additional pollution cost, net profits for a representative firm *j* in each period is given by

$$\pi = (b+r)^{1-\alpha} - \left(q + \frac{a}{N^{1-\beta} n^{\beta}}\right) b - i - dR \qquad (32)$$

$$\text{where } R = \int_0^1 r_j dj \qquad (33)$$

such that *R* represents aggregate non-renewable resource use. d > 0 measures the pollution cost per unit for a representative firm. A firm does not take into account the fact that its own use of the exhaustible resource adds up to the aggregate pollution flow in any given period. However, the representative firm bears a cost for this total pollution flow: This can be thought of as an instantaneous damage cost or a negative effect on firm productivity (similar to a negative effect on worker productivity in Graff Zivin and Neidell (2011)). The problem for a representative firm is then given by

$$\max_{\{b,r,i\}} \int_{t=0}^{\infty} \left((b+r)^{1-\alpha} - \left(q + \frac{a}{N^{1-\beta} n^{\beta}}\right) b - i - dR\right) e^{-\rho t} dt \qquad (34)$$

subject to (7), (3), (6) and (33). n_0 and s_0 are given and the other parameters remain the same as before.

In the equilibrium solution, a firm treats *dR* as constant. As a result, the behavior of the representative firm in competitive equilibrium remains identical with the previous case without pollution cost. The first-order conditions are (11), (19), (14-18) and the paths of *b, r* and *i* remain the same. The energy profile is given by Figure 1; pollution as a damage cost would only affect a firm's net profits in equilibrium.

Including flow pollution, the social planner internalizes the external benefits of investments in knowledge as well as the effect of pollution from aggregate use of the exhaustible resource. So imposing the aggregate consistency conditions $N=n$ and $R=r$ the maximization problem for the planner solution can be written as

$$\max_{\{b,r,i\}} \int_{t=0}^{\infty} \left((b+r)^{1-\alpha} - \left(q + \frac{a}{n}\right) b - i - dr\right) e^{-\rho t} dt \qquad (35)$$

subject to (7), (3), (6) and (33). As before, n_0 and s_0 are given and the other parameters remain unchanged. The subsequent Hamiltonian is

$$H = (b+r)^{1-\alpha} - \left(q + \frac{a}{n}\right) b - i - dr + \lambda_1 \sqrt{i} - \lambda_2 r + \theta_1 b + \theta_2 r + \theta_3 i \qquad (36)$$

where λ_i's denote the respective shadow prices and θ_i's are the Lagrange multipliers associated with the controls. $\lambda_i, \theta_i \geq 0$. The necessary conditions for optimality imply equations (12), (14-17), and (31). The only difference lies in the first-order condition with respect to *r* which now becomes

$$\frac{\partial H}{\partial r} = (1-\alpha)(b+r)^{-\alpha} - d - \lambda_2 + \theta_2 = 0, \theta_2 r = 0 \quad (37)$$

5.1.1 Energy profile and investment in the social planner solution

We follow a similar sequence as before of analyzing cases of simultaneous use of the non-renewable resource and the backstop followed by cases of only exhaustible resource or only backstop use. Since *d* is a constant, the above necessary conditions imply corner solutions for the controls and while investment is given by (19) as before. When $b>0,\ r>0\ (\theta_1 = 0 = \theta_2)$, we get

$$(1-\alpha)(b+r)^{-\alpha} = \lambda_2 + d \quad (38)$$

and (20). Total energy use is given by

$$\hat{e} = \left(\frac{1-\alpha}{q+\frac{a}{n}}\right)^{\frac{1}{\alpha}} = \left(\frac{1-\alpha}{\lambda_2 + d}\right)^{\frac{1}{\alpha}} \quad (39)$$

When we have $r>0,\ b=0\ (\theta_1 > 0, \theta_2 = 0)$, total energy use is given by

$$\hat{e} = \hat{r} = \left(\frac{1-\alpha}{\lambda_2 + d}\right)^{\frac{1}{\alpha}} \quad (40)$$

with $\theta_1 = \left(q+\frac{a}{n}\right) - \lambda_2 - d$. On the other hand, the energy profile with only using the backstop, $b>0,\ r=0,\ (\theta_1 = 0, \theta_2 > 0)$, is given by (27) as before with $\theta_2 = \lambda_2 + d - \left(q+\frac{a}{n}\right)$.

The social planner would produce total energy using the cheaper input in any given time period. The planner would only use the exhaustible resource as long as $\lambda_2 + d < \left(q+\frac{a}{n}\right)$, an indeterminate mix of both the inputs when $\lambda_2 + d = \left(q+\frac{a}{n}\right)$ and only the backstop when $\lambda_2 + d > \left(q+\frac{a}{n}\right)$. But it is important to note that λ_2 now depends on the value of *d*[14]. For a high value of *d* (relative to *q* or the part of the average backstop cost that cannot be influenced through investment in knowledge), the shadow price of the non-renewable resource stock λ_2 falls to zero[15]. That is to say, an additional unit of the exhaustible resource would not have any effect on discounted lifetime net profits. Also note that, the necessary conditions imply that if $\lambda_2(0) = 0$, λ_2 remains at zero forever. However, for $\lambda_2 = 0$, we may still have $d < q + \frac{a}{n_0}$ implying that the planner uses the non-renewable resource at first (until the time when $d = q + \frac{a}{\hat{n}}$). As long as the value of *d* is below a critical threshold such that $\lambda_2(0) > 0$, the time of switch to the backstop technology depends on when the marginal costs of the exhaustible resource and the backstop are equalized and not necessarily whether the initial resource stock is completely exhausted at that time. Intuitively, for any λ_2, if $\left(d > q + \frac{a}{n_0}\right)$, the planner would switch to the backstop from the very beginning. The initial stock of the exhaustible resource s_0 is left intact in this case. Denoting the time of switch from the exhaustible resource to the backstop technology by *T* as before, the following conditions determine *T*

$$\lambda_2(t) \geq 0 \quad (41)$$

$$\int_0^T r(t)\,dt \leq s_0 \quad (42)$$

$$\lambda_2(T)\left(\int_0^T r(t)\,dt - s_0\right) = 0 \quad (43)$$

$$\lambda_2(T) + d = q + \frac{a}{n_T} \quad (44)$$

Equations (41) and (42) are derived from constraints for the maximization problem. Equation (43) indicates that the net stock of the non-renewable resource valued at its shadow price should equal zero at the time of switch *T*. It shows that some stock of the exhaustible resource may be left in the ground if its shadow price falls to zero: The exhaustible resource is completely exhausted only if its shadow price is positive at the time of switch to the backstop. We would have a trivial case when the initial non-renewable resource stock is completely exhausted and its shadow price falls to zero. Equation (44) indicates that the marginal costs of the two energy inputs should be equal at T[16].

Lemma 1 shows how the value of the average cost of pollution *d* relative to the constant portion of the average cost of the backstop *q* determines whether $\lambda_2(0) = 0$.

Lemma 1: *d* > *q* is a necessary but not sufficient condition for $\lambda_2(0) = 0$.

Proof: Given (14), $\lambda_2(0) = 0 \Rightarrow \lambda_2(T) = 0$ for $t = T$. Then equation (44) $\Rightarrow n_T = \frac{a}{d-q}$ where *T* denotes the time of switch to the backstop. As $n_T > 0$, this implies $d > q$. Given the values of the parameters and solving the model numerically, as $d \geq 2.57$ for $\lambda_2(0) = 0$, the above lemma is proved.

Figure 7 shows the optimal energy profiles for various values of the pollution parameter *d*. An increase in unit pollution cost d makes the non-renewable resource less valuable. The shadow price

14 We assume a given d such that the value of $\lambda_2(0)$ justifies the planner using the exhaustible resource at first to produce energy.

15 This is shown in Table 2.

16 Note that $n_T = \hat{n}$; this is mentioned in Table 2.

of the stock of the exhaustible resource at time zero $\lambda_2(0)$ falls when d increases (Table 2 clearly illustrates this effect). However, as long as $\lambda_2 > 0$, a higher d would make the planner extract the non-renewable resource at a slower rate and thus switch later to the backstop (note from (43) that there would be complete exhaustion of the initial stock s_0). This is because of the fact that although the exhaustible resource is still valuable, a higher unit pollution cost makes it less attractive to use in the optimal solution. Thus the point of kink in the V-shaped energy profile showing the time of switch moves to the right and the path of non-renewable resource use gets flatter. Given the parameters, we compute the critical value of $d \geq 2.57$ for $\lambda_2(0) = 0$ and In this case, the path of exhaustible resource extraction would be flat and the time of switch to the backstop would also fall. Finally, whenever $\left(d \geq q + \frac{a}{n_0}\right)$, the planner would switch immediately to the backstop leaving all of the initial exhaustible resource stock in the ground.

We show the energy profiles for the optimal and equilibrium solutions for various values of d in Figure 8. The dashed lines (S.P. wpol) show the planner profiles when pollution is included to the model. The solid lines are paths of the optimal (S.P.nopol) and equilibrium (C.E.) solutions for the model without pollution. Recall that the energy profile for a representative firm in equilibrium doesn't change when pollution is included to the model; for the planner solution, flow pollution would only affect the first phase of the energy profile when using the exhaustible resource. Energy profiles for the planner solution for both the models would eventually merge after the switch is made to the backstop. Figure 8 shows that the time of switch to the backstop increases for the optimal solution when pollution is introduced to the model. This was explained previously in that for $d < 2.57$, a rise in unit pollution cost d makes the planner conserve the resource for longer periods. From the previous model without pollution, T or the time of switch to the backstop equaled 17.8 years for the equilibrium solution and 14.7 years for the optimal one. After including pollution, $T = 15.8$ years for a relatively low value of d = 0.6 and $T = 19.3$ years when $d = 2$ in the planner solution. For both these cases, the non-renewable resource stock is fully exhausted. For a higher unit pollution cost of $d = 3$ ($d \geq 2.57$), the given stock s_0 is not fully exhausted and the time of switch to the backstop falls to 9.8 years. As an illustration, Figure 9 shows the time of switch to the backstop T as a function of the pollution cost d. We see that as long as $d < 2.57$, T rises with a rise in d. However, for $d \geq 2.57$, T falls with a rise in unit pollution cost as $\lambda_2 = 0$.

Turning to the path of investment in knowledge for the planner solution[17], since the path of extraction of the non-renewable resource changes to (40), this leads to a change in investment at every time period until T. Intuitively, from (19) and (31), the path of λ_1 changes during the phase of only non-renewable resource use (when λ_1 grows at the rate of discount ρ). This implies a change in the initial shadow price of the knowledge stock $\lambda_1^*(0)$ and the accumulated stock of knowledge at the time of switch to the backstop or $\hat{n}$. Figure 10 plots investment profiles as a function of the knowledge stock for both the equilibrium and the optimal solutions for various values of d[18]. The dashed lines show investment profiles for the planner solution (S.P.wpol) when pollution is introduced to the model. The solid lines are investment paths for the equilibrium (C.E.) and optimal (S.P.nopol) solutions when pollution is not included. Interestingly, we observe that

17 We do not analyze profit profiles as in the previous model without cost of pollution. The only interesting case would be for the planner solution when the profit profile would not exhibit any discontinuity or jump at the time of switch to the backstop as $\lambda_2 = 0$.

18 Although the investment profile for the equilibrium solution does not change after introducing pollution to the model, we purely plot this for reference.

Figure 7: Planner energy profiles for various values of d

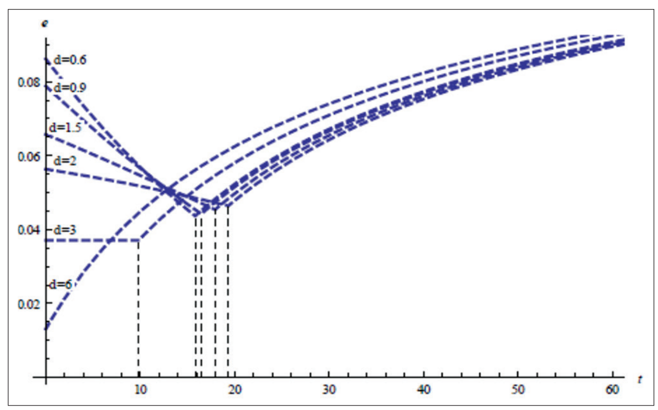


Table 2: Planner solutions for various values of d

Pollution cost	$\lambda_1^*(0)$	$i^*(0)$	$\lambda_2^*(0)$	$r^*(0)$	$\hat{n} = n_T$	T	% of s_0 used
d=0	0.189	0.009	1.521	0.103	2.884	14.655	100
d=0.6	0.178	0.008	1.109	0.086	2.962	15.833	100
d=0.9	0.172	0.007	0.913	0.079	3.002	16.487	100
d=2	0.148	0.005	0.268	0.056	3.162	19.323	100
d=2.57	0.134	0.004	0.0003≈0	0.047	3.263	21.413	100
d=2.6	0.154	0.006	0	0.046	3.125	18.636	85.5
d=3	0.249	0.015	0	0.037	2.5	9.838	36.4
d=4	0.395	0.039	0	0.024	1.667	3.166	7.6
d=6	0.539	0.072	0	-	$1 \equiv n_0$	0	0

Figure 8: Combined energy profiles for d =0.6, d = 2, d = 2.6 and d = 3

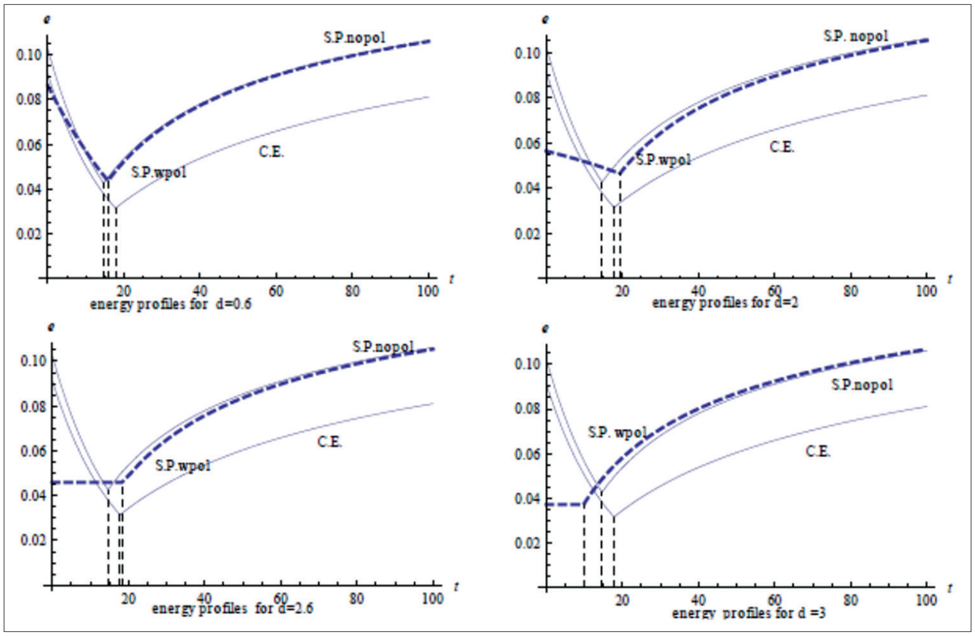


Figure 9: Time of switch to backstop as function of pollution parameter d

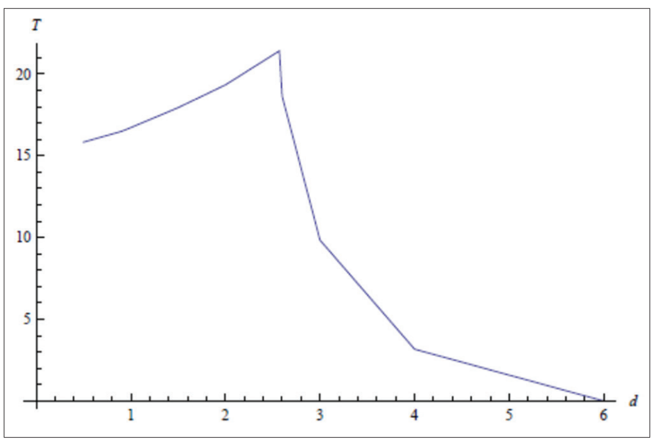
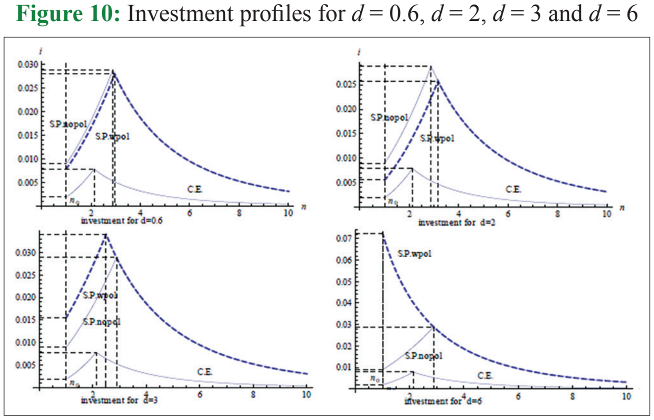


Figure 10: Investment profiles for d = 0.6, d = 2, d = 3 and d = 6



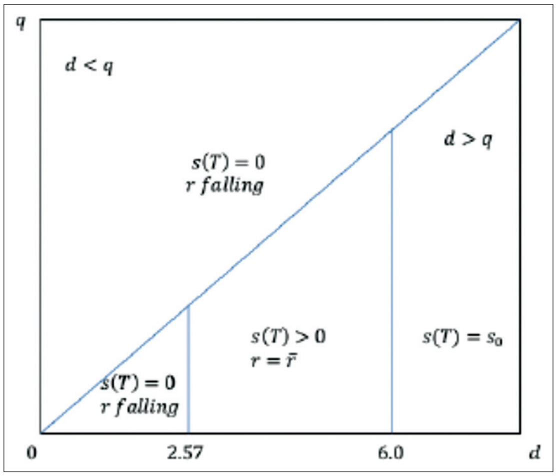
optimum investment is much bigger than investment in the equilibrium solution even after the introduction of pollution. As before, the points of kink in the investment profiles correspond to the accumulated stock of knowledge at the time of switch to the backstop or $\hat{n}$. Optimal investment profiles move down and to the right for $d = 0.6$ and $d = 2$ during the phase of only using r before finally merging with the profile when pollution cost is not included. This is because only the path of exhaustible resource use changes in the planner solution after pollution is introduced: This would change the path of investment initially (hence causing a change in T) before merging with the investment profile of the previous model without pollution. For $0 < d < 2.57$, a rise in the unit pollution cost makes the planner conserve the exhaustible resource over longer periods and invest less as compared with the model without pollution. However, the time effect of extending the life of the exhaustible resource dominates the effect of smaller investments until T and thus the accumulated stock of knowledge at the time of switch $\hat{n}$ increases. For $d = 3$ (since λ_2 falls to zero), investments in knowledge by the planner during the period of only non-renewable resource use would be greater as compared with the previous model. Once again, the time effect of a sooner switch to the backstop dominates the effect of greater investments in knowledge causing a fall in $\hat{n}$. When $d = 6$ (here, given the model parameters $d = q + \frac{a}{n_0}$), $\hat{n} = n_0$ as the backstop is adopted from the very beginning by the planner. Investment falls with accumulation of more knowledge.

5.2 Numerical Solutions with Flow Pollution

Table 2 shows numerical solutions for the social planner including unit pollution cost and for the previous model without pollution[19]. We use values of $\alpha = 0.667$ and $\rho = 0.04$ and fix the other parameter values to be $\beta = 0.33$, $q = 1$, $a = 5$, $s_0 = 1$ and $n_0 = 1$. The results are intuitive and we state the main points from Table 2 briefly. Increase in the pollution cost forces the social planner to conserve the exhaustible resource for longer periods. This is true as long as $\lambda_2^*(0) > 0$ (an extra unit of the

19 $d = 0$ corresponds to the model without pollution or the one analyzed in the previous section.

Figure 11: Relation between d and q and exhaustion of s_0



non-renewable resource stock adds to discounted lifetime net profits). Increase in the life of the exhaustible resource also causes current investment in the backstop to be worth less because of discounting. This is seen through a fall in $\lambda_1^*(0)$ when d increases. Note that initial investment $i^*(0)$ decreases as well. A rise in T or the time of switch to the backstop also entails an increase in the accumulated knowledge stock at the time of switch (through the dominance of the time effect as mentioned previously). But the results change significantly when d rises above its critical value $(d \geq 2.57)$[20]. The planner switches sooner to the backstop and the cost of pollution becomes so high that thisis done without completely exhausting the stock of the non-renewable resource. We note again that $\lambda_1^*(0)=0$ in these cases. $r^*(0)=0$ falls because of the high pollution cost and as investment in the knowledge stock is valued more; this is reflected in a $\lambda_1^*(0)=0$ rising as d increases.

In conclusion, we summarize how the relation between d and q affects the exhaustion of the initial exhaustible resource stock in s_0 Figure 11. In Figure 11, $d=q$ along the diagonal $s(T)$ and represents amount of non-renewable resource stock left in the ground at the date of switch to the backstop T. In the upper triangle when $d < q$, there is complete exhaustion of the initial stock s_0 whereas for $d > q$ (lower triangle), three possible cases arise depending on the value of d given other parameters.

6. SUMMARY OF RESULTS AND FUTURE WORK

We model a continuum of identical firms producing a composite commodity from energy using an exhaustible resource and a perfect substitute backstop technology[21]. We analyze the time of switch from the exhaustible resource to the backstop for the equilibrium and planner solutions[22]. We assume a zero cost of extraction for the non-renewable resource whereas the positive average cost of the backstop falls with knowledge accumulation. A zero cost of extraction not only makes the backstop initially uncompetitive with the exhaustible resource for use by a representative firm but also makes the model tractable; we employ complex numerical methods in Mathematica to solve the model completely which is one the main contributions of this work. Investment in the knowledge stock is thought of as R and D expenditures by a firm which reduces the future average backstop cost. This is similar to Gray and Grimaud (2010) who model R and D spillovers as being partial, but we differ in the respect that investments and the stock of knowledge are purely private from a firm's point of view and the firm does not draw upon from a shared pool of technological knowledge.

The representative firm partially benefitting from its own knowledge accumulation gives rise to difference between the equilibrium and planner solutions. As the social planner fully internalizes the benefits of investing in the knowledge stock, investment is higher in the optimal solution compared to the equilibrium one every period. Introducing flow pollution to the model (Schou 2000), we find the results change considerably for the planner solution. With constant marginal costs for pollution, for a relatively high pollution cost (compared with the average backstop cost) the social planner switches sooner to the backstop and invests greater amounts in knowledge in the initial periods. The planner also leaves some of the exhaustible resource stock unexploited. This occurs because the shadow price of the exhaustible resource stock falls to zero (an additional unit of the stock of the non-renewable resource does not add to discounted lifetime net profits). For a relatively low pollution cost, implying a positive shadow price, the exhaustible resource is conserved for longer periods with lesser investments in knowledge each period.

Some plans for future research are to include extraction costs for the exhaustible resource (based on the cumulative amount extracted) and stock pollution. Stock pollution in the context of an atmospheric ceiling constraint has been modeled by Amigues et al. (2012) and Amigues and Moreaux (2013). In addition, introducing learning by doing (in reducing average backstop cost) would entail the knowledge accumulation function (like (7)) to first be convex and then concave. This might generate interesting possibilities such that larger firms (introducing firm heterogeneity) invest more in knowledge and switch sooner to the backstop. The "critical" knowledge stock[23] where returns to investment change from increasing to diminishing may or may not coincide with the time of switch to the backstop. Finally, introducing correction mechanisms to encourage greater investment by firms in equilibrium (e.g. Acemoglu et al. (2012) introduce research subsidies to increase the productivity of the "clean" sector) is something we want to pursue in future.

20 $d = 2.57$ represents the knife-edge case when $\lambda_2^*(0)=0$ and there is also a complete exhaustion of s_0 at the time of switch.

21 Because of the exhaustible resource and the backstop being perfect substitutes, a firm uses either the exhaustible resource r or the backstop b at any time with an indeterminate mix of the two at one point when their marginal costs are equal.

22 Switching from exhaustible resources to a backstop has been analyzed by Hung and Quyen (1993), Tsur and Zemel (2003, 2005).

23 Boucekkine et al. (2003) call a critical knowledge stock A^* when introducing a learning curve.

REFERENCES

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D. (2012), The environment and directed technical change. American Economic Review, 102(1), 131-166.

Amigues, J.P., Lafforgue, G., Moreaux, M. (2012), IDEI Working Paper. Optimal Timing of Carbon Capture Policies Under Alternative CCS Cost Functions.

Amigues, J.P., Moreaux, M. (2013), IDEI Working Paper. Optimal Growth Under a Climate Constraint.

Boucekkine, R., Saglam, C., Vallée, T. (2003), Working Paper. Technology adoption under embodiment: A two-stage optimal control approach.

Boucekkine, R., Krawczyk, J., Vallée, T. (2011), Environmental quality versus economic performance: A dynamic game approach. Optimal Control Applications and Methods, 32(1), 29-46.

Boucekkine, R., Pommeret, A., Prieur, F. (2013a), Technological vs. ecological switch and the environmental Kuznets Curve. American Journal of Agricultural Economics, 95(2), 252-260.

Boucekkine, R., Pommeret, A., Prieur, F. (2013b), Optimal regime switching and threshold effects. Journal of Economic Dynamics and Control, 37(12), 2979-2997.

Chakravorty, U., Leach, A., Moreaux, M. (2011), Would hotelling kill the electric car? Journal of Environmental Economics and Management, 61(3), 281-296.

Currie, J., Schmieder, J. (2008), Fetal Exposure to Toxic Releases and Infant Health. NBER Working Paper No, 14352.

Dasgupta, P., Heal, G.M. (1974), The optimal depletion of exhaustible resources. Review of Economic Studies, 41, 3-28.

Das Gupta, S. (2015), Dirty and clean technologies. Journal of Agricultural and Applied Economics, 47(1), 123-145.

Energy Information Administration. (2014), Annual Energy Outlook. Available from: http://www.eia.gov/forecasts/aeo.

Graff Zivin, S., Neidell, M. (2011), The Impact of Pollution On Worker Productivity. NBER Working Paper No, 17004.

Gray, E., Grimaud, A. (2010), TSE Working Paper. Scope of Innovations, Knowledge Spillovers and Growth.

Hartley, P., Medlock, III, K.B., Temzelides, T., Zhang, X. (2014), RISE Working Paper. Energy Sector Innovation and Growth.

Hung, N.M., Quyen, N.V. (1993), On R&D timing under uncertainty: The case of exhaustible resource substitution. Journal of Economic Dynamics & Control, 17(5-6), 971-991.

Schou, P. (2000), Polluting non-renewable resources and growth. Environmental and Resource Economics, 16(2), 211-227.

Stiglitz, J. (1974a), Growth with exhaustible natural resources: Efficient and optimal growth paths. Review of Economic Studies, 41, 123-137.

Stiglitz, J. (1974b), Growth with exhaustible natural resources: The competitive economy. Review of Economic Studies, 41, 139-152.

Tsur, Y., Zemel, A. (2003), Optimal transition to backstop substitutes for nonrenewable resources. Journal of Economic Dynamics & Control, 27(4), 551-572.

Tsur, Y., Zemel, A. (2005), Scarcity, growth and R&D. Journal of Environmental Economics and Management, 49(3), 484-499.

APPENDIX

We use the software Mathematica to get numerical solutions to the following parts of the model. Further details are available upon request.

Switching Time and Solutions for $d = 0$

Exhaustion Condition

We find the time T at which the economy switches from the first phase $(r > 0, b = 0)$ to the second $(r = 0, b > 0)$ by combining the first-order conditions and constraints[1]. We cannot find this in closed form however, so we must use numerical methods. Since the stock of the non-renewable resource is fully exhausted at the time of switch (in the case of no flow pollution), using (3) and (25) we write

$$\int_0^T r(t)\,dt = s_0 \tag{45}$$

which implies, $$\int_0^T \left(\frac{1-\alpha}{\lambda_2(0)e^{\rho t}}\right)^{\frac{1}{\alpha}} dt = s_0 \tag{46}$$

Call r_T the extraction rate at the time of switch T. Assuming energy use is continuous, we use (27) to write $r_T = b_T$: The rate of extraction of the exhaustible resource is the same as the rate of backstop production at the time of the switch. Suppose we know r_T. Given s_0, Equations (25), (27) and (46) allow us to find T and $r(0)$ as functions of r_T and s_0. We get

$$T = \left[ln\left(1+\frac{\rho}{r_T\alpha}s_0\right)\right]\frac{\alpha}{\rho} \equiv m(r_T) \tag{47}$$

$$r(0) = r_T + \frac{\rho}{\alpha}s_0 \tag{48}$$

From (27), we write r_T as a function of the stock of knowledge at T as

$$r_T = b_T = \left(\frac{1-\alpha}{q+\frac{a}{n_T}}\right)^{\frac{1}{\alpha}} \equiv h(r_T) \tag{49}$$

Therefore if n_T was given, we could find r_T and given our data on s_0, T and $r(0)$ could be obtained. From (47) and (49) we get

$$T = m\left[h(n_T)\right] = w(n_T) \tag{50}$$

and $w'(n_T) < 0$. This would be important in what follows.

Knowledge Condition and Solution

To find the key value of n_T, we must work our way backwards. That is, we begin in the second phase when $r = 0, b > 0$. Combine (18) and (27) to get

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{a\beta}{n^2}\left(\frac{1-\alpha}{q+\frac{a}{n}}\right)^{\frac{1}{\alpha}} \tag{51}$$

From equations (51) and (19), we plot the phase plane after the switch to backstop in Appendix Figure 1 for the equilibrium solution. The downward sloping curve corresponds to the $\dot{\lambda}_1 = 0$ locus and the $\dot{n} = 0$ locus coincides with the horizontal axis. We see the saddle point would be given by $(n \to \infty, \lambda_1 \to 0)$. The stable arm or the "policy function" (shaded green) that satisfies transversality is found by numerical methods in Mathematica. It asymptotes to the horizontal axis in the limit and $\lambda_1 n$ converges to zero (a constant) when $t \to \infty$. We represent the policy function by

$$\lambda_1 = p(n) \tag{52}$$

The slope of the policy function in the second phase is found by taking the ratio of $\frac{\dot{\lambda}_1}{\dot{n}}$ from (19) and (51).

In the first phase when $r > 0, b = 0$ we combine (19) and (26) to get

$$\lambda_1(t) = \lambda_1(0)e^{\rho t} \tag{53}$$

$$n(t) = \frac{\lambda_1(0)}{2\rho}\left(e^{\rho t} - 1\right) + n_0 \tag{54}$$

for any t given some initial $\lambda_1(0)$. Eliminating time from equation (54), we get

$$\lambda_1 = \lambda_1(0) + 2\rho(n - n_0) \tag{55}$$

given the initial knowledge stock n_0. This is the policy function in the first phase. It is upward sloping and must intersect $p(n)$ at a finite value of n: In other words, at the knowledge stock n_T at the time of switch to the backstop. Combining the policy functions for the first and second phases, we plot the whole policy function in Appendix Figure 2 for any given n_0 and an optimal choice of $\lambda_1(0)$. In Appendix Figure 2, the path of λ_1 has a kink corresponding to the knowledge stock and its shadow price at the time of switch to the backstop $(\hat{n}, \hat{\lambda}_1)$. It must be noted that the investment profile in Appendix Figure 2 is derived from Appendix Figure 2 using (19).

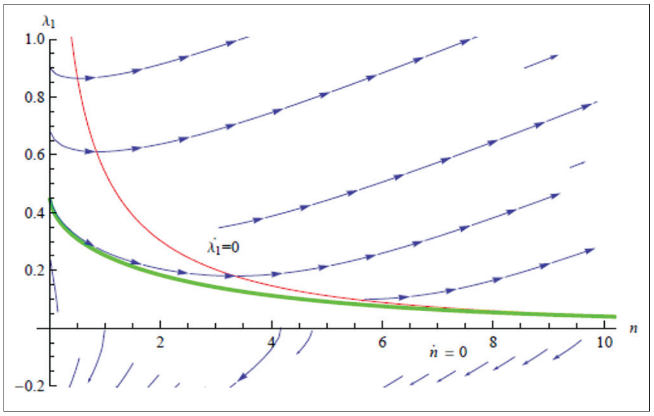
Now given a unique $\lambda_1^*(0)$, we can relate n_T and $x \equiv \lambda_1(0)$. Setting different values of x, we equate (52) and (55) to get an interpolating function in Mathematica in the form

$$n_T = f(x) \tag{56}$$

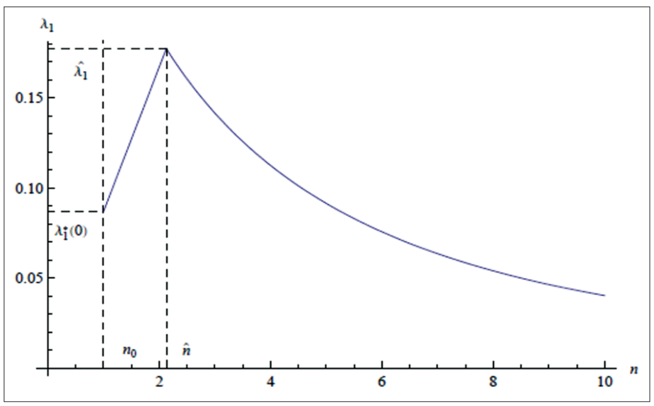
From Appendix Figure 2, we can see that $f'(x) < 0$. From (54) we get

1 We compute the model for the equilibrium solution. The social planner solution can be found by substituting β = 1 in the following equations.

Appendix Figure 1: Phase plane when $r = 0$, $b > 0$ in equilibrium



Appendix Figure 2: Policy function λ_1 in equilibrium



$$T = \frac{1}{\rho} ln\left[\left(\frac{f(x) - n_0}{x}\right) 2\rho + 1\right] \equiv z(x) \tag{57}$$

with $z'(x) < 0$. On the other hand, from (50) and (56), we find

$$T = w(f(x)) \equiv g(x) \tag{58}$$

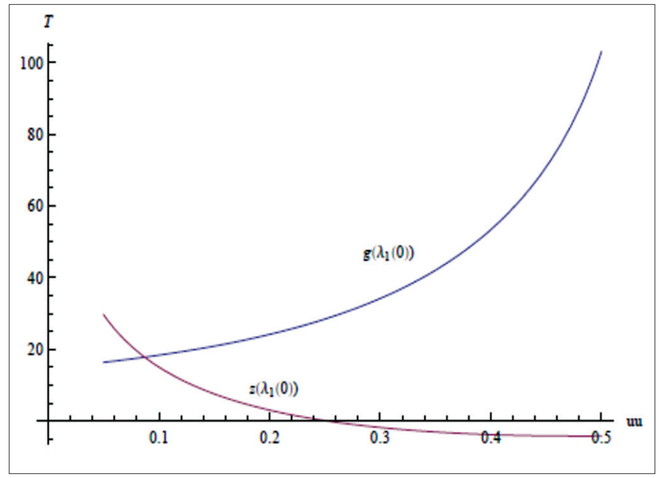
with $g'(x) > 0$. We plot the functions $z(x)$ and $g(x)$ in Appendix Figure 3. From $x^*$and T^*, we get n_T^* from (56) and then $r_T^* \equiv b_T^*$ from (49). We then obtain $r^*(0)$ from (48) going backwards and $\lambda_2^*(0)$ by substituting $r^*(0)$ in (25). From (26), we find $x_T^* \equiv \lambda_1^*(T)$. Finally, solutions for $i^*(0)$ and $i^*(T)$ are found using x^* and x_T^* in (19). As an exposition, we plot the investment profiles over time for the planner (S.P.) and equilibrium (C.E.) solutions in Appendix Figure 4. In Appendix Figure 4, the points of kink represent the time of switch to the backstop and shows that the planner switches sooner compared to a firm in equilibrium.

Switching Time and Solutions for $d > 0$

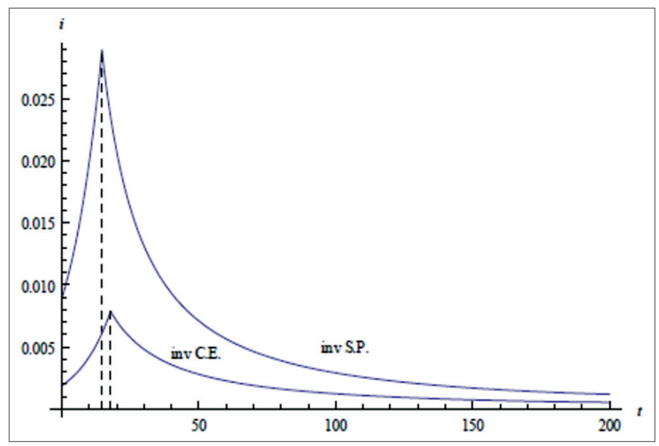
Here the method of obtaining solutions remains largely the same as in the previous subsection when the unit pollution cost was zero[2]. We

2 We again compute the model for the equilibrium solution. For the planner solution use β = 1.

Appendix Figure 3: Profiles of T as a function of $\lambda_1(0)$ in equilibrium: uu denotes $\lambda_1(0)$



Appendix Figure 4: Time paths if investment for the planner and equilibrium solutions



first assume that the non-renewable resource stock is fully exhausted at the time of switch. We once again get T and $r(0)$ (or $\lambda_2(0)$) as a function of r_T or the rate of extraction at T. Using the continuity property from (49), we relate r_T to n_T or the stock of knowledge at the time when a firm switches from using the exhaustible resource to the backstop. Note that in this case, a key relation would be between T and $\lambda_2(0)$. Since we cannot get these relations in closed form, we resort to numerical methods. Now modifying (44) we get

$$\lambda_2(0) = \left(q + \frac{a}{n_T} - d\right) e^{-\rho T} \tag{59}$$

We replace $\lambda_2(0)$ with T from the relation above and replace n_T with $x \equiv \lambda_1(0)$ from (56). We then get an interpolating function in Mathematica similar to $T = g(x)$ with $g'(x) > 0$. Along with (57), these equations give us T^* and x^*. Then $i^*(0)$ is found from (19) using $\lambda_1^*(0)$.

We repeat the above process for various values of the unit pollution cost d. Using the interpolating function between T and $\lambda_2(0)$ found above, we find $\lambda_2^*(0)$ keeps falling with increases in d (Table 2):

At $d = 2.57$, $\lambda_2^*(0) \approx 0$ and $\lambda_2^*(0) < 0$ when $d > 2.57$. To satisfy optimality, we thus assume $\lambda_2^*(0) = 0$ for $d \geq 2.57$. The path of exhaustible resource use either falls (for $\lambda_2^*(0) > 0$) or remains constant (for $\lambda_2^*(0) = 0$).