



## Volatility Transmission in Oil Futures Markets and Carbon Emissions Futures

Tanattrin Bunnag\*

Faculty of Science and Social Sciences, Burapha University, Thailand. \*Email: [ratanan@buu.ac.th](mailto:ratanan@buu.ac.th)

### ABSTRACT

This paper examined the oil futures and the carbon emissions futures volatility comovements and spillovers for crude oil, gasoline and heat oil as well as carbon emissions. The data used in this study was the daily data from 2009 to 2014. The three multivariate GARCH models, namely the vector autoregression model (VAR) (3)-diagonal VECH, the VAR (3)-diagonal Baba, Engle, Kraft and Kroner (BEKK) and the VAR (3)- constant conditional correlations (CCC), were employed. The empirical results showed that the estimates of the VAR (3)-diagonal VECH and the VAR (3)-CCC parameters were statistically significant in a case involving oil except in the case of carbon emissions. This indicates that the short run persistence of shocks on the dynamic conditional correlations was greatest for RGASOLINE with RHEATOIL, while the largest long run persistence of shocks to the conditional correlations for RCRUDE with RGASOLINE. At the same time the VAR (3)-diagonal BEKK parameters were statistically significant in all cases. This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest for RHEATOIL with RCO<sub>2</sub>, while the largest long run persistence of shocks to the conditional correlations for RCRUDE with RCO<sub>2</sub> and RHEATOIL with RCO<sub>2</sub>. Finally, we would choose the best model next by considering the value of log-likelihood, Akaike information criterion, Schwarz information criterion and Hannan-Quinn information criterion. The value of these figures, it could be concluded that we should choose the VAR (3)-diagonal BEKK model in volatility analysis of the oil futures and the carbon emissions futures returns. In addition, we could conclude that oil futures volatility having an impact on carbon emissions futures volatility.

**Keywords:** The Oil Futures and the Carbon Emissions Futures Volatility, Comovements and Spillovers, Multivariate GARCH Models

**JEL Classifications:** C13, C32, G13

### 1. INTRODUCTION

The uncertainty of the current economic conditions caused climate changes, the change in the price of oil. The carbon emission trading has happened. Later, there is an urge to develop quantitative tools to model and understand the origins of variations in carbon emissions prices and effects in oil prices. Information on the movement of these variables has operational and political implications relevant to the main players in the market such as polluters and regulators. As Stern (2006) pointed out, this is one of the first steps in order to deal with climate change and we may say that this has been one of the principal contributions of Phase I of the European Union Emission Trading Scheme (EU ETS).

Due to the reasons above, it is main cause of the EU ETS launch, in January 2005, of the EU ETS has been the establishment of a

price for carbon emissions. Carbon emissions trading are emissions trading specifically for carbon dioxide (calculated in tonnes of carbon dioxide equivalent or tCO<sub>2</sub>e) and currently make up the bulk of emissions trading. It is one of the ways countries can meet their obligations under the Kyoto protocol to reduce carbon emissions and thereby mitigate global warming.

As it is well known, the EU ETS is organized in three phases. Phases I was considered as a pilot phase and it run from January 1, 2005 to December 31, 2007. On the other hand, Phases II started from January 1, 2008 and run until December 31, 2012. Finally, Phase III of the EU ETS started from January 1, 2013 and will probably last until December 31, 2020.

Trading exchanges have been established to provide a spot market in permits, as well as futures and options market to help discover a market price and maintain liquidity. Carbon prices are normally

quoted in Euros per ton of carbon dioxide or its equivalent. However, in this study, we choose to use futures market due to trading together quite a lot.

Currently there are exchanges trading in carbon credits: the European Climate Exchange (ECX), NASDAQ OMX Commodities Europe, PowerNext, Commodity Exchange Bratislava and the European Energy Exchange. Many companies now engage in emissions abatement, offsetting, and sequestration programs to generate credits that can be sold on one of the exchanges. But the market we are interested in that is ECX because trading volume in large quantities.

The ECX manages the product development and marketing for ECX Carbon Financial Instruments (ECFI), listed and admitted for trading on the intercontinental exchange (ICE) Futures Europe electronic platform. It listed on the London Stock Exchange. ECX futures is the most liquid, pan-European platform for carbon emissions trading, with its futures contract based on the underlying EU allowances (EUAs) and Certified Emissions Allowances (CERs) attracting over 80% of the exchange-traded volume in the European market. ECX contracts (EUA and CER futures, options and spot contracts) are standardized exchange-traded products and all trades are cleared by ICE Clear Europe.

The purpose is to analyze the oil futures and the carbon emissions futures volatility comovements and spillovers among major oil including crude oil (West Texas Intermediate market), gasoline and heat oil as well as carbon emissions by using multivariate GARCH, namely the diagonal VECH, the diagonal Baba, Engle, Kraft and Kroner (BEKK) and constant conditional correlations (CCC) model and choose the best way for such analysis. In addition to see if the oil future returns do have an impact on carbon emissions future returns, it could also be the case of oil futures volatility having an impact on carbon emissions futures volatility or not. We can explain more in the next section, which is related to the literature reviews, research methodology and empirical results.

## 2. LITERATURE REVIEW

Since the beginning of the EU ETS, the interest in studying the carbon emissions markets from a financial point of view has exponentially increased.

Uhrig-Homburg and Wagner (2007) analyzed the relationship between spot and futures prices in the EU ETS. Their empirical evidence suggested that, after December 2005, spot and futures prices were linked by the cost-of-carry approach. Alberola and Chevalier (2009) focused in the study of the intra-period banking during Phase I and the effects of inter-period banking restrictions between Phase I and II of the EU ETS. Furthermore, a variety of articles including Mansanet-Bataller et al. (2007) and Alberola et al. (2008) have focused their attention on the determinants of carbon emissions prices. They provide evidence that lagged energy prices (oil and natural gas) as well as weather variables may explain carbon emissions price for the first period of the EU ETS.

Concerning carbon emission prices' determinants for Phase II of the EU ETS, Mansanet-Bataller and Pardo (2011) find that the contemporary energy variables, specifically oil, gas and coal, have the expected impact on carbon emission prices. That is, the increase in the prices of oil and natural gas that makes the prices of carbon emissions increase. On the contrary, the increase in the price of coal that makes the price of carbon emissions reduction. So, in the Phase II of EU ETS, increasing the prices of fuel is directly transmitted to carbon emission prices. The effect of energy prices on carbon emission has further been confirmed by Bunn and Fezzi (2007). They studied the impact of the EU-ETS on the wholesale electricity market in the United Kingdom. The results of a cointegrated vector autoregression model (VAR) estimation highlight the essential role of energy prices, especially that of natural gas, in determining the price of emission allowances. In addition, Chavallier (2011) analyzes the time-varying correlations in oil, gas and carbon dioxide futures prices using BEKK, CCC and DCC-MGARCH models and identify dynamic correlations between energy and carbon emission market.

Look back to the conditional volatility of the petroleum futures by using multivariate GARCH and no carbon emissions are involved. Manera et al. (2012) analyze the conditional volatility of future prices for four energy commodities (crude oil, heat oil, gasoline and natural gas) using CCC and DCC multivariate GARCH models. They find that the spillovers between commodities and the conditional correlations among commodities are high and time-varying. As well as Bunnag (2015) examined comovements and spillovers in petroleum futures (crude oil, gasoline, heat oil and natural gas) using three multivariate GARCH models, namely the VAR (1)-diagonal VECH, the VAR (1)-diagonal BEKK and the VAR (1)-CCC models. The empirical results overall showed that the estimates of the multivariate GARCH parameters were statistically significant in almost all cases except in the case of gasoline with natural gas. This indicates that the short run persistence of shocks on the dynamic conditional correlations was greatest for crude oil with heat oil, while the largest long run persistence of shocks to the conditional correlations for crude oil with gasoline.

Finally, Mansanet-Bataller and Soriano (2012) have focused on price volatility transmission between carbon emissions prices and energy market using the BEKK model. The results show that carbon emissions prices are directly affected by their own volatility and have the conditional correlation between carbon emissions with the energy market such as oil and natural gas.

However, in this study we use the popular multivariate GARCH include the diagonal VECH, the diagonal BEKK and the CCC model as detailed below.

## 3. RESEARCH METHODOLOGY

### 3.1. Multivariate GARCH Models

The basic idea to extend univariate GARCH models to multivariate GARCH models is that it is significant to predict the dependence in the comovement of the oil futures and carbon emissions futures returns. To recognize this feature through a multivariate model

would generate a more reliable model than separate univariate models.

In the first place, one should consider what specification of a multivariate GARCH model should be imposed. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariances. On the other hand, as the number of parameters in a multivariate GARCH model increase rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured. So it is important to get balance between the parsimony and the flexibility when designing the multivariate GARCH model specification. Another feature that multivariate GARCH models must satisfy is that the covariance matrix should be positive definite.

Several different multivariate GARCH model formulations have been proposed in the literature, and the most popular of these are the diagonal VECM, the diagonal BEKK and CCC models. Each of these is discussed briefly in turn below; for a more detailed discussion, Kroner and Ng (1998).

### 3.2. The Diagonal VECM Model

The first multivariate GARCH model was introduced by Bollerslev et al. in 1988, which is called VECM model. It is much general compared to the subsequent formulations. In the VECM model, every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns. The model can be expressed below:

$$VECM(H_t) = c + \sum_{j=1}^q A_j VECM(\epsilon_{t-j} \epsilon'_{t-j}) + \sum_{j=1}^p B_j VECM(H_{t-j}), \quad (1)$$

Where  $VECM(H_t)$  is an operator that stacks the columns of the lower triangular part of its argument square matrix,  $H_t$  is the covariance matrix of the residuals,  $N$  presents the number of variables,  $t$  is the index of the  $t^{th}$  observation,  $c$  is an  $\frac{N(N+1)}{2} \times 1$  vector,  $A_j$  and  $B_j$  are  $\frac{N(N+1)}{2} \times \frac{N(N+1)}{2}$  parameter matrices and  $\epsilon$  is an  $N \times 1$  vector.

The condition for  $H_t$  is to be positive definite for all  $t$  is not restrictive. In addition, the number of parameters equals to  $(p+q) \times \left( \frac{N(N+1)}{2} \right)^2 + \frac{N(N+1)}{2}$ , which is large. Furthermore, it demands a large quantity of computation.

The diagonal VECM model, the restricted version of VECM, was also proposed by Bollerslev et al. (1988). It assumes the  $A_j$  and  $B_j$  in equation (1) are diagonal matrices, which makes it possible for  $H_t$  to be positive definite for all  $t$ . Also, the estimation process proceeds much smoothly compared to the

complete VECM model. However, the diagonal VECM model with  $(p+q+1) \times N \times \frac{N(N+1)}{2}$  parameters is too restrictive since it does not take into account the interaction between different conditional variances and covariances.

### 3.3. The Diagonal BEKK Model

To ensure positive definiteness, a new parameterization of the conditional variance matrix  $H_t$  was defined by Baba et al. (1990) and became known as the BEKK model, which is viewed as another restricted version of the VECM model. It achieves the positive definiteness of the conditional variance by formulating the model in a way that its property is implied by model structure.

The form of the BEKK model is as follows:

$$H_t = CC' + \sum_{j=1}^q \sum_{k=1}^K A'_{kj} \epsilon_{t-j} \epsilon'_{t-j} A_{kj} + \sum_{j=1}^p \sum_{k=1}^K B'_{kj} H_{t-j} B_{kj} \quad (2)$$

Where  $A_{kj}$ ,  $B_{kj}$  and  $C$  are  $N \times N$  parameter matrices, and  $C$  is a lower triangular matrix. The purpose of decomposing the constant term into a product of two triangle matrices is to guarantee the positive semi-definiteness of  $H_t$ . Whenever  $K > 1$ , an identification problem would be generated for the reason that there are not only single parameterizations that can obtain the same representation of the model.

The first order BEKK model is:

$$H_t = CC' + A' \epsilon_{t-1} \epsilon'_{t-1} A + B' H_{t-1} B \quad (3)$$

The BEKK model also has its diagonal form by assuming  $A_{kj}$ ,  $B_{kj}$  matrices are diagonal. It is a restricted version of the diagonal VECM model. The most restricted version of the diagonal BEKK model is the scalar BEKK one with  $A=aI$  and  $B=bI$  where  $a$  and  $b$  are scalars.

Estimation of a BEKK model still bears large computations due to several matrix transpositions. The number of parameters of the complete BEKK model is  $(p+q)KN^2 + \frac{N(N+1)}{2}$ . Even in the diagonal one, the number of parameters soon reduces to  $(p+q)KN + \frac{N(N+1)}{2}$ , but it is still large. The BEKK form is not linear in parameters, which makes the convergence of the model difficult. However, the strong point lies in that the model structure automatically guarantees the positive definiteness of  $H_t$ . Under the overall consideration, it is typically assumed that  $p=q=K=1$  in BEKK form's application.

### 3.4. CCC Model

The CCC model was introduced by Bollerslev in 1990 to primarily model the condition covariance matrix indirectly by estimating the conditional correlation matrix. The conditional correlation is assumed to be constant while the conditional variances are varying.

Consider the CCC model of Bollerslev (1990):

$$y_t = E\langle y_t | F_{t-1} \rangle + \varepsilon_t, \quad \varepsilon_t = D_t \eta_t \tag{4}$$

$$\text{var}\langle \varepsilon_t | F_{t-1} \rangle = D_t \Gamma D_t$$

Where  $y_t = (y_{1t}, \dots, y_{mt})'$ ,  $\eta_t = (\eta_{1t}, \dots, \eta_{mt})'$  is a sequence of independently and identically distributed (i.i.d) random vectors,  $F_t$  is the past information available at time  $t$ ,  $D_t = \text{diag}(h_{1t}^{1/2}, \dots, h_{mt}^{1/2})$ ,  $m$  is the number of returns, and  $t = 1, \dots, n$ . As  $\Gamma = E\langle \eta_t \eta_t' | F_{t-1} \rangle = E(\eta_t \eta_t')$ , where  $\Gamma = \{ \rho_{ij} \}$  for  $i, j = 1 \dots m$ , the CCC matrix of the unconditional shocks,  $\eta_t$ , is equivalent to the constant conditional covariance matrix of the conditional shocks,  $\varepsilon_t$ , from (4),  $\varepsilon_t \varepsilon_t' = D_t \eta_t \eta_t' D_t$ ,  $D_t = (\text{diag } Q_t)^{1/2}$ , and  $E\langle \varepsilon_t \varepsilon_t' | F_{t-1} \rangle = Q_t = D_t \Gamma D_t$ , where  $Q_t$  is the conditional covariance matrix.

The CCC model assumes that the conditional variance for each return  $h_{ij}$ ,  $i=1, \dots, m$ , follows a univariate GARCH process, that is:

$$h_t = \omega_t + \sum_{j=1}^r \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^s \beta_{ij} h_{i,t-j} \tag{5}$$

Where  $\alpha_{ij}$  represents the ARCH effect, or short run persistence of shocks to return  $i$ ,  $\beta_{ij}$  represents the GARCH effect, and  $\sum_{j=1}^r \alpha_{ij} + \sum_{j=1}^s \beta_{ij}$  denotes the long run persistence.

### 3.5. Model Estimation for Multivariate GARCH

Under the assumption of conditional normality, the parameters of the multivariate GARCH models of any of the above specifications can be estimated by maximizing the log-likelihood function.

$$\ell(\theta) = -\frac{TN}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T (\log |H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t) \tag{6}$$

Where  $\theta$  denotes all the unknown parameters to be estimated,  $N$  is the number of the oil futures and carbon emissions futures prices and  $T$  is the number of observations and all other notation is as above. The maximum-likelihood estimates for  $\theta$  is asymptotically normal, and thus traditional procedures for statistical inference are applicable.

## 4. DATA

The data used in this study is the daily data from November 4, 2009 to October 29, 2014. We will get 1352 observations. The data is derived from [www.quandl.com](http://www.quandl.com) and [www.investing.com](http://www.investing.com) which trade in Chicago Mercantile Exchange and ECX respectively. Moreover, data analysis can be carried out using EViews 8. The three oil futures and carbon emissions futures return is defined as:

$$R_t = \log \left( \frac{FP_t}{FP_{t-1}} \right) \tag{7}$$

Where  $FP_t$  is the oil futures and carbon emissions futures price at time  $t$  and  $FP_{t-1}$  is the oil futures and carbon emissions future price at time  $t-1$ . The  $R_t$  of equation (7) will be used in observing the volatility of the oil and carbon emissions between the selected oil and carbon emissions over the period 2009-2014. We can create the variables of the return on the oil futures and carbon emissions futures as follows:

The returns of crude oil futures = RCRUDE, the returns of gasoline futures = RGASOLINE, the returns of heat oil futures = RHEATOIL and the returns of carbon emissions futures = RCO<sub>2</sub>.

In addition, we can show the movement of the daily three oil futures prices and returns as well as carbon emissions futures prices and returns according to Figures 1 and 2.

The descriptive statistics are given in Table 1. The daily future returns of carbon emissions (RCO<sub>2</sub>) display the greatest variability with the mean of -0.068%, a maximum of 22.430%, and a minimum of -43.070%. Furthermore, the skewness, the kurtosis and the Jarque-Bera Lagrange multiplier statistics of all oil futures and carbon emissions futures returns are statistically significant, thereby implying that the distribution is not normal.

## 5. UNIT ROOT TESTS

Standard econometric practice in the analysis of financial time series data begins with an examination of unit roots. The augmented Dickey-Fuller and Phillips-Perron tests are used to test for all the oil futures and carbon emissions futures returns under the null hypothesis of a unit root against the alternative hypothesis of stationarity. The results from unit root tests are presented in Table 2. The tests yield negative values in all cases for levels, such that the individual returns series reject the null hypothesis at the 1% significance level, so that all returns are stationary.

## 6. EMPIRICAL RESULTS

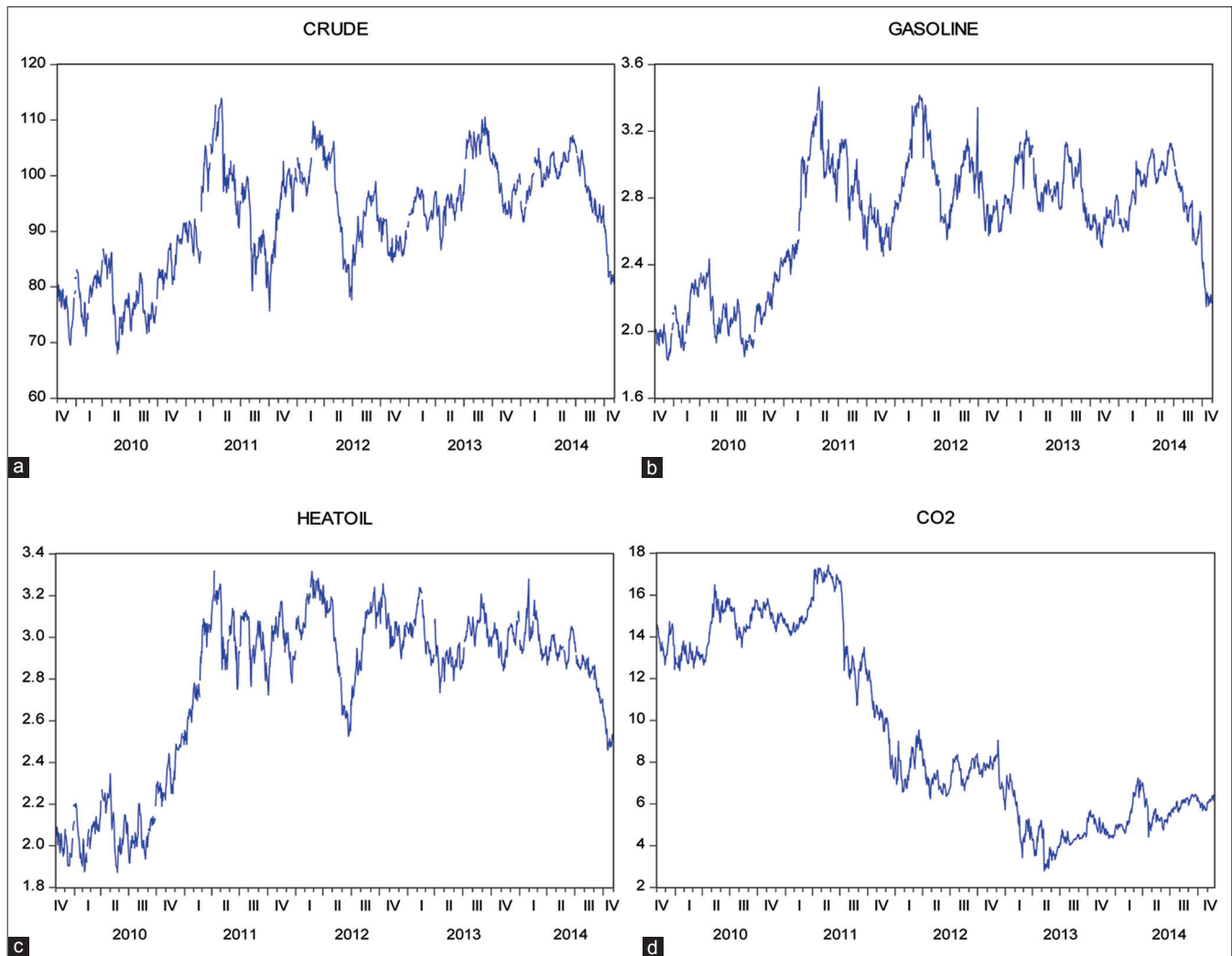
An important task is to model the conditional mean and conditional variances of the return series. Therefore, the appropriate multivariate conditional volatility model given as vector autoregression model VAR (3)-diagonal VECM, VAR (3)-diagonal BEKK and VAR (3)-CCC models is estimated. The conditional mean comes from VAR which can display the source as follows:

**Table 1: Descriptive statistics**

Returns	RCRUDE	RGASOLINE	RHEATOIL	RCO <sub>2</sub>
Mean	3.00E-05	6.88E-05	0.000164	-0.000687
Median	0.000297	7.45E-05	0.000230	0.000000
Maximum	0.0894	0.0968	0.0549	0.2243
Minimum	-0.0903	-0.1349	-0.0865	-0.4307
SD	0.0164	0.0182	0.0142	0.0349
Skewness	-0.1494	-0.4337	-0.3416	-1.3161
Kurtosis	5.6752	8.3840	5.7794	25.2900
Jarque-Bera	378.3093	1551.6820	427.7041	26301.1600

Beside, the return series will be used to construct the conditional mean and the conditional variances in next, SD: Standard deviation

**Figure 1:** The daily three oil futures and carbon emissions futures prices



**Table 2: Unit root tests**

Returns	ADF test				PP test			
	Constant		Constant and trend		Constant		Constant and trend	
	I (0)	I (1)	I (0)	I (1)	I (0)	I (1)	I (0)	I (1)
RCRUDE	-35.712***	-17.304***	-35.717***	-17.297***	-35.712***	-811.866***	-35.717***	-810.068***
RGASOLINE	-36.319***	-19.835***	-36.353***	-19.827***	-36.343***	-549.900***	-36.457***	-550.837***
RHEATOIL	-34.294***	-15.271***	-34.323***	-15.264***	-34.288***	-316.475***	-34.309***	-316.093***
RCO <sub>2</sub>	-18.460***	-16.177***	-18.457***	-16.170***	-35.987***	-400.763***	-35.977***	-400.487***

\*\*\*Significance at the 1% level, ADF: Augmented Dickey-Fuller, PP: Phillips-Perron

**6.1. VAR**

Let  $Y_t = (Y_{1t}, Y_{2t}, \dots, Y_{nt})'$  denote a  $k \times 1$  vector of oil futures and carbon emissions futures return series variables. The basic vector autoregressive model of order  $p$ , VAR ( $p$ ), is

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \dots + \Pi_p Y_{t-p} + \mu_t, \quad t = 1, \dots, T, \quad (8)$$

Where  $\Pi_i$  are  $k \times k$  matrices of coefficients,  $c$  is a  $k \times 1$  vector of constants and  $\mu_t$  is an  $k \times 1$  unobservable zero mean white noise vector process with covariance matrix  $\Sigma$ .

As in the univariate case with AR processes, we can use the lag operator to represent VAR ( $p$ )

$$\Pi(L)Y_t = c + \mu_t,$$

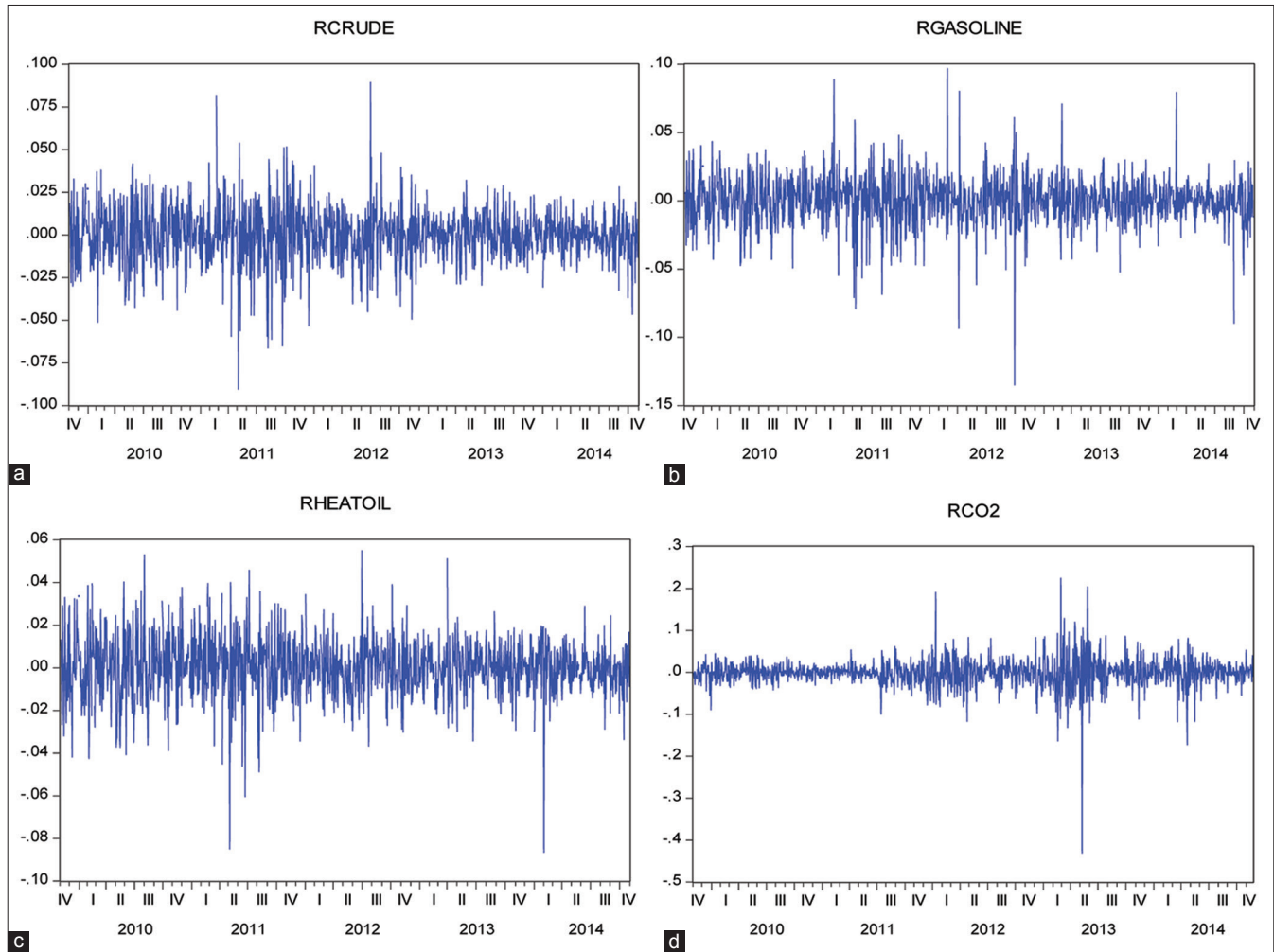
Where  $\Pi(L) = I_n - \Pi_1 L - \dots - \Pi_p L^p$

If we impose stationarity on  $Y_t$  in (8), the unconditional expected value is given by  $\mu = (I_n - \Pi_1 - \dots - \Pi_p)^{-1}c$

**6.2. Lag Length Selection**

A reasonable strategy how to determine the lag length of the VAR model is to fit VAR ( $p$ ) models with different orders  $p=0, \dots, p_{max}$  and choose the value of  $p$  which minimizes some model selection criteria. Model selection criteria for VAR ( $p$ ) could be base on Akaike information criteria (AIC), Schewarz-Bayesian

**Figure 2:** The daily three oil futures and carbon emissions futures returns



information criteria and Hannan-Quinn (HQ) information criteria.

Before we construct the conditional mean, the first thing to do is to find the right lag of VAR model as shown in the Table 3. From the various criteria are found to be selected lag that 1 and 3. Most of them will choose lag 3. We therefore conclude that lag 3 should be suitable for the conditional mean.

After all multivariate conditional volatility models in this paper are already estimated. The next step, we will have to explain that the results of each model and select the best model. The VAR (3)-diagonal VECH estimates of the conditional correlation between the volatilities of the three oil futures and the carbon emissions futures returns base on estimating the univariate GARCH (1,1) model for each the oil and the carbon emissions are given in Table 4. The estimates of the VAR (3)-diagonal VECH parameters that  $\theta_1$  and  $\theta_2$  are statistically significant in the case of  $\rho_{(RCR\_RGA)}$ ,  $\rho_{(RCR\_RHE)}$  and  $\rho_{(RGA\_RHE)}$  except in the case of  $\rho_{(RCR\_RCO)}$ ,  $\rho_{(RGA\_RCO)}$  and  $\rho_{(RHE\_RCO)}$ . This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest for RGASOLINE with RHEATOIL at 0.111 ( $\theta_1$ ), while the largest long run persistence of shocks to the conditional correlations is 0.979 ( $\theta_1 + \theta_2$ ) for RCRUDE with RHEATOIL.

**Table 3: Lag order selection**

Lag	LR	FPE	AIC	SC	HQ
0	NA	7.08e-15	-21.229	-21.213	-21.223
1	286.594	5.77e-15	-21.435	-21.352*	-21.404*
2	32.478	5.76e-15	-21.435	-21.287	-21.380
3	37.447*	5.74e-15*	-21.440*	-21.226	-21.360
4	19.547	5.79e-15	-21.430	-21.150	-21.325
5	10.975	5.89e-15	-21.414	-21.068	-21.284
6	16.444	5.96e-15	-21.401	-20.990	-21.247
7	11.388	6.06e-15	-21.385	-20.907	-21.205
8	10.569	6.16e-15	-21.368	-20.824	-21.164

\*Lag order selected. LR: Sequential modified LR test statistic, FPE: Final prediction error, AIC=Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion

The VAR (3)-diagonal BEKK estimates of the conditional correlation between the volatilities of the three oil futures and the carbon emissions futures returns are given in Table 5. The estimates of the diagonal BEKK parameters that  $\theta_1$  and  $\theta_2$  are statistically significant in all cases. This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest at 0.068 for RHEATOIL with RCO<sub>2</sub>, while the largest long run persistence of shocks to the conditional correlations is 0.992 ( $\theta_1 + \theta_2$ ) for RCRUDE with RCO<sub>2</sub> and RHEATOIL with RCO<sub>2</sub>.

Finally, in Table 6 presents the estimates for the VAR (3)-CCC model, with  $p=q=r=s=1$ . The ARCH and GARCH estimates of

Table 4: VAR (3) – diagonal VECM model estimates

VAR (3)	RCR	RGA	RHE	RCO	$\rho$ (RCR_RGA)	$\rho$ (RCR_RHE)	$\rho$ (RCR_RCO)	$\rho$ (RGA_RHE)	$\rho$ (RGA_RCO)	$\rho$ (RHE_RCO)
RCR. (-1)	-0.062* (0.035)	0.104** (0.047)	0.009 (0.027)	-0.147** (0.065)	-	-	-	-	-	-
RCR. (-2)	-0.010 (0.035)	0.050 (0.046)	0.028 (0.029)	-0.149*** (0.056)	-	-	-	-	-	-
RCR. (-3)	-0.004 (0.039)	0.050 (0.050)	0.026 (0.031)	0.027 (0.066)	-	-	-	-	-	-
RGA. (-1)	0.394*** (0.016)	0.054 (0.034)	0.369*** (0.012)	-0.050 (0.040)	-	-	-	-	-	-
RGA. (-2)	0.048** (0.021)	-0.020 (0.035)	0.064*** (0.016)	-0.018 (0.040)	-	-	-	-	-	-
RGA. (-3)	0.067*** (0.023)	0.022 (0.031)	0.070*** (0.020)	-0.002 (0.043)	-	-	-	-	-	-
RHE. (-1)	-0.130*** (0.041)	-0.065 (0.054)	-0.147*** (0.037)	0.176** (0.072)	-	-	-	-	-	-
RHE. (-2)	-0.051 (0.039)	-0.067 (0.059)	-0.088** (0.037)	0.116* (0.071)	-	-	-	-	-	-
RHE. (-3)	-0.012 (0.044)	-0.073 (0.056)	-0.064* (0.038)	-0.051 (0.088)	-	-	-	-	-	-
RCO. (-1)	0.009 (0.008)	-0.006 (0.013)	0.003 (0.008)	-0.023 (0.031)	-	-	-	-	-	-
RCO. (-2)	0.007 (0.007)	0.019 (0.014)	0.001 (0.007)	-0.055* (0.031)	-	-	-	-	-	-
RCO. (-3)	0.003 (0.010)	-0.004 (0.013)	-8.84E-05 (0.008)	-0.024 (0.029)	-	-	-	-	-	-
$\omega$	2.50E-06***	8.36E-05***	2.94E-06***	7.70E-06***	-	-	-	-	-	-
(constant)	(6.78E-07)	(1.01E-05)	(5.79E-07)	(2.60E-06)	-	-	-	-	-	-
$\alpha$	0.047*** (0.006)	0.205*** (0.024)	0.063*** (0.006)	0.114*** (0.013)	-	-	-	-	-	-
$\beta$	0.936*** (0.008)	0.559*** (0.041)	0.913*** (0.008)	0.884*** (0.011)	-	-	-	-	-	-
$\alpha + \beta$	0.983	0.764	0.976	0.998	-	-	-	-	-	-
$\theta_0$	-	-	-	-	3.43E-06***	1.74E-06***	-7.07E-07	3.39E-06***	2.73E-07	-9.10E-07
(constant)	-	-	-	-	(1.24E-06)	(3.95E-07)	(8.39E-07)	(1.88E-08)	(4.27E-06)	(1.99E-08)
$\theta_1$	-	-	-	-	0.096*** (0.009)	0.056*** (0.005)	0.006 (0.010)	0.111*** (0.010)	0.016 (0.034)	0.021 (0.014)
$\theta_2$	-	-	-	-	0.836*** (0.015)	0.923*** (0.006)	0.912*** (0.046)	0.823*** (0.014)	0.663*** (0.117)	0.870*** (0.069)
$\theta_1 + \theta_2$	-	-	-	-	0.932	0.979	0.918	0.934	0.679	0.891
Log-likelihood	14066.84	-	-	-	-	-	-	-	-	-
AIC	-22.375	-	-	-	-	-	-	-	-	-
SIC	-22.039	-	-	-	-	-	-	-	-	-
HQ	-22.249	-	-	-	-	-	-	-	-	-

Standard error in parenthesis. \*\*\*Significance at the 1% level, \*\*Significance at the 5% level and \*Significance at the 10% level, RCR.: The returns of crude oil, RGA.: The returns of gasoline, RHE.: The returns of heat oil and RCO.: The returns of carbon emissions, AIC: Akaike information criterion, SIC: Schwarz information criterion, HQ: Hannan-Quinn information criterion

Table 5: VAR (3) – diagonal BEKK model estimates

VAR (3)	RCR.	RGA.	RHE.	RCO.	$\rho$ (RCR_RGA.)	$\rho$ (RCR_RHE.)	$\rho$ (RCR_RCO.)	$\rho$ (RGA_RHE.)	$\rho$ (RGA_RCO.)	$\rho$ (RHE_RCO.)
RCR. (-1)	-0.063* (0.036)	0.085** (0.045)	0.011 (0.029)	-0.067 (0.063)	-	-	-	-	-	-
RCR. (-2)	0.011 (0.034)	0.035 (0.046)	0.040 (0.030)	-0.137*** (0.057)	-	-	-	-	-	-
RCR. (-3)	-0.0009 (0.040)	0.029 (0.045)	0.026 (0.030)	0.003 (0.065)	-	-	-	-	-	-
RGA. (-1)	0.389*** (0.016)	0.103*** (0.026)	0.365*** (0.012)	-0.069* (0.038)	-	-	-	-	-	-
RGA. (-2)	0.044** (0.021)	-0.035 (0.031)	0.050*** (0.016)	-0.005 (0.041)	-	-	-	-	-	-
RGA. (-3)	0.028 (0.023)	-0.016 (0.029)	0.038* (0.020)	0.012 (0.041)	-	-	-	-	-	-
RHE. (-1)	-0.111*** (0.041)	-0.019 (0.052)	-0.129*** (0.037)	0.081 (0.073)	-	-	-	-	-	-
RHE. (-2)	-0.025 (0.035)	-0.005 (0.051)	-0.064* (0.037)	0.063 (0.073)	-	-	-	-	-	-
RHE. (-3)	0.007 (0.043)	-0.026 (0.051)	-0.051 (0.036)	-0.017 (0.085)	-	-	-	-	-	-
RCO. (-1)	0.011 (0.008)	-0.002 (0.013)	0.003 (0.007)	-0.029 (0.028)	-	-	-	-	-	-
RCO. (-2)	0.007 (0.007)	0.016 (0.014)	0.001 (0.007)	-0.044 (0.029)	-	-	-	-	-	-
RCO. (-3)	0.001 (0.009)	-0.003 (0.012)	-0.003 (0.008)	-0.016 (0.027)	-	-	-	-	-	-
$\omega$	1.93E-06***	3.38E-06***	1.58E-06***	8.86E-06***	-	-	-	-	-	-
(constant)	(4.76E-07)	(7.16E-07)	(3.55E-07)	(1.93E-06)	-	-	-	-	-	-
$\alpha^2$	0.047***	0.040***	0.053***	0.085***	-	-	-	-	-	-
$\beta^2$	0.943***	0.951***	0.937***	0.913***	-	-	-	-	-	-
$\alpha^2 + \beta^2$	0.990	0.991	0.990	0.998	-	-	-	-	-	-
$\theta_0$					6.48E-07**	1.02E-06***	-3.63E-07	8.18E-07***	-1.66E-08	-5.82E-07
(constant)					(2.95E-07)	(2.70E-07)	(6.17E-07)	(2.27E-07)	(7.62E-07)	(5.98E-07)
$\theta_1$					0.044***	0.050***	0.064***	0.046***	0.059***	0.068***
$\theta_2$					0.947***	0.940***	0.928***	0.944***	0.931***	0.924***
Log-likelihood		14060.49			0.991	0.990	0.992	0.990	0.990	0.992
AIC		-22.384								
SIC		-22.097								
HQ		-22.276								

Standard error in parenthesis. \*\*\*Significance at the 1% level, \*\*Significance at the 5% level and \*Significance at the 10% level, RCR.: The returns of crude oil, RGA.: The returns of gasoline, RHE.: The returns of heat oil and RCO.: The returns of carbon emissions, BEKK.: Baba, Engle, Kraft and Kroner, AIC: Akaike information criterion, SIC: Schwarz information criterion, HQ: Hannan-Quinn information criterion, VAR: Vector autoregression model



Table 6: VAR (3) – CCC model estimates

VAR (3)	RCR.	RGA.	RHE.	RCO.	$\rho$ (RCR, RGA.)	$\rho$ (RCR, RHE.)	$\rho$ (RCR, RCO.)	$\rho$ (RGA, RHE.)	$\rho$ (RGA, RCO.)	$\rho$ (RHE, RCO.)
RCR. (-1)	-0.084** (0.041)	0.086* (0.052)	-0.007 (0.035)	-0.060 (0.063)	-	-	-	-	-	-
RCR. (-2)	0.010 (0.041)	0.042 (0.051)	0.023 (0.035)	-0.138*** (0.055)	-	-	-	-	-	-
RCR. (-3)	0.005 (0.046)	0.041 (0.052)	0.036 (0.038)	-0.009 (0.064)	-	-	-	-	-	-
RGA. (-1)	0.398*** (0.018)	-0.020 (0.036)	0.377*** (0.014)	-0.002 (0.039)	-	-	-	-	-	-
RGA. (-2)	0.057** (0.026)	-0.040 (0.042)	0.061*** (0.022)	0.018 (0.039)	-	-	-	-	-	-
RGA. (-3)	0.059** (0.029)	0.003 (0.038)	0.073*** (0.027)	0.036 (0.041)	-	-	-	-	-	-
RHE. (-1)	-0.105** (0.047)	-0.087 (0.060)	-0.130*** (0.045)	0.039 (0.073)	-	-	-	-	-	-
RHE. (-2)	-0.052 (0.046)	-0.061 (0.066)	-0.070* (0.043)	0.051 (0.071)	-	-	-	-	-	-
RHE. (-3)	-0.016 (0.052)	-0.083 (0.062)	-0.082* (0.046)	-0.022 (0.083)	-	-	-	-	-	-
RCO. (-1)	0.008 (0.011)	-0.004 (0.015)	-0.001 (0.011)	-0.035 (0.032)	-	-	-	-	-	-
RCO. (-2)	0.011 (0.010)	0.017 (0.017)	0.004 (0.010)	-0.042 (0.031)	-	-	-	-	-	-
RCO. (-3)	0.007 (0.012)	0.003 (0.015)	-0.002 (0.011)	-0.006 (0.028)	-	-	-	-	-	-
$\omega$	8.45E-06***	0.0001***	8.23E-06***	8.46E-06***	-	-	-	-	-	-
(constant)	(2.08E-06)	(2.53E-05)	(1.68E-06)	(2.26E-06)	-	-	-	-	-	-
$\alpha$	0.043*** (0.009)	0.137*** (0.024)	0.055*** (0.007)	0.123*** (0.015)	-	-	-	-	-	-
$\beta$	0.914*** (0.016)	0.508*** (0.093)	0.888*** (0.015)	0.873*** (0.012)	-	-	-	-	-	-
$\alpha+\beta$	0.957	0.645	0.943	0.996	-	-	-	-	-	-
CCC	-	-	-	-	0.315*** (0.023)	0.725*** (0.012)	-0.022 (0.031)	0.350*** (0.024)	-0.061* (0.032)	-0.014 (0.032)
Log-likelihood	13853.91									
AIC	-22.054									
SIC	-21.766									
HQ	-21.946									

Standard error in parenthesis, \*\*\*Significance at the 1% level, \*\*Significance at the 5% level and \*Significance at the 10% level, RCR.: The returns of crude oil, RGA.: The returns of gasoline, RHE.: The returns of heat oil and RCO.: The returns of carbon emissions, CCC: Constant conditional correlations, AIC: Akaike information criterion, HQ: Hannan-Quinn information criterion, SIC: Schwarz information criterion, VAR: Vector autoregression model

the conditional variance between the three oil futures and the carbon emissions futures returns are statistically significant in all cases. The ARCH ( $\alpha$ ) estimates are generally small ( $<0.2$ ), and the GARCH ( $\beta$ ) estimates are generally high (more than 0.5) and close to one. Therefore, the long run persistence ( $\alpha+\beta$ ), is generally to one, indicating a near long memory process. This indicates a near long memory process. In addition, since  $\alpha+\beta < 1$ , all oil and carbon emission satisfies the second moment and log-moment condition, which is a sufficient condition for the quasi-maximum likelihood to be consistent and asymptotically normal. VAR (3)-CCC estimates of the CCC between RCRUDE and RHEATOIL with the highest in 0.725. This indicates that the standardized shock on the CCC for RCRUDE with RHEATOIL is 0.725.

Furthermore, we will choose the best model next by considering the value of log-likelihood, AIC, Schwarz information criterion (SIC) and HQ. From the Tables 4-6, we found that the VAR (3)-diagonal VECH model is highest log-likelihood equal 14066.84. But the VAR (3)-diagonal BEKK has AIC, SIC and HQ lowest is equal  $-22.384$ ,  $-22.097$  and  $-22.276$ , respectively. Thus, it can be concluded that we should choose the VAR (3)-diagonal BEKK model in volatility analysis of the oil futures and the carbon emissions futures returns.

However, we can show the movement of the conditional covariance and the conditional correlation of the three oil futures and the carbon emissions futures returns in each model according to Figures 3-7, respectively.

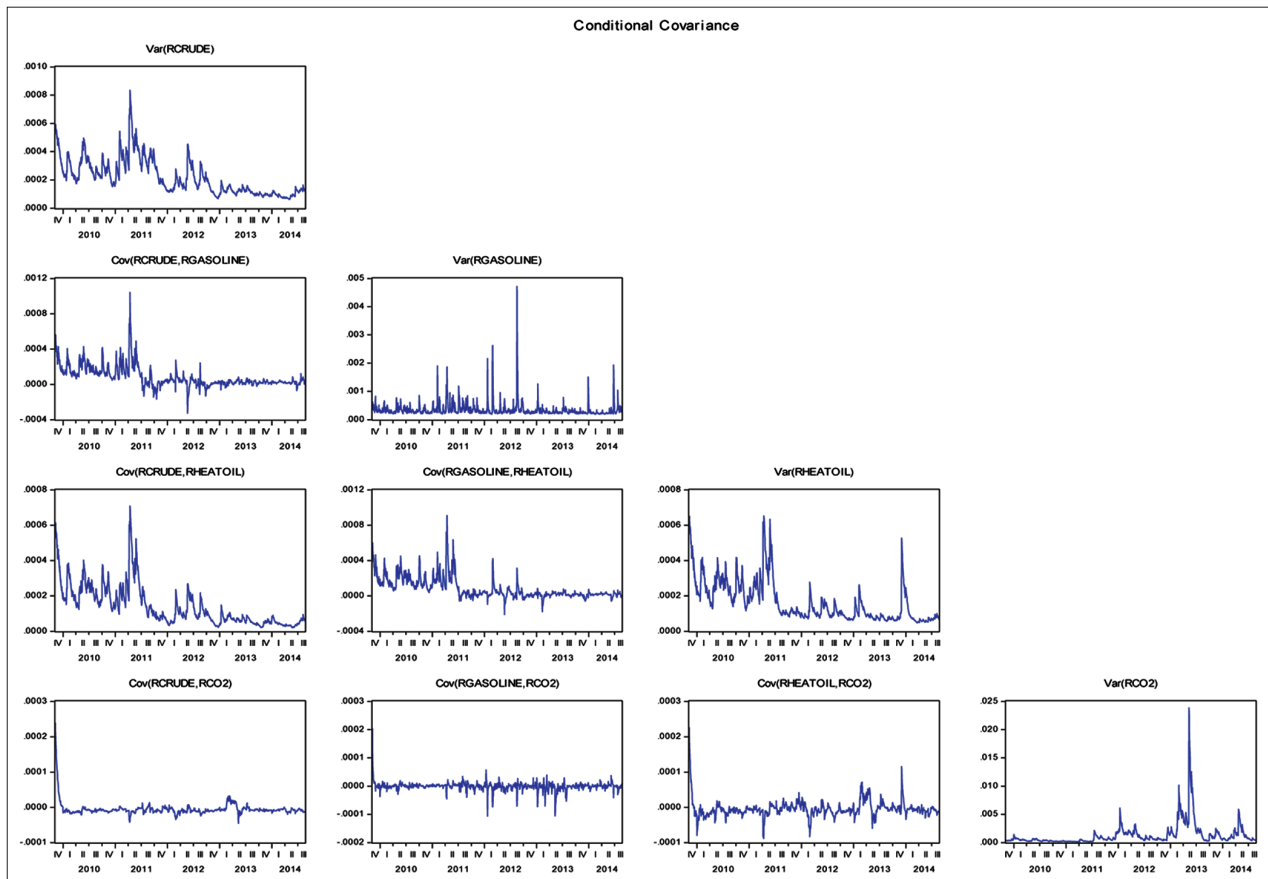
## 7. MULTIVARIATE GARCH DIAGNOSTIC TESTS

The multivariate GARCH models consist of the VAR (3)-diagonal VECH, the VAR (3)-diagonal BEKK and the VAR (3)-CCC model. We can diagnostic check on the system residuals to determine efficiency of estimator according to the Table 7. We found that system residuals have no autocorrelations up to lag 6 and are not normally distributed. Therefore, it can be concluded that the estimators of multivariate GARCH model are efficient.

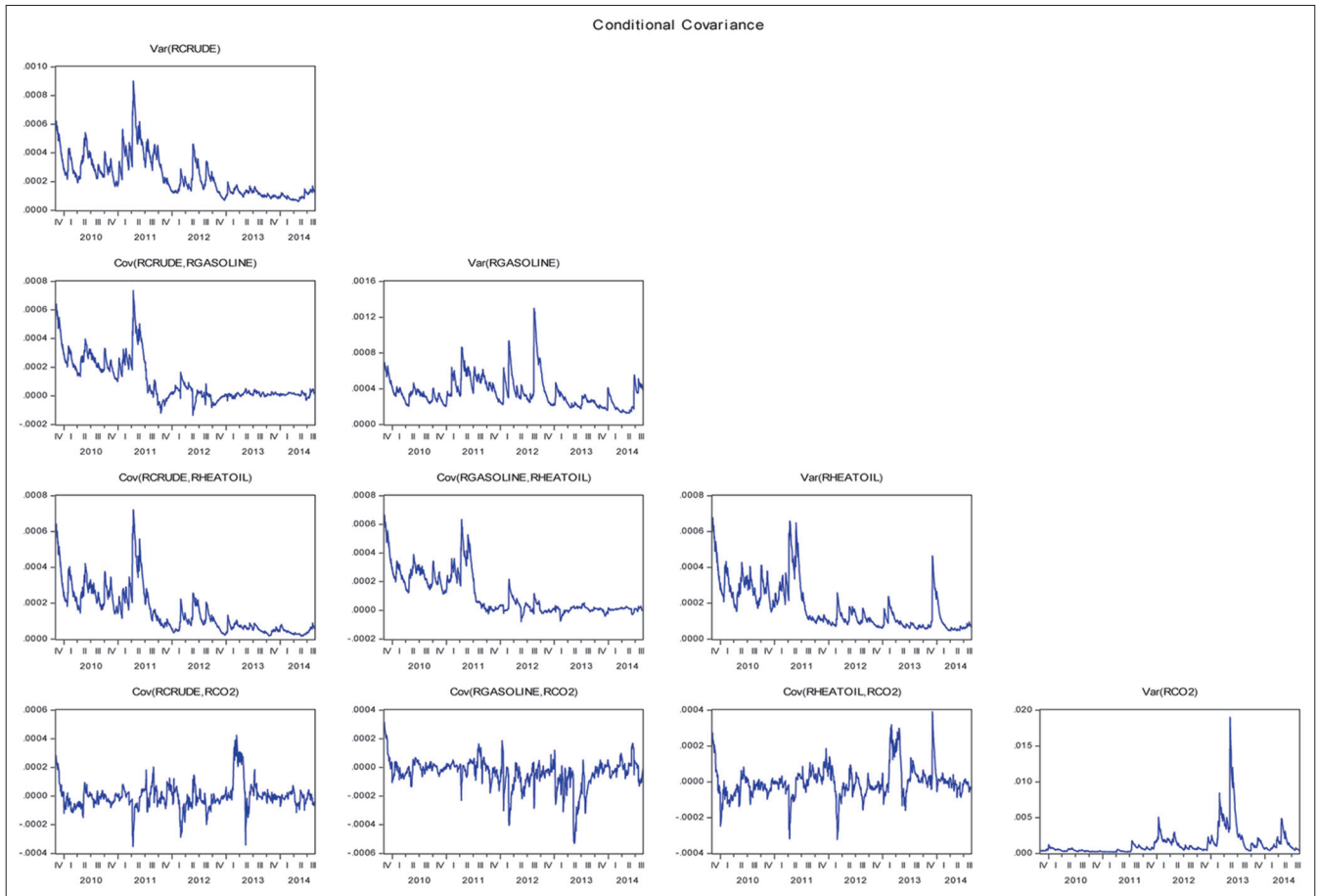
## 8. CONCLUSION

This paper investigates volatility comovements and spillovers for crude oil, gasoline and heat oil futures as well as carbon emissions futures. The empirical results showed that the estimates of the VAR (3)-diagonal VECH and the VAR (3)-CCC parameters were statistically significant in a case involving oil except in the case of carbon emissions. This indicates that the short run persistence of shocks on the dynamic conditional correlations was greatest for RGASOLINE with RHEATOIL, while the largest long run persistence of shocks to the conditional correlations for RCRUDE with RGASOLINE. At the same time the VAR (3)-diagonal BEKK parameters were statistically significant in all cases. This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest for RHEATOIL with RCO<sub>2</sub>, while the largest long run persistence of shocks to the conditional correlations for RCRUDE with RCO<sub>2</sub> and RHEATOIL with RCO<sub>2</sub>.

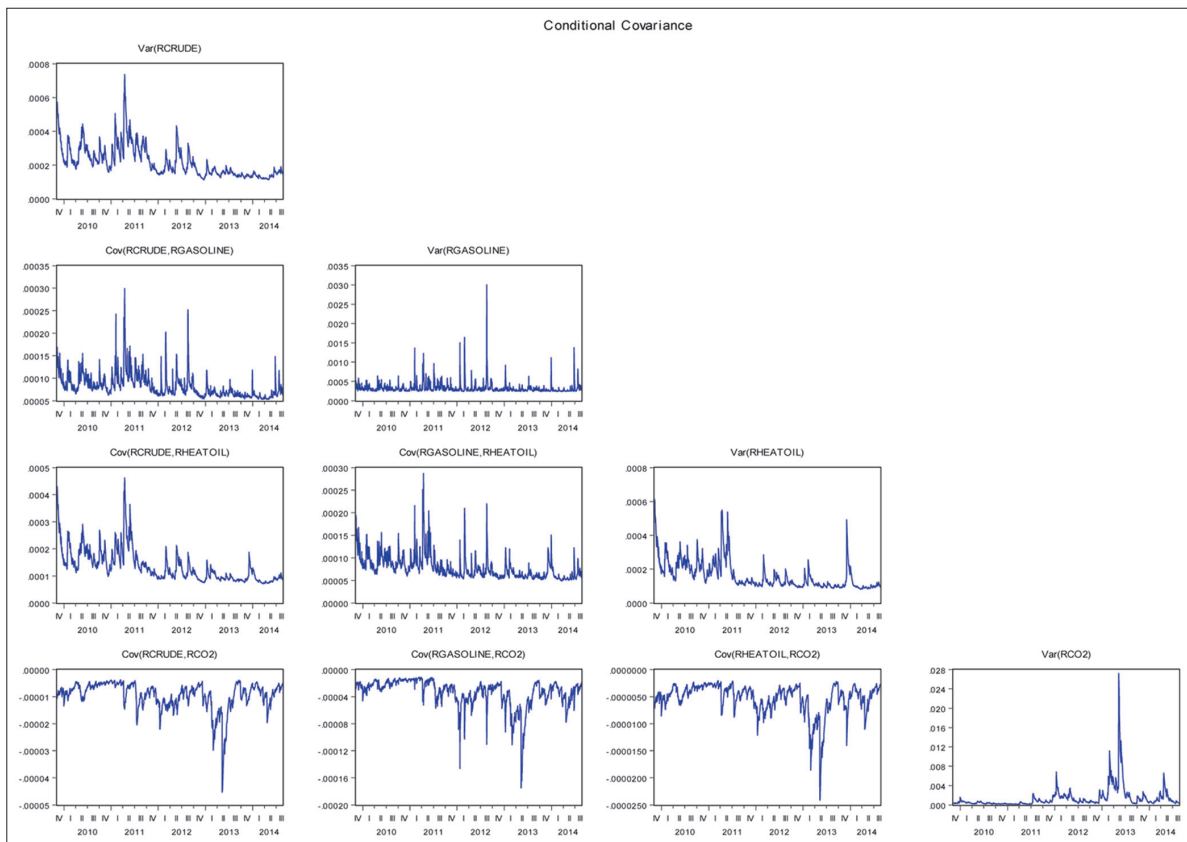
Figure 3: Conditional covariance (vector autoregression model (3) - diagonal VECH estimates)



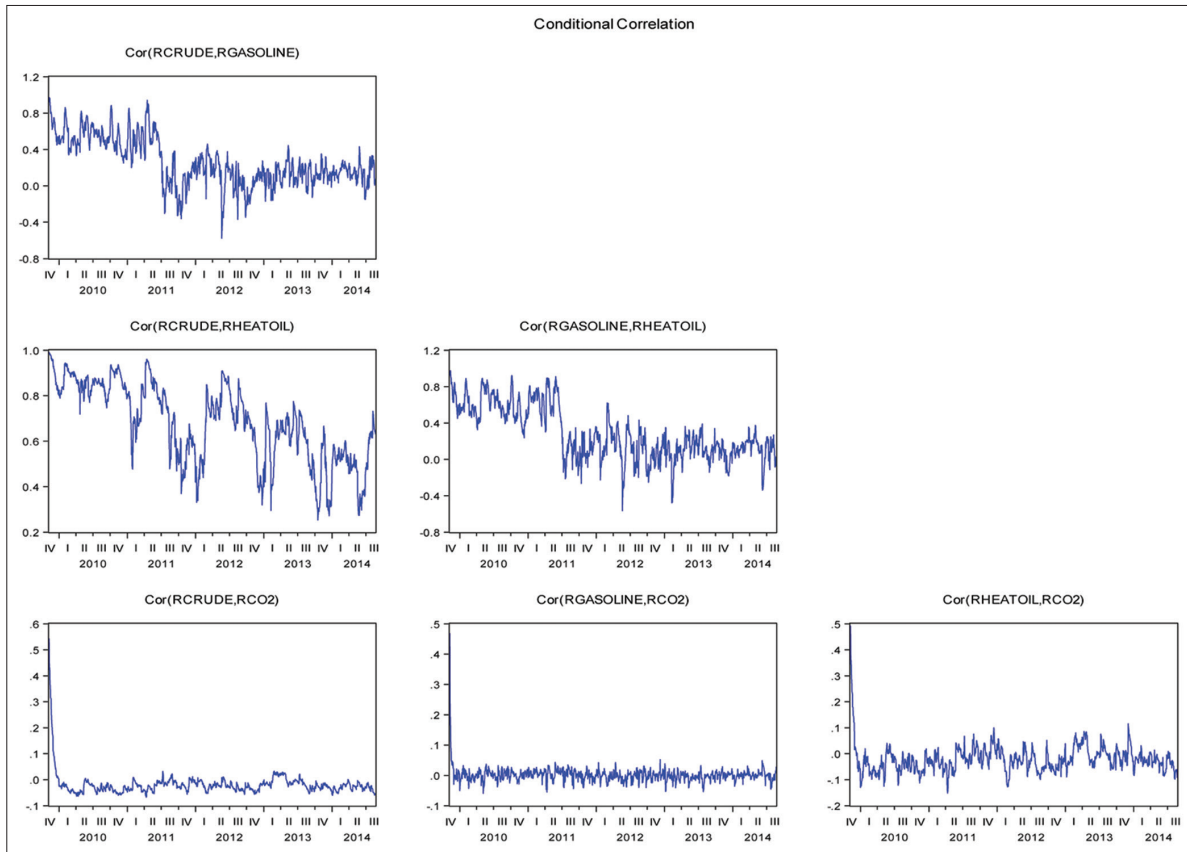
**Figure 4:** Conditional covariance (vector autoregression model (3) - diagonal Baba, Engle, Kraft and Kroner estimates)



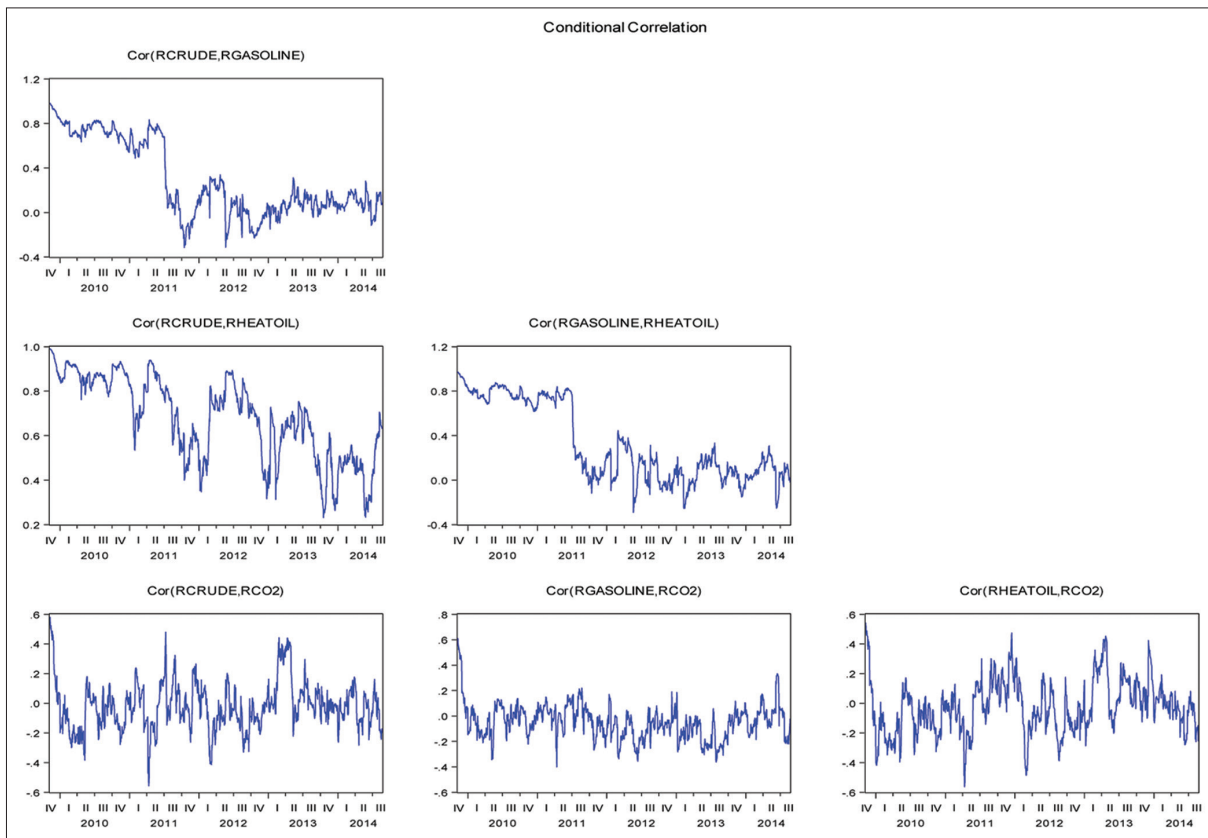
**Figure 5:** Conditional covariance (vector autoregression model (3) - constant conditional correlations estimates)



**Figure 6:** Conditional correlation (vector autoregression model (3) - diagonal VECM estimates)



**Figure 7:** Conditional correlation (VAR (3) - diagonal Baba, Engle, Kraft and Kroner estimates)



**Table 7: Multivariate GARCH diagnostic tests**

Test	Lags	Value	Probability	Test	Value	Probability
<b>VAR (3) – diagonal VECH</b>						
System residual tests for autocorrelations	1	19.273	0.254	System residual normality tests		
$H_0$ =No residual autocorrelation (Q-Statistics)	2	27.300	0.703	$H_0$ =Multivariate normal	49.281	0.000
	3	33.656	0.942	Skewness (Chi-square)	1174.560	0.000
	4	49.420	0.910	Kurtosis (Chi-square)	1223.841	0.000
	5	62.166	0.930	Jarque-Bera		
	6	82.134	0.842			
<b>VAR (3) – diagonal BEKK</b>						
System residual tests for autocorrelations	1	27.306	0.038	System residual normality tests		
$H_0$ =No residual autocorrelation (Q-Statistics)	2	34.926	0.330	$H_0$ =Multivariate normal	54.618	0.000
	3	41.297	0.742	Skewness (Chi-square)	1644.597	0.000
	4	57.081	0.717	Kurtosis (Chi-square)	1699.216	0.000
	5	67.964	0.829	Jarque-Bera		
	6	88.761	0.687			
<b>VAR (3) – CCC</b>						
System residual tests for autocorrelations	1	10.060	0.863	System residual normality tests		
$H_0$ =No residual autocorrelation (Q-Statistics)	2	14.645	0.996	$H_0$ =Multivariate normal	52.250	0.000
	3	19.088	0.999	Skewness (Chi-square)	1476.766	0.000
	4	30.385	0.999	Kurtosis (Chi-square)	1529.017	0.000
	5	41.607	0.999	Jarque-Bera		
	6	62.988	0.996			

CCC: Constant conditional correlations, BEKK: Baba, Engle, Kraft and Kroner, VAR: Vector autoregression model

Finally, we would choose the best model next by considering the value of log-likelihood, AIC, SIC and HQ. We found that the VAR (3)-diagonal VECH model is highest log-likelihood equal 14066.84. But the VAR (3)-diagonal BEKK has AIC, SIC and HQ lowest is equal -22.384, -22.097 and -22.276, respectively. Thus, it could be concluded that we should choose the VAR (3)-diagonal BEKK model in volatility analysis of the oil futures and the carbon emissions futures returns. In addition, we could conclude that oil futures volatility having an impact on carbon emissions futures volatility. Such results can be useful as the management the volatility of the oil and carbon emissions for investors, including polluters and regulators.

## REFERENCES

- Alberola, E., Chevalier, J., Cheze, B. (2008), Price drivers and structural breaks in European carbon prices 2005-2007. *Energy Policy*, 30(3), 787-797.
- Alberola, E., Chevalier, J. (2009), European carbon prices and banking restrictions: evidence from phase I 2005-2007. *The Energy Journal*, 30(3), 107-136.
- Baba, Y., Engle, R.F., Kraft, D., Kroner, K. (1990), Multivariate Simultaneous Generalized Arch. San Diego, University of California: Unpublished Manuscript.
- Bollerslev, T., Engle, R.F., Wooldridge, A. (1988), Capital asset pricing model with time vary covariance. *Journal of Political Economy*, 96, 116-131.
- Bollerslev, T. (1990), Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics*, 72, 498-505.
- Bunn, D.W., Fezzi, C. (2007), Interaction of European Carbon Trading and Energy Prices. FEEM Working Paper, London Business School.
- Bunnag, T. (2015), Hedging petroleum futures with multivariate GARCH models. *International Journal of Energy Economics and Policy*, 5(1), 105-120.
- Chavallier, J. (2011), Time-varying correlations in oil, gas and  $CO_2$  prices: an application using BEKK, CCC and DCC-MGARCH models. *Applied Economics*, 44(32), 4257-4274.
- Kroner, K.F., Ng, V.K. (1998), Modeling asymmetric comovement of asset returns. *Review of Financial Studies*, 11(4), 817-844.
- Manera, M., Nicolini, M., Vignati, I. (2012), Returns in Commodities Futures Markets and Financial Speculation: a Multivariate GARCH Approach. Working Paper Series, Fondazione Eni Enrico Mattei.
- Mansanet-Bataller, M., Pardo, A., Valor, E. (2007),  $CO_2$  prices, energy and weather. *The Energy Journal*, 28(3), 73-92.
- Mansanet-Bataller, M., Pardo, A. (2011), What you should know about carbon markets. *Energies*, 1(1), 120-153.
- Mansanet-Bataller, M., Soriano, P. (2012), Volatility transmission in the  $CO_2$  and energy markets. *Environmental Economics*, 3(4), 75-81.
- Stern, N. (2006), *The Economics of Climate Change*. Cabinet Office-HM Treasury, Cambridge.
- Uhrig-Homburg, M., Wagner, M. (2007), Futures price dynamics of  $CO_2$  emission certificates-an empirical analysis. Working Paper, University of Karlsruhe.